**KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY**

***Department of Mechanical Engineering***

B. Sc. Engineering 1st Year 1st Term Examination, 2021

**Math 1105**

(Mathematics I)

Time: 3 Hours | Total Marks: 210

**N.B.**: i) Answer any THREE questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.
iii) Assume reasonable data if any missing.

## SECTION-A

1(a) Define continuity of a function at a point. A function $f(x)$ is defined by– 15

$$f(x)=|x+2|+|x-1|$$

Show that $f(x)$ is continuous at $x=-2$ but $f'(1)$ does not exist.

1(b) Evaluate $\lim_{x\to 1}\left(1-x^2\right)^{1/\ln(1-x)}$. 10

1(c) Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$. 10

2(a) If $y=\sin^{-1}x$, then find the relations between $y_{n+2}$, $y_{n+1}$ and $y_n$. Also, find the value of $(y_n)_0$. 13

2(b) Find the extreme values of $U=\frac{2}{x}+\frac{18}{y}$ where $x+y=1$. 11

2(c) Find all possible asymptotes of the curve $(y+3)\left(x^2-3x-4\right)+3x+7=0$. 11

3(a) State Rolle's theorem. Is Rolle's theorem applicable for the function $f(x)=\frac{2}{2+|x|}$ in $-1\le x\le 1$? Justify your answer. 13

3(b) If $u=F(y-z,\ z-x,\ x-y)$, then find the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$. 12

3(c) Calculate the value of cos 61° using the value of cos 60°. 10

4(a) Show that the two curves $xy=4$ and $x^2-y^2=15$ cut orthogonally to each other. 11

4(b) Find where the tangent is parallel to the $x$-axis for the curve $y=x^3-3x^2-9x+15$. 12

4(c) Find the radius of curvature of the curve $y=x\,e^{-x}$ at its extreme value. 12

## SECTION-B

5. Integrate the followings:

(a) $\int\frac{dx}{x^4\sqrt{x^2-1}}$ 12

(b) $\int\sqrt{\frac{x}{x-a}}dx$ 11

(c) $\int\frac{dx}{\sin x+7\cos x+5}$ 12

6. Evaluate the followings:

(a) $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx$ 12

(b) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ 12

(c) $\underset{n\to\infty}{Lim}\left[\frac{\sqrt{n+1} + \sqrt{n+2} + \cdots\cdots\cdots + \sqrt{2n}}{n\sqrt{n}}\right]$ 11

7(a) Define Gamma function and Beta function. Using Gamma and Beta function evaluate 12

$\int_0^1 x^2\left(1-x^3\right)^{\frac{3}{2}} dx$

7(b) Find the reduction formula for $\int cosec^n x\, dx$ and hence find the value of $\int cosec^3 x\, dx$. 12

7(c) Find the area of the region bounded by the curve $y^2 = 2ax - x^2$ and $y^2 = ax$. 11

8(a) State and prove Walli's formula. 07

8(b) Find the volume of the solid produced by the revolution of the loop of the curve $a^2y^2 = x^3(2a-x)$. 14

8(c) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. 14