OPTIMIZATION OF MULTI-ITEM INVENTORY SYSTEM

by

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Declaration

This is to certify that the thesis work entitled "*Optimization Of Multi-Item Inventory System* " has been carried out by *Subrata Talapatra* in the department of *Industrial Engineering and Management*, Khulna University of Engineering & Technology, Khulna, Bangladesh. The above research work and any part of the work has not been submitted anywhere for the award of any degree or diploma.

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Approval

This is to certify that the thesis work submitted by *Subrata Talapatra* entitled "*Optimization Of Multi-Item Inventory System* " has been approved by the Board of Examiners for the partial fulfillment of the requirements for the degree of *Masters of Science in Engineering* in the Department of *Industrial Engineering and Management*, Khulna University of Engineering & Technology, Khulna. Bangladesh in September, 2009.

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ABSTRACT

Inventory is the lifeblood of a business organization. Industry uses various types of materials for its smooth running. A significant amount of total assets is invested as inventory. In many companies, the management has the lack of control over quantity of material to order and their lead-time. As a result, it is not possible to maintain schedule. Hence management of inventory becomes primary concern everywhere. Khulna Shipyard Ltd follows the traditional method of purchasing multiple inventory items. Demand for replenishment of inventory items are placed when they are needed. Sufficient additional stock is not maintained to meet the lead time demand. Therefore, shortage of materials frequently occurs during production. So, appropriate method of inventory replenishment for the shipyard is developed in this research.

The present study is an outcome of scientific planning of ordering right inventory items in the right quantity in right place at right time to facilitate uninterrupted production of Shipyard.

To provide continuous supply of inventory during the production period a standard method for purchasing multiple inventory items for joint replenishment was developed. This method is consisting of some models and policies. EOQ model for single inventory system is used with necessary adjustment for multi-items. This model will provide optimal ordering quantity of each item. The ROP models will provide information when to order the items. The policies of grouping inventory items are based on the sequential need of the items in the assembly line of the end product. Safety stock policy can be adopted for providing protection against the variability in both demand and lead time. Continuous review type (R,Q) inventory replenishment policy is also introduced in the new procurement method. In (R,Q) policy, if the inventory position is reviewed continuously at a point, if the point is less than R, an order for Q quantity is placed immediately. Where R is the re-order point and Q is the order quantity.

Material demand and replenishment policy is almost similar for all the shops of KSY. Machine shop is selected as representative of all the shops for applying the new

purchased method in the procurement of three components of Gun Metal. Material demand and lead time Data of Material demand and lead time for last three years from 2006 to 2009 was collected from the three individual departments of the organization named Machine Shop Indoor, Costing Department and Main Store in this regard. Calculation for determining EOQ, ROP and safety stock for joint replenishment of inventory items is done according to the procedure mentioned in the chapter-4.

It is observed from the Figure-5.1 that, monthly demand for Gun Metal fluctuates. This is due to the fact that number of ships to be repair or renewed are not constant, this varies over time. Again, the requirement of Gun Metal is not same for the ships during their each schedule repair/maintenance. Monthly average consumption of Gun Metal in the shipyard is found 101 Kg.

Again it is observed that, lead time for Gun Metal procurement also varies due to unavailability of material in the market, transportation delays, complexities arise in procurement process and so on. Average lead time for the procurement of Gun Metal components is found 41 days.

Fixed ordering cost for purchasing inventory items in KSY is calculated TK 21,100 according to the procedure mentioned in the chapter 4. Inventory carrying cost is found 25 % of the purchase price of items in the group. EOQ of each item is computed using the formula (4.3.1.3). Safety stock for Gun Metal is obtained 889.72 Kg by using the formula (4.3.4.1). Safety stock depends on the variability of the demand and lead time, desired service level, average demand rate and average lead time. This stock is also proportional to the supply disruption, demand forecast error, square root of the lead time.

The order for replenishment of Gun Metal is triggered when the quantity level falls below the 1036.48Kg.This quantity is influenced by the demand rate, lead time and degree of stock out risk.

The frequency distribution of monthly demand for Gun Metal and lead time for the procurement illustrated in the separate figure in chapter 5. The finding comes out from the analysis of EOQ model is presented in the figure 5.3.

No scientific method was found for purchasing inventory items in the Khulna shipyard. So, it is to be recommended that, to use this new purchasing method, the shipyard needs to change their method of ordering, ordering cost and ordering cycle in the following ways:

The method of grouping inventory items based on the type of production, inventory level, work load, customer service. In the study, company producing one unit of certain product may require a multiple units of different components in a sequence assembling procedure to fabricate the end product. So, grouping of items based on the sequence of assembly of the end product.

Ordering cost of the multi-item inventory system has two components. One is fixed cost and another is variable or marginal cost which includes the cost of processing some specific purchase activities of an individual items. So, Marginal cost must be calculated for determining the economic order quantity of each item.

When inventory item are ordered in a group, the number of order cycle is then fixed not for the individual items, but for the group items as a whole. Order cycle for each item can be calculated in terms of integral multiple of the shortest order cycle among the individual item. When the cycle length is a multiple of some basic cycle time, reduces the major ordering costs.

If the developed purchase method is implemented for the inventory replenishment of the shipyard, then the company will be benefited in many ways.
This method will ensure continuous supply of materials to facilitate uninterrupted production. Inventory level will be optimal. Holding cost of materials can be reduced. Even out the production load.

Finally some recommendations are made based on the data and information collected from the shops. This model can be used in procurement process of other shops by collecting the data of the monthly demand, lead time and purchase price of different inventory items reverent to that purchase.

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EOQ:	Economic Order Quantity.
ROP:	Re-order Point.
ERP:	Enterprise Resource Planning.
MRP:	Material Requirement Planning.
MRR:	Material Receive Report
KSY:	Khulna Shipyard Ltd.
CPFR:	Collaborative Planning Forecasting and Replenishment.
$\overline{LT}$:	Average Lead Time.
$\overline{d}$:	Average Demand Rate.
σd:	Standard Deviation of Demand.
σ_{LT}:	Standard deviation of Lead Time.
Z:	Normal Standard deviation.

CHAPTER 1

Introduction

1.1 General

An inventory is the stock or store of goods. It is the lifeblood of a business and is essential to running it efficiently and profitably because it represents a significant portion of total assets. It is not only necessary for operations, but also contributes to customer satisfaction. Although, the amount and dollar values of inventories carried by different firms vary widely, a typical firm probably has about 30 percent of current assets and perhaps as much as 90 percent of its working capital invested in inventory. An operation system's performance is affected directly by the size of inventories held There is generally a penalty associated with having either too much or too little inventory.

In most industries, inventory is the foundation of conducting business. Consider the manufacturing industry, where it is necessary to coordinate both inventory-producing and inventory-consuming activities. There are inventories for multiple processing stages at multiple locations in the course of turning raw materials into components, producing spare parts, and ultimately creating finished goods. In the retail industry, companies maintain large volumes of different items at various locations. They must monitor quantities, estimate usage, and place orders for replenishment. Slow-moving items are discontinued, while new items are introduced. In the service industry, inventories are critical in providing the services that customers require. When doing business with a company, customers often will not tolerate product unavailability or delays in delivery. In some cases, a shortage may be only a small inconvenience, while sometimes it may cause a severe problem (such as interrupting production-line activity at a computer manufacturer). On some occasions, sporadic shortages can be expected, but frequent shortages may ultimately erode a company's reputation and reduce their market share. Inversely, overabundant, slow-moving inventories can place a serious strain on a company's available capital and the company's ability to take advantage of financial opportunities. Frequent shortages or excessive inventories are telltale signs of a company headed in the wrong direction. The scope of inventory-dependent operations is tremendous. In March 2002, U.S. businesses alone maintained about $1.117 trillion worth of inventories or roughly 1.38 times their total monthly sales. Thus, effective management of inventory can have a big impact on profitability. Recently, much success has come to retailers that focus their operations on keeping their inventories lean. Less has become more, and intelligent inventory replenishment planning is a major key toward realizing that goal. In order to compete effectively in today's business world, it is imperative that adequate inventories are maintained efficiently.

1.2 Reasons for holding inventory

Although, the increase of inventory requires additional space to store it and hide problems in manufacturing process causing quality problems. There is also a possibility for spoilage or obsolescence of inventory. Besides these, the uncertainties in production give rise to the need of keeping inventories. The major reason of keeping of inventories is mentioned bellow:

1) To take advantage of the scale economy (i.e., setup cost & switch over times),
2) To ship products over a long lead-time.
3) To protect against uncertainty (i.e., demand surge, price increase, random lead times)

1.3 Concept of multi-item inventory management system

Many of the classical inventory models concern with single-item model. In fact, such this model seldom occurs. The multi-item inventory models are more realistic than the single item model. Because, most the firm carries various amount of inventories of different kinds. So, this study concern with multi- item inventory models. The purpose remains the same for single-item as well as for multi-item inventory. It is either maximization of total average profit or minimization of total average cost during a given cycle. Consequently, the analysis for a single-item inventory is almost parallel to that of multi-item inventory. Also, the results obtained are almost identical in single-item and in multi-item inventory. Multi-item inventory problems are characterized by the nature of interaction of the quantities and timing of orders for different items. The set–up cost of ordering individual items is such that a part of the cost can be saved by jointly ordering more than one item at a time.

Chakravarty and Goyal [1] has classified the multi- inventory items into several groups with a common order interval for each group. They establish that the optimal groups will be consecutive by *hD/A* where *h, D* and *A* are the holding cost, demand rate and setup cost of an item, respectively.

Inventory decisions are more important for the high-margin products with spiky random fluctuations in demand. As on one side the shortage costs are significant, on the other side, to maintain appropriate service levels and avoid shortages necessitates excessive inventory, which in turn affects the liquidity by blocking working capital. The problem gets more complicated for multi-inventory management system.

The model developed by Pandey [2] presents a new scheme to arrive at maintaining the low inventory replenishment levels and tries to improve the pull in the system. The model is simulated and the results are compared.

One major theme in the continuing development of multi- inventory theory is to incorporate more realistic assumptions about the fluctuation of demand and lead time without any shortage of stock into inventory models. In most industrial contexts, demand is uncertain and hard to forecast. Many demand histories behave like random walks that evolve over time with frequent changes in their directions and rates of growth or decline. The lead time also fluctuates for various reasons.

1.4 Requirements for effective multi-item inventory management

Management has two basic functions concerning inventory. One is to establish a system of keeping track of items in inventory, and the other is to make decisions about how much and when to order. To be effective, management must have the following:

1. A system to keep track of the inventory on hand and on order.
2. A reliable forecast of demand that includes an indication of possible forecast error.
3. Knowledge of lead times and lead time variability.
4. Reasonable estimates of inventory holding costs, ordering costs, and shortage costs.

1.5 Impact of demand fluctuation on multi-item inventory management system

Fluctuation of demand has a great impact on multi-item inventory management system. Among them the most important are the following:

1) Inefficient production.

2) Imbalanced finished goods inventory.

3) Transportation costs are higher than necessary.

4) Workers utilization of central distribution is low.

5) Equipment utilization of central distribution is low.

1.6 Impact of long and uncertain lead time on multi-item inventory management system

Long and un-certain lead time have a bad impact on multi-item inventory management system. Among them the most important are the following:

1) Inventory increases as the lead time increases.

2) Larger forecast errors.

3) Higher markdown and lost sales.

4) Higher transportation cost due to

5) Emergence shipments.

1.7 Factors that are affecting the safety stock of the multi-item inventory system

Safety stock is the additional stock in hand that is carried to tackle the probability of stock out of inventory during the un-certain lead time. This safety stock is affected by too many factors, among them the most important are the following:

1) Safety-stock is proportional to demand forecast error.

2) Safety-stock is proportional to supply disruption.

3) Safety-stock is proportional to the square root of the lead-time.

1.8 Background of the research

Khulna Shipyard Ltd. is one of the renowned shipyards in Bangladesh. Quantity of work it provides is the best one. For this reason, number of ships to be repair or renewed are increasing day by day. The schedule of ship repair becomes very difficult to maintain and even sometimes it becomes impossible. Now a day the shipyard is loosing their customer because they are unable to repair or renovate the ships within the schedule date. So, the shipyard is facing a severe problem regarding this. In preliminary study, it is found that in most of the cases the schedules are not possible to maintain due to the unavailability of material during the production. This is due to the fact that, KSY follows the traditional method of purchasing inventory items. Demand for replenishment of inventory items are placed when they are needed. Sufficient additional stock is not maintained to meet the demand of inventory items during lead time. Availability of material during production can be ensured to facilitate uninterrupted production. This is possible only when the optimum quantities of items are ordered at right time and the minimum safety stock is maintained to meet the lead time demand. EOQ, ROP models can be helpful in determining the optimum quantity to order and when to order respectively. Safety stock should be calculated against both uncertainties in demand and lead time.

1.9 Objectives of the research

Schedule for repair or renovation of ships becomes very difficult to maintain at Khulna Shipyard Ltd due the unavailability of material during the production.
Therefore, the objective of the present study is to find out the actual reason for unavailability of material during the production and necessary improvement of the existing system. The specific objectives of the research are as follows.

1) To Study the actual reason for variability in demand and lead times patterns of inventory items.

2) To ensure continuous supply of materials to facilitate uninterrupted production.

3) To find out EOQ, ROP and Safety stock level of different inventory items.

4) To find out the criteria of grouping the items for joint replenishment in multi-inventory system in KSY.

1.10 Scope of the study

Data was collected from the three relevant departments of Khulna shipyard regarding the monthly demand of Gun Metal by Machine Shop, lead time and purchase price of its components etc. This was done to study the present procurement process and then developing a new replenishment inventory model for ordering optimum quantity of Gun Metal components jointly at right time with safety stock. The analysis of model, comments, suggestions are limited to apply in the purchase process of Machine Shop of Khulna Shipyard Ltd.

CHAPTER 2

Literature Review

2.1 General

For conducting research work, the availability of information is one of the major issues, without this any significant development is not possible on that particulate issue. To conduct this research several documents and literature are search. A limited number of references was found to know about inventory management system in Bangladesh. However, some important studies related to the multi-item inventory management are found in the internet. Some important studies on multi-item inventories in the past are discussed here. Inventory is a list for goods and materials, or those goods and materials themselves, held available in stock by a business. The word "inventory" was first recorded in 1601. The French term inventories, or "detailed list of goods," dates back to 1415. Inventory management is primarily about specifying the size and placement of stocked goods. Inventory management is required at different locations within a facility or within multiple locations of a supply network to protect the regular and planned course of production against the random disturbance of running out of materials or goods. The scope of inventory management also concerns the fine lines between replenishment lead time, carrying costs of inventory, asset management, inventory forecasting, inventory valuation, inventory visibility, future inventory price forecasting, physical inventory, available physical space for inventory, quality management, replenishment, returns and defective goods and demand forecasting. Management of the inventories, with the primary objective of determining controlling stock levels within the physical distribution function to balance the need for product availability against the need for minimizing stock holding and handling costs. There are a lot of research work performed for the solution of inventory management problems, but till there are some unanswered questions or open issues worthy of further research.

2.2 Present state-of-the-art of the research topic

Many of the classical inventory models concern with single-item model. In fact such model seldom occurs. The multi-item inventory models are more realistic than the single item model. So, this study concern with Multi-item inventory models. Most of the classical studies are concerned with a single-item inventory model. The analysis for a single-item inventory is almost parallel to that of multi-item inventory. The results obtained are also identical in single-item and in multi-item inventory. The purpose remains the same for single-item as well as for multi-item inventory. It is either maximization of total average profit or minimization of total average cost during a given cycle.

Some of the most recent studies are cited here, in order to give an ideạ on the wide range of optimal control applications in the multi-item inventory production system. Multi-item classical inventory models under resource constraints are available in well-known books .Ben-Daya and Raouf [5] have developed approach for a more realistic and general SPIP (Single Period Inventory Problem).They also consider a multi- item with budgetary and floor-or shelf-space constraints. They assume that, the demand of the items follows uniform probability distribution. They have also discussed a multi-item inventory model with stochastic demand subject to the restrictions on available space and budget.

Bhattacharya [6] has studied a two- item inventory for deteriorating items with a linear stock-dependent demand rates. From the linear demand rate, it is found that more is the inventory, more is the demand. So, a control parameter is introduced, such that it maintains the continuous supply to the inventory. An objective function is formed to calculate the net profit with respect to all possible profits and all possible loss (taken with negative sign). A necessary criterion for the steady state optimal control problem for optimizing the objective function subjected to the constraints given by the ordinary differential equations of the inventory. It also considers a particular choice of parameters satisfying the above necessary conditions. Under this choice, the optimal values of control parameters are calculated, the optimal amount of inventories is also found out. Finally, with respect to these optimal values of control parameters and those of the optimal inventories, the optimal value of the objective function is determined.

Lenard and Roy [7] have define another approach for the determination of inventory policies based on the notion of efficient policy surface and extend this notion to multi-item inventory control by defining the concepts of family and aggregate items. Different mathematicians like Worell and Hall [8] and others have applied different programming methods to solve multi-items inventory problems. Kar and others [9] have considered, density-dependent demand rate for multi-item inventory as in the single-item inventory case. Sulem [10] has been determined the optimal ordering policy for impulse control of a deterministic two product inventory system subject to constant demand rates, linear storage and shortage costs and economies of joint ordering.

Rosenblatt [11] has discussed multi-item inventory system with budgetary constraint comparison between the lagrangian and the fixed cycle approach. Lastly, he has presented the multi-item inventory model and derives the optimal control of this model.

Gohary and Elsayed [12] studied the problem of optimal control of multi-item inventory model with deteriorating items. They developed a mathematical model (non-linear deferential equations was solved numerically) for two-item inventory and the mathematical formulation of the optimal control problem of multi-item inventory model with deteriorating items. Optimal production rates and optimal inventory levels are also derived. Finally, the critical economic conditions that ensure the inventories having no shortage and the economic conditions impose on the production rates are also discussed.

In most of the situations demand is probabilistic. Hala, Egypt [13] has studied probabilistic safety stock for multi-items inventory system having varying order cost and zero lead-time subject to two linear constraints. The optimal values of maximum inventory level and minimum expected total cost variables are obtain by using the geometric programming approach. They also investigated the probabilistic safety stock in the multi-item, single source inventory model with zero lead-time and varying order cost under two constraints, one of them on the expected holding cost and the other on the expected cost of safety stock.

Real life inventory systems are always capacitated by multiple resources such as storage areas and capitals invested. In such model, the demand for each item from customers is stochastic and stationary with constant lead time for each item. The optimization problem is to find the best inventory policies for each item, satisfying all resource constraints.

Qiu et al [14] first solved a relaxed optimization problem to obtain the initial inventory policies for all items, regardless of resource constraints. For each solution, eight neighbor solutions are defined. Then a heuristic algorithm is applied to the original optimization problem to improve the initial solution by searching for the best neighbor solutions continuously. Numerical examples show that in most cases the algorithm generates high quality solutions. A case study with actual data from a large distribution center was performed to show the cost can be reduced by 8.1% compared with current inventory policies.

Recently Abou-El-Ata, et al [15] studied the probabilistic multi-item inventory model with varying order cost under the following assumptions: (1) the maximum inventory level of each item is a constant multiple of the average quantity ordered; (2) the order cost is a continuous increasing function of the replenishment quantity, which itself is proportional to some number of periods covered by the replenishment quantity. No shortages are to be allowed. And the optimal maximum inventory level is derived using a geometric programming approach.

One major theme in the continuing development of inventory theory is to incorporate more realistic assumptions about product demand into inventory models. In most industrial contexts, demand is uncertain and hard to forecast.

Stephen [16] has considered a class of non-stationary demand processes, for which an exponential-weighted moving average provides the minimum mean square forecast. Then a single-item inventory model assuming a deterministic replenishment lead-time for this family of demand was build. From the analysis of this model, they determine the safety stock requirements for a single item, and then explore implications from applying the model to a multi-stage or supply chain setting. This work is related to a series of papers that develop optimal inventory policies when the demand distribution depends upon some unknown parameter. In the first part of this paper, a model for a single-item inventory system with a deterministic lead time is presented. Safety stock required for the case of non-stationary demand is much greater than for stationary demand. Further-more, the relationship between safety stock and the replenishment lead-time becomes convex when the demand process is non-stationary.

The non-stationary demand process in the single-item inventory context was examined in most of the research works. But, there are a lot of unanswered questions regarding the Multi-item inventory context for further research. They are:

What happens if the replenishment lead-times are probabilistic?

What happens if the demand is variable and no shortage of inventory is allowed?

In the present study, the actual reason for unavailability of material during the production in Khulna shipyard is find out and necessary improvement of the existing system is made. In this regard, a standard method for purchasing inventory items for multiple –item joint replenishment was developed. This method consists of some models and policies.

CHAPTER 3

Methodology

3.1 General

This chapter deals with the presentation of all the methods implemented to gather data and how the actual research work has been conducted. The methodology used in this research covers the collection of information, screening of the information to make the information more valid and reliable and analyzing them to reach at correct decision.

3.2 Selection of sample

Selection of Sample is the process used for categorizing the samples available and can be either probabilistic or non probability sampling.

The for different materials demand and replenishment policy is almost similar their for different shops of KSY. As a result, all the shops of the organization were not taken into consideration while carrying out the research work. Data was collected from the three individual departments of the organization named Machine Shop Indoor, Costing Department and Main Store. Store manager, other personnel were interviewed based on the level of technical knowledge, education, experience, intimacy with the inventory management problem.

3.3 Period of research

In research work, data was collected during the last three years from 2006 to 2009 for forecasting the material demand, lead time for material replenishment.

3.4 Instrument of the research

Research study was executed in terms of following models:

Optimal quantity for inventory items was calculated by using the Modified EOQ (Economic Order Quantity) model (4.3.1.1) discussed in the chapter-4.

Reorder point (ROP) model in terms of quantity was used to find out the exact time for ordering inventory in the inventory management system. Joint order of several items may save a part of ordering cost in the multi-item inventory system. This Reorder point (ROP) is easy to find-out by using a mathematical formula (4.3.2.1) discussed in the chapter-4.

Safety stock is the additional stock in hand to reduce the risk of running out of the stock during lead time. This safety stock level is easy to find-out by using a mathematical formula (4.3.4.1) discussed in the chapter no-4.

3.5 Procedures of data collection

A lot of inquiry is required for research work in particular field. Collection of data is the main base of inquiry. Inquiry is performed through data collection to conclude about particular decision. There are two kinds of data, one is primary data, which is collected from the field level and another is secondary data, which is collecting form the presently available primary data. Collecting primary data is time consuming and costly.

Method of data collection

In this research, data was collected by direct inquiry. The collected data was then written down.

Method of presentation of statistical data

Statistical data can be presented in different ways. Two statistical measures of probabilistic data are: mean and standard deviation.

Method of calculation Statistical Mean

Mean is calculated by the following steps

1) A frequency table is constructed from the statistical data table.

2) Use the formula Mean, M = $\Sigma fx / \Sigma f$ for calculating mean for a frequency distribution.

Method of calculation Standard Deviation

The standard deviation is calculated by the following steps:

1) Determine the mean (average) of a set of numbers.
2) Determine the difference of each number and the mean.
3) Square each difference
4) Calculate the average of the squares.
5) Calculate the square root of the average.

Rules for forming frequency distribution table from unclassified raw data

Frequency table can be constructed from the raw data by the following steps:

1) Classify the nature of the variable.
2) Making class interval of raw data.
3) Frequency distribution through class interval.
4) Find out the width of the class interval.

Steps are followed to draw Histogram Diagram

Measured data form samples taken during the present study can be plotted in order to determine the shape of the frequency distribution. The frequency distribution can give a visual clue to the process average and dispersion of data set. Most common method of presentation of statistical data is Histogram. Histogram diagram is drawn by the following steps:

1) Data are group into class intervals.
2) The class limits, class boundaries, tally marks and the frequency for each class are calculated and shown in the table.
3) The class limit is marked on the horizontal axis.
4) The frequency is marked on vertical axis.
5) A rectangle is constructed on each class interval that has heights proportional to the class frequency.

3.6 Data screening and analysis

The collected data was screened to cut down redundant information. When any contradictory was arisen, the field was revisited to take interview again of the respective respondents. In such situation, if the questionnaire did not cover required quarries, the respondents were directly asked different necessary questions. The collected data (shortage of inventory and to avoid it) were analyzed and were arranged according to weight given by the respondents.

3.7 Interpretation of the results

The entire process of collection data and its interpretation is written down in correct sequence and logical manner so that, the reader gets the complete picture of what had been done to get the desired results and to arrive at a conclusion.

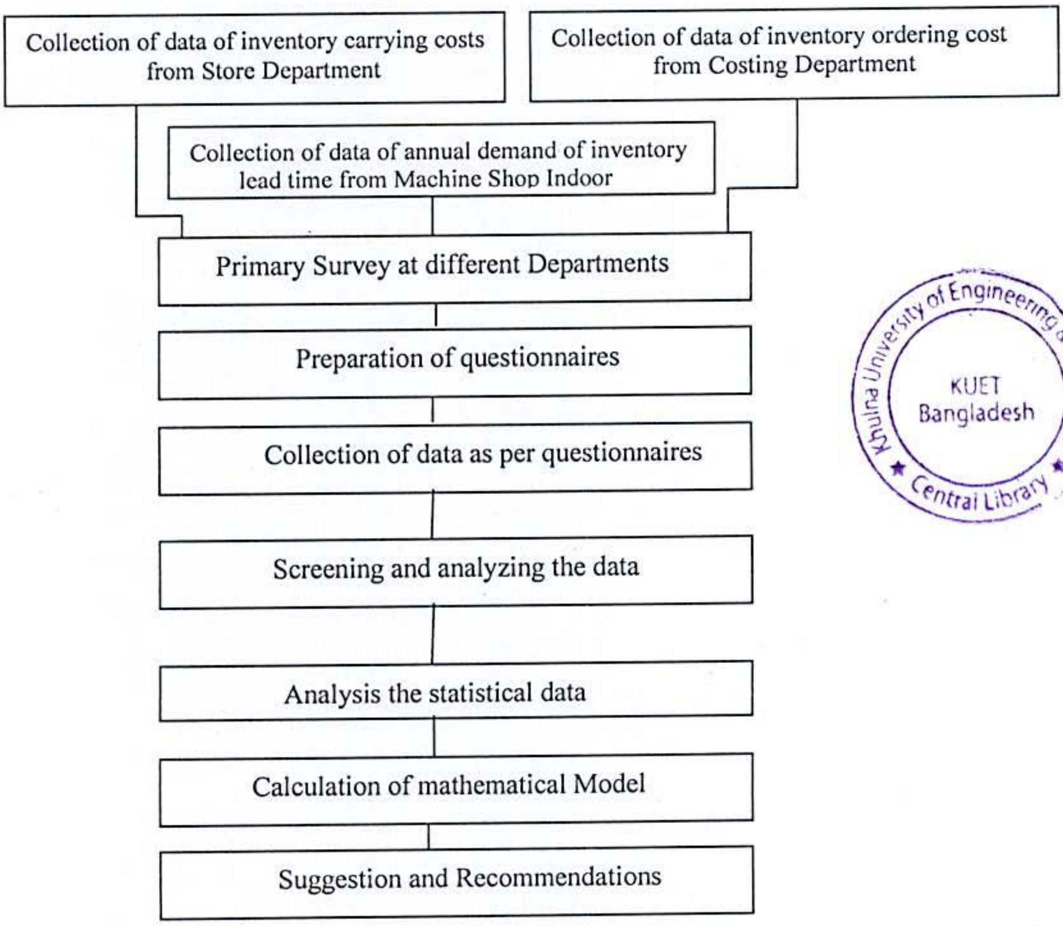

Figure-3.1: The steps of the study are shown in block diagram.

Development of Models

4.1 General

In this thesis, purchase model of inventory for multi- item joint replenishment with continue review policy by (Q, R) rule is analyzed in a multi-inventory system under variability of both demand and lead time considering no shortage. The multi- item inventory system has m independent items P_1, P_2........ P_m. For each item it is assumed that the deterministic EOQ model can be used. The demand D_i and lead time L_i for individual items P_i, where i =1,2,3....m are changing over time. No shortage of inventory is allowed through the production. Each group of items is ordered under common order cycle from a single supplier. An economic order quantity is then fixed not for individual items; consecutive order is also obtained from this model.

4.2 Basic economic order quantity model

In relation to the technique used to determine quantity to order, the EOQ model provides useful guideline for ordering decisions. A basic problem for manufacturers deal with several types of items is what quantity of a given item to order. A large order quantity reduces ordering frequency and hence ordering cost, but increases the holding cost of inventory. A smaller order quantity reduces holding cost, but increase ordering cost, risk of stock out of inventory during lead time. The objective is to select an order quantity that minimizes both holding and ordering cost. This formula allows us to determine the followings:

1) The optimal quantity to order.
2) When it should be ordered.
3) The total cost.
4) The average inventory level.
5) How much should be ordered each time.
6) The maximum inventory level.

The Economic Order Quantity model enables us to determine the lowest-cost order quantity directly. Total cost curve is relatively flat near the EOQ. In the Economic Order Quantity equation a large change of demand rate or item cost result in small changes in order quantity. So, the model is remarkably robust. EOQ model has some major limitations which are mentioned bellow:

1) Demand rate is assumed to be constant, but in reality it changes over time.
2) Place of the items changes through out the procurement cycle.
3) EOQ formula is not an integer formula.
4) EOQ model is beneficial only for repetitive purchasing.

So, some adjustments are needed to make the EOQ model more realistic. Recently several authors have concentrated their effort in improvement of models to determine the Economic Order Quantity for multi-item inventory system.

4.3 Necessary adjustments in EOQ model

EOQ model is used for single inventory item. It can not be applied directly to the multi-inventory items. So, necessary adjustment has to be made in EOQ model.

4.3.1 Development of purchase model of inventory for multi-item joint replenishment without shortage

The variables involved in the purchase model are listed below. In this model, shortage is not permitted.

C_o is the fixed ordering cost in taka for a group of items.

Coi is the marginal cost of ordering for the item *i*

D_i is the annual demand in units of the item *i*

D_r is the annual demand in taka for the group of items ordered.

d_{ri} is the annual demand in taka for the item *i* in the group of item order.

P_i is the purchase price per unit of the item *i* in the group of items ordered.

I is the inventory carrying cost is percentage of the unit .

Q_r is the economic order quantity (EOQ) in taka for the group of items ordered.

q_{ri} is the economic order quantity (EOQ) in taka for the item *i*.

Q_i is the economic order quantity (EOQ) in unit for the item i

N is the number of order cycles per year.

m is the number of items in the group.

T is the time between consecutive orders.

The formula for the total cost of this inventory system is as given below:

TC= Total ordering cost + Total carrying cost + Total purchase cost.

$$=[Dr/Q_r]\{C_o+\sum_{i=1}^{n}C_{oi}\}+(Q_r/2)I+\sum_{i=1}^{n}Pi\,Di$$

Differentiating the function TC with respect to Q_r, we get

$$\frac{d}{dQ_r}(TC)=[-Dr/Q_r^{-2}]\{C_o+\sum_{i=1}^{n}C_{oi}\}+I/2$$

Again differentiating the function TC with respect to Q_r, we get

$$\frac{d^2}{dQ_r^2}(TC)=[Dr/Q_r^{-3}]\{C_o+\sum_{i=1}^{n}C_{oi}\}$$

As per the principles of minima, the second derivative is positive. Hence, equating the first derivative to 0, we get

$$Q_r=\sqrt{\frac{2(C_o+\sum_{i=1}^{n}C_{oi})\,Dr}{I}} \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.1.1)$$

The formulas to get the EOQ in taka for the items Q_r and the EOQ in units for the item are Q_r given below,

$$q_{ri}=\frac{d_{ri}}{Dr}\times Q_r \quad \text{where } i=1,2,3.........m \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.1.2)$$

$$Q_i=\frac{q_{ri}}{P_i} \quad \text{where } i=1,2,3.........m \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.1.3)$$

Farther, the formula to obtain N and T are given below:

$$N=Dr/Q_r \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.1.4)$$

$$T = {Q_r}/{Dr} \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.1.5)$$

N is computed for the group of items ordered. But the number of intervals for the individual items may differ from N. There are many procedures to obtain the individual interval multiples n_i where i =1,2,3.........m. An algorithm called Silver's Algorithm [17] is persecuted to find such n_i values

Silver's Algorithm

Step 1: Find the ratios C_{oi}/d_{ri} for i =1,2,3.........m and find their minimum. Let the item corresponding to this minimum be K.

Step 2: Set the interval multiple for the item K to i (i.e. $n_k = i$).

Step 3: Compare the interval multiple n_i for the remaining items i =1,2,3.........m $i \neq k$ using the following formula and round off them to the nearest integer (Note if n_i is less than 1 ,then round off it to 1 irrespective of the value)

$$n_i = \sqrt{\left(\frac{C_{oi}}{d_{ri}} \times \frac{d_{rk}}{(C_o + C_{ok})}\right)} \text{ for } i=1,2,3 \quad i \neq k \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.1.6)$$

where K is the number for witch C_{oi}/d_{ri} is the minimum and the value of n_i is 1.

4.3.2 Determination of re-order point

Re-order point is defined as the predetermined inventory level below which a replenishment order is triggered. One common way of controlling multi-inventory system is to apply a re-order point. EOQ models answer the question of how much to order, but now the question when to order. The latter part is the function of models that identify the re-order point in terms of quantity. ROP models should involve service and safety stock considerations. Re-ordering point has two components. One, account for average demand during lead time and other, account for deviations from average. Re-order point can be calculated by the following formula [18]

Re-order point = Mean demand over lead time + Safety Stock

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)} \quad \ldots. \quad \ldots \quad \ldots \quad \ldots.(4.3.2.1)$$

Re-order point model is particularly helpful in dealing with situations that include variation in either demand rate and /or lead time. Re-order point is influenced by four factors: demand rate, lead time requires replenishing inventory amount of uncertainty in the demand rate and in the replenishment lead time, and management policy regarding the acceptable level of customer service.

4.3.3 Inventory replenishment policy for uncertain demand and lead time

There are different type of inventory replenishment policy, they include (R,Q) policy, two bin or min-max policy, order up to policy etc. Among them (R,Q) policy is the most popular in practice. In (R,Q) policy, if the inventory position at the review point is less than R, we place an order for Q quantity. Where R is the re-order point and Q is the order quantity. If the control system is of the continuous review type, order Q unit is placed as soon as the inventory position decline to R. The review interval corresponds to the times inventory levels are checked and reorder decision made. Today, most of the system is continuous review, which means that transaction is reported as they occur.

4.3.4 Determination of safety stock

Safety stock is defined as minimum amount of inventory which one would like to have on hand at all the times. The re-order point must be grater than average demand during the replenishment lead time. The difference between the average demand during lead time and the re-order point is also called safety stock. It is the trade off between the probability of stock out during a replenishment order cycle and investment of funds in inventory. Safety stock provides protection against irregularities or uncertainties in an item's demand. An important management question concerns about the amount of safety depends upon the how much demand requirement is needed to fulfill and how much protection is desirable? Safety stock against both uncertainties a demand and lead time can be calculated by the following Formula [18].

Safety Stock quantity = $Z\sqrt{(\overline{LT} \times \sigma_d^2 + \overline{d}^2 \times \sigma_{LT}^2)}$ ... .(4.3.4.1)

4.3.5 Inventory groupings for joint replenishment in multi-inventory system

In multi-inventory system, it is needed to define objectives with regard to inventory level, work load, Customer Service and therefore simple methodology are needed to enable decision making regarding ordering items in a group. Joint replenishment is the well known case in the multi-item inventory problem. A joint stock replenishment policy is an inventory control strategy under which groups of item from single supplier are jointly re-ordered on a single order. An EOQ is then fixed not for the individual items, but for the item group as a whole. Where producing one unit of a certain product may require a multiple units of different components that must be assembling to fabricate the end product. Grouping of items are based on the sequential need of items in the assembly line. It is desirable to form a certain number of items where all items of one group share the same order cycle in a multi-item inventory system. This yields simplified stock control, better supplier relations, more reliable demand fulfillment, improved warehouse planning. The main objective of establishing a joint replenishment policy to minimize the total cost of inventory by reducing ordering cost. In this study, inventory items are grouped based on their sequential demand in the assembly line.

4.4 Procedure for determination of EOQ, ROP, safety stock for joint replenishment order in multi-item inventory system

In a multi- item joint replenishment inventory model, the order quantity distribution are analyzed of each item in their joint order. Following procedure is maintained for determining EOQ, ROP, and Safety Stock for joint replenishment order.

1) Tabulating annual demand, unit price and marginal cost of ordering of n individual items.
2) Find out, the fixed and summation of marginal ordering cost of each items for computing total ordering cost.
3) Find out, the inventory holding cost as percentage of the purchase price of items in the group.
4) EOQ of all items put together in a group is computed in money by the formula (4.3.1.1)
5) EOQ in units of each item is computed using the formula (4.3.1.3)

6) No of order cycle per year for each group of items and time between two consecutive orders.
7) Find out, order cycle for each item in terms of the integral multiple of shortest order cycle.
8) Each group has a common order cycle for all items in the group. Optimal order point is obtained for the groups by using the previously mentioned formula.
9) Safety stock for each group is obtained by using the formula mentioned earlier.
10) Find out the mean and standard deviation of fluctuating demand and lead time by using the formula mentioned previously.
11) Classifying the items into a few groups with common order cycles for all the items in a particular group.

4.5 Determination of ordering cost of multi-item inventory system

Order cost are all costs associated with ordering inventory items. It is important to understand that these are the costs associated with the frequency of the orders and not the quantity ordered. In real practice, the manufacturing organization purchases multi-item jointly to replenish them. Under this situation, the cost has both fixed and variable components. The fixed cost includes common activities involved in purchasing all the items in a group. Variable or marginal includes the cost of processing some specific purchase activities of an individual item. ordering cost include the cost to creating the purchase order or requisition, any approval steps, reviewing quotation, to process the receipt, incoming inspection, invoice processing, vendor payment and in some cases a portion of the inbound freight may also be included in ordering cost. The order cost is calculated by the following formula.

$$\text{Ordering Cost for Q units} = C_0 + \sum_{i=1}^{n} C_{0i} \quad \cdots\cdots \quad (4.5.1)$$

4.6 Determination of holding cost of multi-item inventory system

Holding cost is known as carrying cost, all costs that accrue as a result of holding inventory. Holding cost is composed of several components. They include the cost of

1) Providing the physical space to store the items.
2) Taxes and insurance.
3) Breakage, spoilage, deterioration, pilferage and obsolescence
4) Opportunity cost of alternative investment.
5) Material handling.
6) Cost of money invested (i.e., Bank loan)

Inventory carrying cost is usually express in percentage of the purchase price of items in the group. Normally carrying cost is 20 to 25% of the inventory purchase cost.

Results and Discussions

5.1 Introduction

In this section, the model developed in the previous chapter is applied for the procurement of three components of Gun Metal demanded by the Machine shop of Khulna shipyard. The findings are presented and illustrated with figure and tables.

5.2 Determination of probability distribution for demand

Demand data for the past three years are shown in the table -5 of appendix-A. Mean demand and standard deviation are calculated according to the procedure mentioned in the methodology chapter by the following steps. Finally, Probability distribution of monthly demand for Gun Metal is shown in the figure-5.1

$$\therefore \text{Mean, M} = \Sigma f X \Big/ \Sigma f = \frac{10\times1+30\times2+50\times3+70\times4+ 90\times6+110\times9+130\times4+150\times3+170\times2+190\times1}{35}$$

$$=100.85 \approx 101 \text{ Kg}$$

Table-1: Table for calculating standard deviation of demand data for Gun Metal.

Demand	Frequency f	Midpoint X	$d = \frac{X-A}{C}$	fd	fd^2
0-20	1	10	-4	-4	16
20-40	2	30	-3	-6	18
40-60	3	50	-2	-6	12
60-80	4	70	-1	-4	4
80-100	6	90 =A	0	0	0
100-120	9	110	1	9	9
120-140	4	130	2	8	16
140-160	3	150	3	9	27
160-180	2	170	4	8	32
180-200	1	190	5	5	25
	$\Sigma f = 35$			$\Sigma fd = 19$	$\Sigma fd^2 = 159$

$$\therefore \text{SD} = C\sqrt{\left[\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right]}$$

$$= 20\sqrt{\left[\frac{159}{35} - \left(\frac{19}{35}\right)^2\right]}$$

$$= 41.23 \text{ Kg}$$

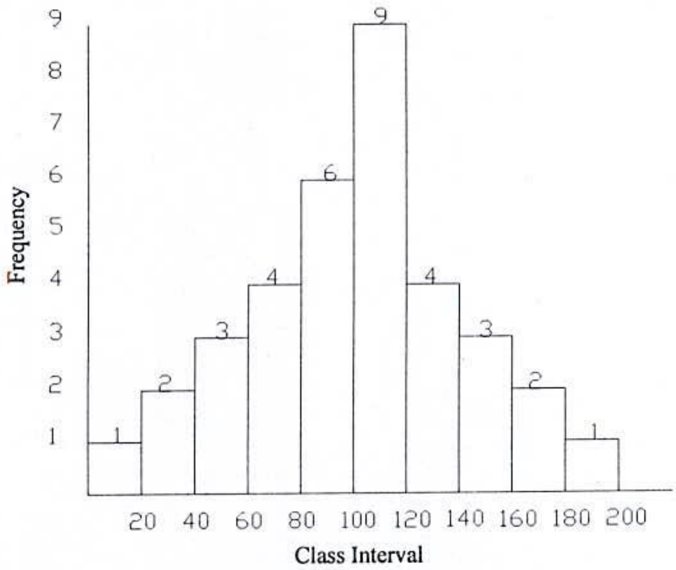

Figure-5.1: Frequency distribution of monthly demand for Gun Metal.

It is observed that, monthly demand for Gun Metal fluctuates. This is due to the fact that number of ships to be repair or renewed are not constant, but varies time to time. Again, the requirement of Gun Metal is not same for the ships during their each schedule repair/maintenance. Mean demand and standard deviation of demand for Gun Metal are found 101Kg and 41.23Kg respectively.

5.3 Determination of probability distribution for lead time

Lead time data for Gun Metal procurement for the last three years are shown in the table -6 of appendix-B. Mean demand and standard deviation of lead time are calculated according to the procedure mentioned in the chapter-3 by the following steps. Finally, Figure-5.2 shows a frequency distribution of lead time for the procurement of Gun Metal.

$\therefore$ Mean, M = $\sum fX \Big/ \sum f = \dfrac{25\times6+35\times10+45\times12+55\times5+65\times2}{35} = 41.3 \approx 41$ Days

Table-2: Table for calculating standard deviation of lead time data for the procurement of Gun Metal.

Lead Time	Frequency f	Midpoint X	$d = \dfrac{X-A}{C}$	fd	fd^2
0-20					
20-30	6	25	1	6	36
30-40	10	35	0.5	5	25
40-50	11	45 =A	0	0	0
50-60	6	55	0.5	3	9
60-70	2	65	1	2	4
	$\sum f = 35$			$\sum fd = 16$	$\sum fd^2 = 74$

$$SD = C\sqrt{\left[\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right]}$$

$$= 20\sqrt{\left[\frac{74}{35} - \left(\frac{16}{35}\right)^2\right]}$$

= 7.21 Days

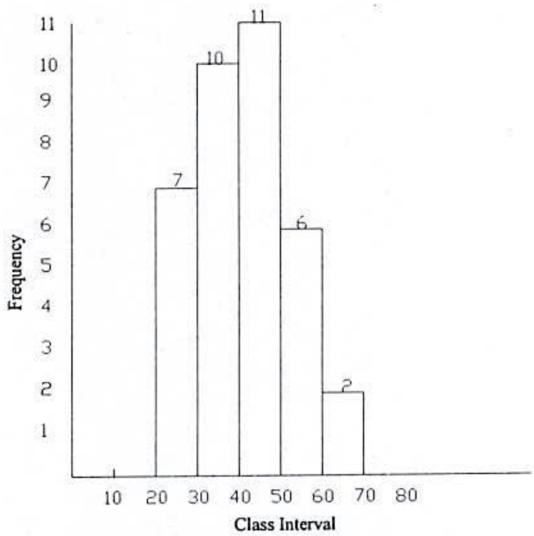

Figure-5.2: Frequency distribution of lead time for the procurement of Gun Metal.

It is observed that, the lead time for Gun Metal procurement also varies due to unavailability of material in the market, transportation delays, complicities arise in procurement process and so on. Mean demand and standard deviation of lead time for the procurement of Gun Metal were found 41Days and 7.21Days respectively.

5.4 Determination of EOQ, ROP, safety stock for joint replenishment order in multi-item inventory system

Each shops of KSY need a variety of materials. But, all the materials are not needed at the same time. Materials are needed according to the sequences of assembly in the production line. It is found that, purchase process is similar for all shops of KSY. The purchase model that is developed follows joint stock replenishment policy where grouping of inventory items are done based on the sequential need of items in the assembly line. This model can also be used in any purchase process performed by the different shops of KSY.

Calculation of fixed ordering cost

Calculation of fixed ordering cost for purchasing inventory items in KSY is calculated according to the procedure mentioned in the chapter-4 by the following steps

1. Raising indent from the shops:

a) Man- Hours.

One project engineer working for half of a working day $1\times300\times0.5 = 150$ TK.

b) Materials and others:

Computer compose & Print out $5\times6 = 30$ TK.

2. Preparing demand with full specification for purchasing inventory by the store department.

a) Man- Hours.

Three personnel including one Asst. store officer for one working day for preparing demand with

$1\times300\times1 = 300$ TK.
$1\times300\times1 = 300$ TK.
$1\times300\times1 = 300$ TK.

Total = 530 TK.

b) Materials and others:

Computer Compose, Print out & Photocopy $20\times6 + 30\times1.50 = 120 + 45 = 165$ TK.

3. Costing Department Calculating the Cost involved in purchasing inventory

a) Man- Hours.

Two staff of Costing department calculating the cost involvement in purchasing items for half of a working day $2\times150\times0.5$ =150 TK.

b) Materials and others:

Computer compose & Print out 20×6 =120 TK.

4. Finance Department check the necessary budget related to the purchase

a) Man- Hours.

One staff of Finance department looking for necessary budget for the purchase for half of a working day $1\times150\times0.5$ =75 TK.

b) Materials and others:

Computer compose & Print out 5×6 =30 TK.

5. Purchase Department arranges tender, Competitive statement, prepare purchase order, take purchase permission form higher management etc.

a) Man- Hours.

Four personnel including MD and purchase Manager Working for processing the purchase of items $1\times250\times3$(Hours.) + $2\times150\times3$=750 + 900 + 900 = 2550 TK.

b) Materials and others:

Computer compose , Print out, Photocopy, Fax, Tender advertisement cost 120×6 +80×1.5 + $10\times22\times5$ + 5000×3 =720 + 120 + 1100 + 15,000 = 16,940 TK.

6. Quality Control Department checks the purchase items and prepares MRR (Material Receive Report) by store department.

a) Man- Hours.

One personnel from Quality department inspect the quality of purchased item for half of a working day $1\times300\times0.5$ = 150 TK.

Two staff from the store department prepare MRR for each purchase for half of a working day $2\times150\times0.5$ = 150 TK.

b) Materials and others:

Computer compose & Print out 10×6 =60 TK.

Total cost of the Man-Hours for executing a purchase process,
150+ 530 + 150 + 75 + 2550 + 150 + 150 = 3,725 TK.

Total cost of the Materials and others for executing a purchase process,
30+ 165 + 120 + 30 + 1690 + 60 = 17,345 TK.

Total cost for executing a purchase process, 3,725+ 17,345 = **21,100 TK.**

Calculation of holding cost

Calculation of holding cost of inventory items in KSY is presented bellow. It is calculated according to the procedure mentioned in the "Development of Models" chapter by the following steps

Average Inventory Value	7,52,38,342 TK
Interest on Bank Loan	97,80,985 TK
Material Handling	37,61,917 TK
Tax (ATI) 4.5%	33,85,725 TK
Vat 2.5 %	18,80,959 TK
Total Annual Cost	1,88,09,586 TK

$$\text{Annual Cost of Carrying Inventory} = \frac{\text{Total Annual Cost}}{\text{Average Inventory Value}}$$

$$= \frac{1,88,09,586}{7,52,38,342} = 25\%$$

Inventory carrying cost is found **25%** of the purchase price of inventory items.

Economic order quantity calculation for multi-inventory items

In this section, we are planning a new proposal for ordering optimal purchase quantity of three components of Gun Metal jointly to meet the lead time demand in a local Shipyard. Calculation for determining EOQ, ROP and safety stock for coordinated replenishment of Gun Metal components is done according to the procedure mentioned in the chapter-4. The important data are included in the following tables 3.

Table -3 Demand Data for the Gun Metal components with additional details

Item	Annual Demand in Unit, Kg	Price Unit, TK. P_i	Annual Demand in TK, d_{ri}	Cost of shipment	Marginal Cost of ordering Items *i*,TK C_{0i}
1. Copper	1212	450	5,45,400	358	358
2. Tin	3	380	1140	588	230
3. Zinc	150	3200	4,80,000	688	100
			$D_r = 10{,}36{,}800$		$\Sigma C_{0_i} = 688$

Finally it is found that,

Fixed ordering Cost, C_0 =21,100 TK.

Sum of Marginal Costs of ordering the items, ΣC_{oi} = 688 TK.

Annual Demand in TK. of all items in a group D_r = 10,36,800 TK.

Inventory Carrying Cost % in decimal, I = 0.25

Therefore, the EOQ in TK. Of all the items put together in the group Q_r is computed as shown below,

$$Q_r = \sqrt{\frac{2(C_o + \sum_{i=1}^{3} C_{oi})D_r}{I}} = \sqrt{\frac{2(21{,}100 + 688)\times 10{,}36{,}800}{0.25}} = 424914.7 \text{ TK.}$$

From the value of Q_r, the value of q_{ri} and Q_i for each of the items are computed as shown,

$$q_{ri} = \frac{d_{ri}}{Dr}\times Q_r \qquad q_{r1} = \frac{d_{r_1}}{Dr}\times Q_r = \frac{545400}{1036800}\times 424915 = 2{,}23{,}523 TK.$$

$$\therefore q_{r2} = \frac{d_{r_2}}{Dr}\times Q_r = \frac{11400}{1036800}\times 424915 = 4{,}672 TK.$$

$$\therefore q_{r3} = \frac{d_{r_3}}{Dr}\times Q_r = \frac{480000}{1036800}\times 424915 = 1{,}96{,}720 TK.$$

Similarly the EOQ in units of each items *i* is computed using the formula

$Q_i = \frac{q_{ri}}{Pi}$ $\quad Q_1 = \frac{q_{r1}}{P_1} = \frac{223523}{450} = 497\text{Kg}$

$$Q_2 = \frac{q_{r2}}{P_2} = \frac{4672}{380} = 12\text{Kg}$$

$$Q_3 = \frac{q_{r3}}{P_3} = \frac{196720}{3200} = 61\text{Kg}$$

No of order Cycles per year $N = \frac{Dr}{Q_r} = \frac{1036800}{424915} = 2$ (Avg. order frequency)

Time between consecutive Orders $T = 1/_N = 1/_2 = 0.5 Years$

The calculation for determining the interval multiples of the individual items as per Silver's Algorithm are summarized in the table-4

Formula for the interval multiples of the items is given below.

$$n_i = \sqrt{\left(\frac{C_{oi}}{d_{ri}} \times \frac{d_{rk}}{(C_o + C_{ok})}\right)} \text{ for } i = 1,2,3\ldots\text{m} \quad i \neq k$$

Table -4 Determination of interval multiples of individual items

Items	d_{ri}	C_{0i}	C_{0i}/d_{ri}	$d_{rk}/(C_0 + C_{0k})$	ni	ni^*
1. Copper	5,45,400	358	0.00064 (minimum)	22.64	0.1213	1
2. Tin	1,140	230	0.2017	22.64	2.136	2
3. Zinc	4,80,000	100	0.000208			1

Calculation of Re-order point

Re-order point for copper

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.36 \times 41 + 3.39\sqrt{(41 \times 41.23^2 + 3.36^2 \times 7.21^2)}$$

= 1036.48 Kg

Re-order point for Tin

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 0.0083 \times 41 + 3.39\sqrt{(41 \times 41.23^2 + 0.0083^2 \times 7.21^2)}$$

$$= 113.84 \text{ Kg}$$

Re-order point for Zinc

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 0.42 \times 41 + 3.39\sqrt{(41 \times 41.23^2 + 0.42^2 \times 7.21^2)}$$

$$= 912.18 \text{ Kg}$$

Safety Stock Calculation

Safety Stock for Copper

$$= Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 3.36^2 \times 7.21^2)}$$

$$= 898.72 Kg$$

Safety Stock for Tin

$$= Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 0.0083^2 \times 7.21^2)}$$

$$= 113.5 Kg$$

Safety Stock for Zinc

$$= Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 0.42^2 \times 7.21^2)}$$

$$= 894.96 Kg$$

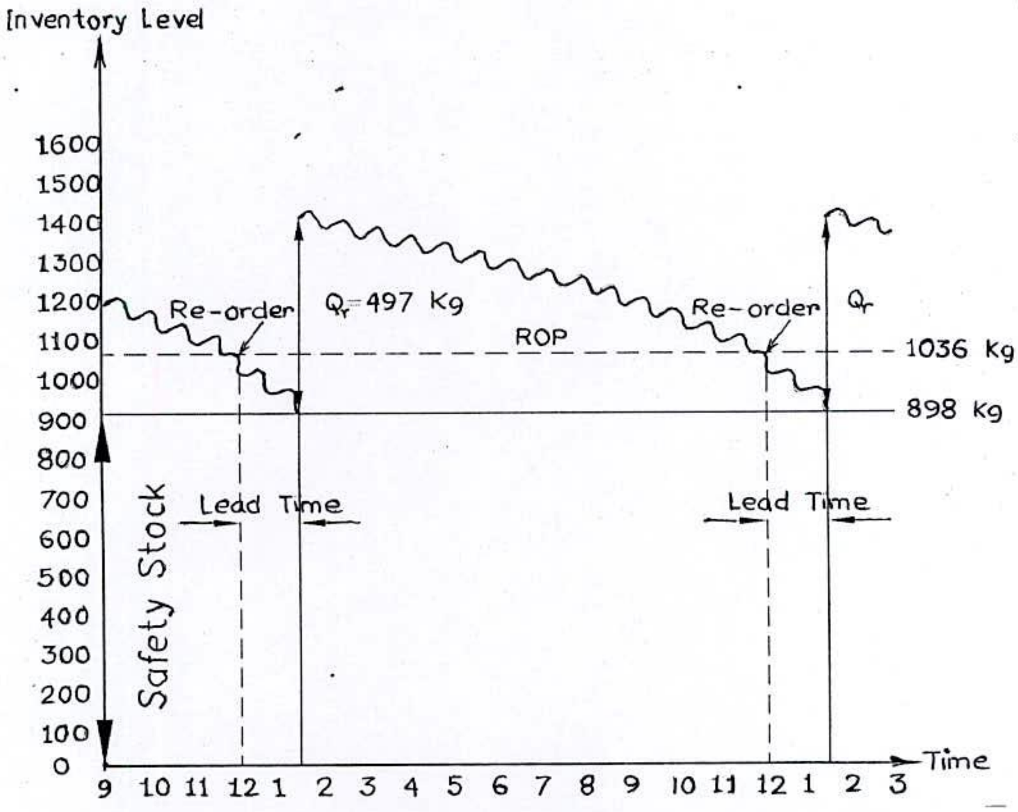

Figure-5.3 Inventory system for Copper with EOQ, ROP and safety stock.

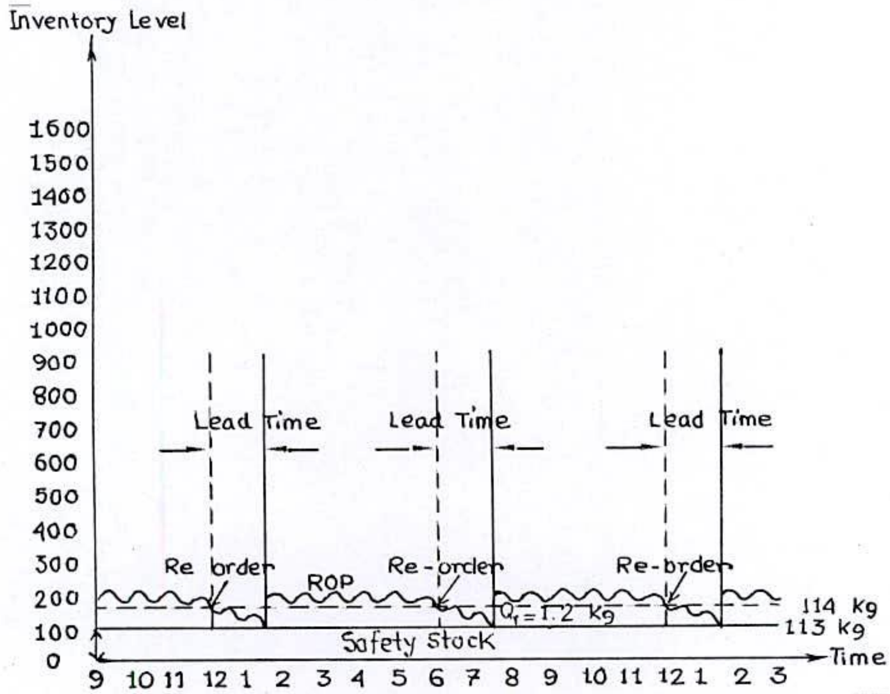

Figure-5.4 Inventory system for Tin with EOQ, ROP and safety stock.

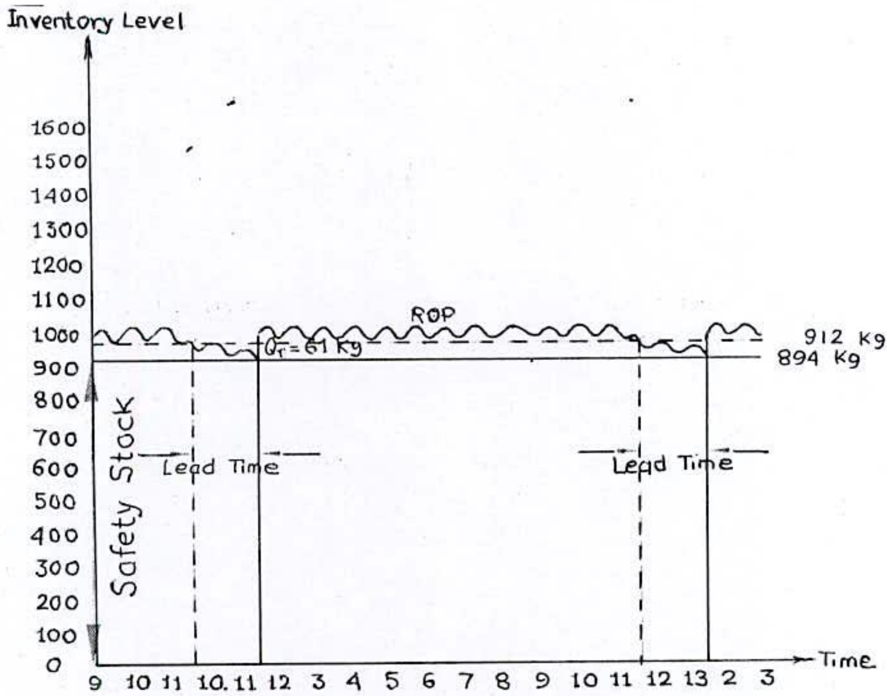

Figure-5.5 Inventory system for Zinc with EOQ, ROP and safety stock

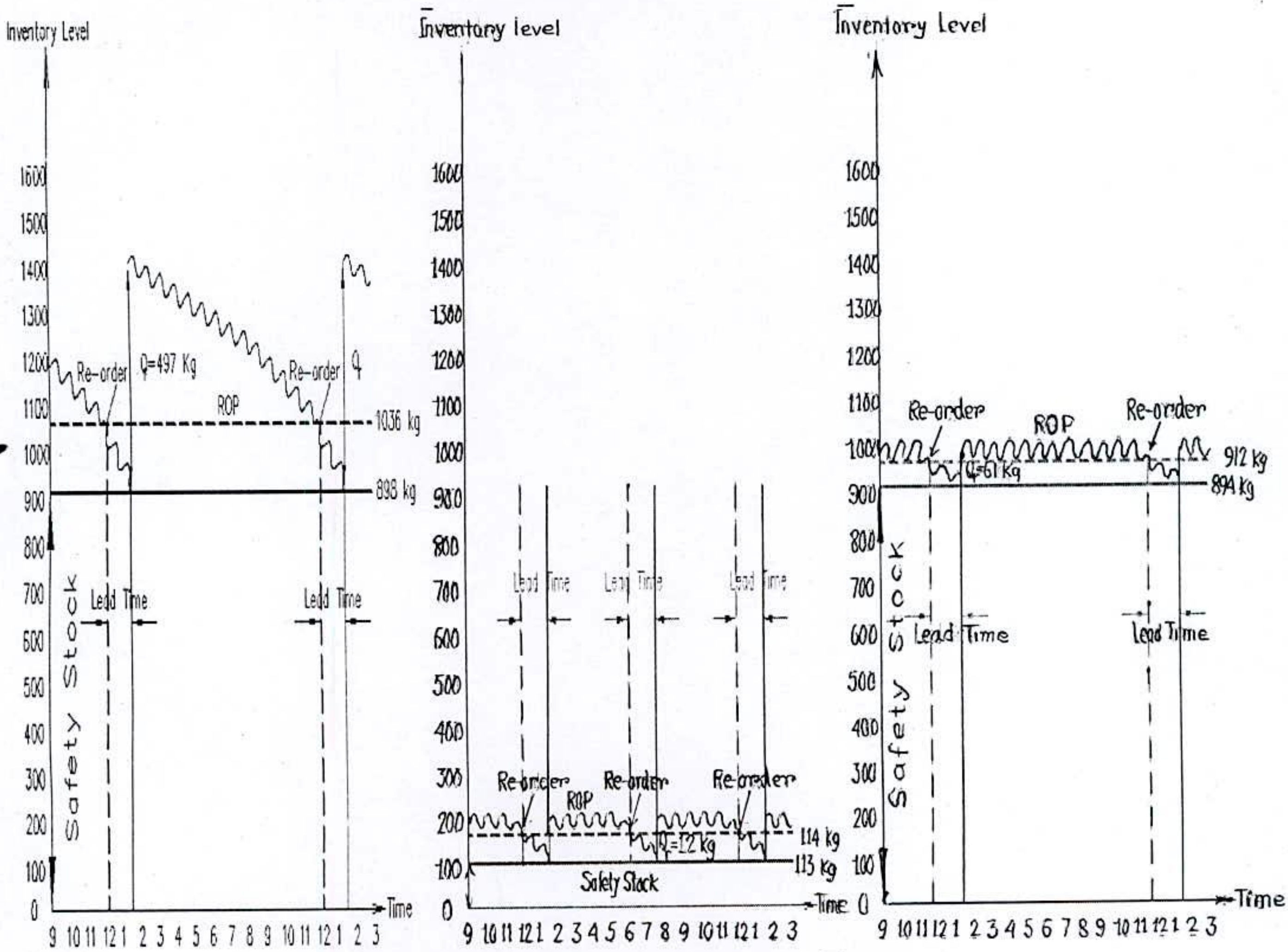

Figure-5.6 Comparison of EOQ, ROP and safety stock between Gun Metal components.

It is observed that, Safety stock depends on the variability of the demand and lead time, desired service level, average demand rate and average lead time. This stock is also proportional to the supply disruption, demand forecast error, square root of the lead time. From the above figure it is found that, it is necessary to carry additional 898 Kg, 113 Kg, and 894 Kg Copper, Tin and Zinc respectively to reduce the risk of running out of stock during lead time. Again, the order of replenishment of Copper, Tin and Zinc is triggered when the inventory level falls below the 1036 Kg, 113 Kg, and 912 Kg respectively. This quantity is influenced by the demand rate, lead time and degree of stock out risk.

6.5 Cost reduction by implementing the new purchase method instead of present purchase method

Calculation of total cost of inventory items in the new purchase method

$$\left(\frac{Dr}{Qr}\right)\{C_o + \sum_{i=1}^{3} C_{oi}\} + \left(\frac{Qr}{2}\right) \times I$$

$$= \left(\frac{1036800}{424915}\right)\{21100 + 688\} + \left(\frac{424915}{2}\right) \times 0.25$$

$$= 106276 \text{ TK.}$$

Calculation of total cost of inventory items in the traditional purchase method

Table -5 Determination of total cost of individual inventory items

Items	Order per year Ti	Marginal ordering cost of items	Ordered quantity (yearly) $Q_i = T_i + D_i$	Holding cost of items $\frac{Q_i}{2} \times H\ P_i$	Ordering cost of items $(C_o + C_{oi})$	Total cost of individual inventory items
1. Copper	1	358	1212	68175	21458	89633
2. Tin	1	230	3	143	21330	21473
3. Zinc	1	100	150	60000	21200	81200
				Total cost of inventory items		192306

It is observed from the above calculation that, **38%** of total inventory cost can be reduced by implementing the new purchase method instead of present purchase method.

5.5 Results

In the present study it is found that, average lead time for inventory replenishment is 41 days and monthly average consumption of Gun Metal in the local company is 101 Kg. Gun Metal is the alloy of copper, Zink and tin. Ordering cost in this multi-item joint replenishment inventory model is reduced by ordering the above items in a group according to their percentage of composition in Gun Metal alloy. Inventory carrying cost is found 25 % of the purchase price of items in the group. The ordering cost for each procurement process is 21,100 TK. Economic order quantity for copper, Zink and tin are found 497Kg, 12Kg and 61 Kg respectively. Interval multiple of ordering individual inventory items like Copper, Tin and Zinc are found 1, 2, 1 respectively. Safety stock for Copper, Tin and Zinc are 898 Kg, 113 Kg, and 894 Kg respectively.

CHAPTER 6

Conclusions and Recommendations

The study was undertaken to apply the purchase model of inventory for multi-item joint replenishment without shortage in Khulna Shipyard Ltd. The analysis, comments, suggestion, action plan etc are based on the data and information collected from the company.

6.1 Conclusions

KSY follows the traditional method for purchasing inventory items. Demand for replenishment of inventory items are placed when they are needed. These items are not arrived just in time. Sufficient additional stock is not maintained to meet the demand of inventory items during lead time. Replenishment lead time varies due to the unavailability of material in the market, transportation delays etc. Therefore, the production is halted and it is not possible to maintain schedule accordingly.

The model developed for purchasing multi-items of inventory in a joint replenishment policy will be very much helpful for the KSY for taking inventory management dissensions like how much to order inventory items at what time. This model will also provide the decision of maintaining minimum safety stock. To use the model, the shipyard needs to change their method of ordering, ordering cost and ordering cycle in the following ways:

The method of grouping inventory items based on the type of production, inventory level, work load, customer service. In the present study it was found that, company producing one unit of certain product may require a multiple units of different components in a sequence assembling procedure to fabricate the end product. So, grouping of items has to be done based on the sequence of assembly of the end product.

Ordering cost of the multi-item inventory system has two components. One is fixed cost and another is variable or marginal cost which includes the cost of processing some specific purchase activities of an individual items. So, Marginal cost must be calculated for determining the economic order quantity of each item.

When inventory item are ordered in a group, the number of order cycle is then fixed not for the individual items, but for the group items as a whole. Order cycle for each item can be calculated in terms of integral multiple of the shortest order cycle among the individual item. When the cycle length is a multiple of some basic cycle time, reduces the major ordering costs.

Furthermore, the following benefits can be obtained as implementation of the replenishment inventory model in the operation of the company where both demand and lead time is variable. These are as follows:

1) EOQ model is beneficial for find out the optimal quantity of ordering of each item. The ROP models will give decision about the right time for placing orders. Safety stock model will decide minimum amount of inventory items which would like to have on hand to meet the demand during lead time. So, continuous supply of materials can be provided to facilitate uninterrupted production.
2) Minimum investment in inventory and keep it at an optimal level.
3) Replenishment orders for a group of items at the same time to exploit quantity discounts offered by the suppliers.
4) This will set the ordering period of various items in such a way that it will even out the production load.

6.2 Limitations of the present study

1. The replenishment rules that has been discussed assuming that items are purchased from single supplier at single location. But, in actual practice, items are purchased from multiple suppliers at different locations.

2. Limitation of storage space is not considered in this model. But in reality, more storage space is required for ordering more items.

3. The Safety stock model becomes impractical if the variation in demand during lead time is too much.

4. The modified EOQ model becomes impractical if the variability in demand is too high.

5. Purchase price is assumed to be constant during the purchase process. But, in reality, the purchase price always changes.

6.3 Recommendations

There are some scopes to improve the inventory control system of Khulna Shipyard Ltd.. These are as follows:

1. An adoption of internet based procurement policy can be adopted to reduce the average length of purchase and order cost.

2. Enterprise resource planning (ERP) and web based technologies has accurate prediction of demand, thereby facilitating the calculation of appropriate reorder point.

3. Collective planning, forecasting and replenishment (CPFR) system can be used to compress the lead times among trading partners and increase forecast accuracy.

4. Material requirement planning (MRP) technique can be adopted to determine when and how much to order according to the requirements of assembles line of an end product.

REFERENCES

[1] Chakravarty and Goyal (1986), Multi-item inventory grouping with dependent set-up cost and group overhead cost. Engineering Costs and Production Economics. 10, pp. 13-23.

[2] Seth,D. and Pandey, M.K. (2009), Production Planning & Control: The Management of Operations 20, pp. 242-253.

[3] Harris, F. W. Operations Cost (Factory Management Series), Chicago: Shaw (1915).

[4] Grubbström, R. W. and Erdem, A. (1999), The EOQ with backlogging derived without derivatives , International Journal of Production Economics 59, pp. 529-530

[5] Ben-Daya, M. and Raouf, A. (1993), On the constrained multi-item single-period inventory problem, International journal of Production Management 13, pp.104-112.

[6] Bhattacharya, D. K. (2005), Production, manufacturing and logistics on multi-item inventory, European Journal of Operational Research 162, pp. 786-791.

[7] Lenard, J. D. and Roy, B. (1995), Multi-item inventory control: A multi- criteria view, European Journal of Operational Research 87, pp. 685-692.

[8] Worell, E. M. and Hall, M. A. (1982), The analysis of inventory control model using polynomial geometric programming, International journal of production management 20, pp. 657-667.

[9] Kar, S., Bhunia, A. K. and Maiti, M. (2001), Inventory of multi- deteriorating items sold from two shops under single management with constraints on space and investment, Computers and Operations Research 28, pp. 1203-1221.

[10] Sulem, A. (1986), Explicit solution of a two-dimensional deterministic inventory Problem, Mathematics of Operations Research 11, pp. 134-146.

[11] Rosenblatt, M. J. (1981), Multi-item inventory system with budgetary constraint: A comparison between the lagrangian and the fixed cycle approach, International Journal of Production Research 19, No.4.

[12] Gohary, A. and Elsayed, A. (2008), Optimal Control of a Multi-item Inventory Model, International Mathematical Forum 27, pp.1295-1312.

[13] Hala, A.F. (2005), Preiodic Review Probabilistic Multi-Item Inventory System with Zero Lead Time Under Constrainnts And Varying Order Cost, American Journal of Applied Science 2(8), pp. 1213-1217

[14] Qin et al., (2007), Channel coordination and volume discounts with price-sensitive demand. International Journal of Production Economics 105, pp.43-53.

[15] Abou-El-Ata, Fergany, H.A. and El-Wakeel, M.F. (2000), Probabilistic Multi-Item Inventory Model With Varying Order Cost Under Two Restrictions, International Journal of Production Economics 83, pp. 223-231.

[16] Stephen,C.G. (1999), A single Item inventory model for a non-stationary demand process, Manufacturing & Service Operations Management 1(1), pp. 50-61.

[17] Panneerselvam, R. (2006), Operation Research, Prentice Hall of India Pvt. Ltd., New Delhi-110001, pp. 259-266.

[18] Stephenson,J. (2009), Operational Management, McGraw-Hill Book Company, New York, pp.543-553.

Table -6 Monthly Consumption of Gun- Metal during September, 2006 to July ,2009.

Year	Name of the Month	Monthly Usage (Kg)	Remarks
2009	July	122	
	June	98	
	May	125	
	April	95	
	March	130	
	February	85	
	January	90	
2008	December	115	
	November	82	
	October	115	
	September	75	
	August	119	
	July	70	
	June	112	
	May	68	
	April	135	
	March	45	
	February	155	
	January	200	
2007	December	55	
	November	109	
	October	175	
	September	110	
	August	95	
	July	35	
	June	165	
	May	65	
	April	150	
	March	50	
	February	145	
	January	108	
2006	December	30	
	November	105	
	October	5	
	September	116	

APPENDIX–B

Table -7 Monthly Consumption of M.S. Plate during September, 2006 to July ,2009

Year	Name of the Month	Monthly Usage (Kg)	Remarks
2009	July	1220	
	June	980	
	May	1250	
	April	950	
	March	1300	
	February	850	
	January	900	
2008	December	1150	
	November	820	
	October	1150	
	September	750	
	August	1190	
	July	700	
	June	1120	
	May	680	
	April	1350	
	March	450	
	February	1550	
	January	2000	
2007	December	550	
	November	1090	
	October	1750	
	September	1100	
	August	950	
	July	350	
	June	1650	
	May	650	
	April	1500	
	March	500	
	February	1450	
	January	1080	
2006	December	300	
	November	1050	
	October	50	
	September	1160	

APPENDIX–C

Table -8 Monthly Usage Painting area (Sq. M.) during September, 2006 to July ,2009

Year	Name of the Month	Monthly Usage Painting area (Sq. M.)	Remarks
2009	July	122	
	June	98	
	May	125	
	April	95	
	March	130	
	February	85	
	January	90	
2008	December	115	
	November	82	
	October	115	
	September	75	
	August	119	
	July	70	
	June	112	
	May	68	
	April	135	
	March	45	
	February	155	
	January	200	
2007	December	55	
	November	109	
	October	175	
	September	110	
	August	95	
	July	35	
	June	165	
	May	65	
	April	150	
	March	50	
	February	145	
	January	108	
2006	December	30	
	November	105	
	October	5	
	September	116	

Table -9: Lead Time of Gun- Metal, M.S. Plate, Red Oxide Primer during September, 2006 to July, 2009.

Year	Name of the Month	Monthly Usage (Kg)	Remarks
2009	July	45	
	June	22	
	May	37	
	April	41	
	March	55	
	February	35	
	January	27	
2008	December	42	
	November	27	
	October	64	
	September	52	
	August	42	
	July	57	
	June	43	
	May	36	
	April	27	
	March	55	
	February	31	
	January	47	
2007	December	42	
	November	38	
	October	64	
	September	31	
	August	45	
	July	36	
	June	52	
	May	37	
	April	41	
	March	32	
	February	58	
	January	42	
2006	December	27	
	November	31	
	October	22	
	September	42	

Sample calculation for determining EOQ, ROP and safety stock for co-ordinate replenishment of multiple inventory items needed for M. S. Plate welding and cutting.

Table -10 Demand Data of inventory items needed for M. S. Plate welding and cutting with additional details

Item	Annual Demand in Unit, Kg	Price Unit, TK. $P\,i$	Annual Demand in TK, dri	Cost of shipment TK	Marginal Cost of ordering Items i,TK C_{0i}
1.M.S. Plate	12000	85	1020000	358	358
2. LP Gas	3	900	518400	588	230
3. Oxygen	1620	1500	2430000	688	100
			$D_r = 3968400$		$\Sigma C_{0_i} = 688$

N.B.: For 1 ton M. S. Plate welding and cutting 48 kg. LP Gas and 135 kg. Oxygen Gas are needed.

Fixed Ordering Cost $C_0 = 21{,}100$ TK.

Sum of Marginal Costs of ordering the items $\Sigma C_{0_i} = 688$ TK.

Annual Demand in TK. Of all items in a group $D_r = 3968400$TK.

Inventory Carrying Cost % in decimal (I) = 0.25

Demand for M. S. Plate is normally distributed with mean $\overline{d}$, 1 ton per month and Standard Deviation σ_d, 7.21 ton

Therefore, the EOQ in TK. Of all the items put together in the group Q_r is computed as shown below,

$$Q_r = \sqrt{\frac{2(C_0 + \sum_{i=1}^{3} C_{0_i}) D_r}{I}} = \sqrt{\frac{2(21{,}100 + 688) \times 3968400}{0.25}} = 831690 \text{ TK.}$$

From the value of Q_r, the value of q_{ri} and Q_i for each of the items are computed as shown,

$$q_{ri} = \frac{d_{ri}}{Dr} \times Q_r \qquad \therefore q_{r1} = \frac{d_{r1}}{Dr} \times Q_r = \frac{1020000}{3968400} \times 831690 = 213769TK.$$

$$\therefore q_{r2} = \frac{d_{r_2}}{Dr} \times Q_r = \frac{518400}{3968400} \times 831690 = 108645TK.$$

$$\therefore q_{r3} = \frac{d_{r_3}}{Dr} \times Q_r = \frac{2430000}{3968400} \times 831690 = 509275TK.$$

Similarly the EOQ in units of each items *i* is computed using the formula

Similarly the EOQ in units of each items *i* is computed using the formula

$$Q_i = \frac{q_{ri}}{Pi} \qquad Q_1 = \frac{q_{r1}}{P_1} = \frac{213769}{85} = 2500\text{Kg}$$

$$Q_2 = \frac{q_{r2}}{P_2} = \frac{10864}{900} = 120\text{Kg}$$

$$Q_3 = \frac{q_{r3}}{P_3} = \frac{509275}{1500} = 339\text{Kg}$$

No of order Cycles per year $N = \frac{Dr}{Q_r} = \frac{13968400}{831690} = 5$ (Avg. order frequency)

Time between consecutive Orders $T = 1/N = 1/5 = 0.2Years$

Formula for the interval multiples of the items is

$$ni = \sqrt{(\frac{C_{0i}}{d_{ri}} \times \frac{d_{rk}}{(c0 + C_{0k})})} \text{ for } i = 1,2,3 \quad i \neq k$$

The calculation for determining the interval multiples of the individual items as per Silver's Algorithm are summarized in the table -11.

Table -11 Determination of interval multiple of individual items needed for M. S. plate welding and cutting

Items	d_{ri}	C_{0i}	C_{0i}/d_{ri}	$d_{rk}/(C_0+C_{0k})$	n_i	n_i^*
1.M.S. Plate	1020000	358	0.00035	114.62	0.2	1
2. LP Gas	518400	230	0.00044	114.62	0.22	1
3. Oxygen Gas	2430000	100	0.000041 (minimum)			1

Calculation of Re-order point:-

Re-order point for M.S. Plate

$$=\bar{d}\times\overline{LT}+Z\sqrt{(\overline{LT}\times\sigma d^2+\bar{d}^2\times\sigma LT^2)}$$

$$=33.33\times 41+3.39\sqrt{(41\times 41.23^2+33.33^2\times 7.21^2)}$$

= 2576 Kg

Re-order point for LP Gas

$$=\bar{d}\times\overline{LT}+Z\sqrt{(\overline{LT}\times\sigma d^2+\bar{d}^2\times\sigma LT^2)}$$

$$=1.6\times 41+3.39\sqrt{(41\times 41.23^2+1.6^2\times 7.21^2)}$$

= 961 Kg

Re-order point for Oxygen Gas

$$=\bar{d}\times\overline{LT}+Z\sqrt{(\overline{LT}\times\sigma d^2+\bar{d}^2\times\sigma LT^2)}$$

$$=4.5\times 41+3.39\sqrt{(41\times 41.23^2+4.5^2\times 7.21^2)}$$

= 1086 Kg

Safety Stock Calculation

Safety Stock for M.S. Plate

$$= Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 33.33^2 \times 7.21^2)}$$

$$= 1210Kg$$

Safety Stock for LP Gas

$$= Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 1.6^2 \times 7.21^2)}$$

$$= 895Kg$$

Safety Stock for Oxygen Gas

$$= Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 4.5^2 \times 7.21^2)}$$

$$= 901Kg$$

$$q_{ri} = \frac{d_{ri}}{Dr} \times Q_r \qquad \therefore q_{r1} = \frac{d_{r1}}{Dr} \times Q_r = \frac{24960}{61760} \times 103754 = 41931TK.$$

$$\therefore q_{r2} = \frac{d_{r_2}}{Dr} \times Q_r = \frac{2592018400}{61760} \times 103754 = 18278TK.$$

$$\therefore q_{r3} = \frac{d_{r_3}}{Dr} \times Q_r = \frac{10880}{61760} \times 103754 = 18278TK.$$

Similarly the EOQ in units of each items i is computed using the formula

$$Q_i = \frac{q_{ri}}{Pi} \qquad Q_1 = \frac{}{P_1} = \frac{41931}{130} = 322 \text{ ltr.}$$

$$Q_2 = \frac{q_{r2}}{P_2} = \frac{43544}{180} = 242 \text{ ltr.}$$

$$Q_3 = \frac{q_{r3}}{P_3} = \frac{18278}{160} = 114 \text{ ltr.}$$

No of order Cycles per year $N = \frac{Dr}{Q_r} = \frac{61760}{103754} = 0.6 \approx 1$ (Avg. order frequency)

Time between consecutive Orders $T = \frac{1}{N} = \frac{1}{0.6} = 1.67 Years$

Formula for the interval multiples of the items is

$$n_i = \sqrt{\left(\frac{C_{0i}}{d_{ri}} \times \frac{d_{rk}}{(c_0 + C_{0k})}\right)} \text{ for } i = 1,2,3 \quad i \neq k$$

The calculation for determining the interval multiples of the individual items as per Silver's Algorithm are summarized in the table -13.

Table -13 Determination of interval multiple of individual items needed for painting of ship

Items	d_{ri}	C_{0i}	C_{0i}/d_{ri}	$d_{rk}/(C_0 + C_{0k})$	n_i	n_i^*
1.Red Oxide Primer	24960	358	0.0143	1.2	0.132	1
2. Finish Coat	25920	230	0.0089 (minimum)			1
3. Thinner T6	10880	100	0.0092	1.2	0.105	1

Calculation of Re-order point:-

Re-order point for Red Oxide Primer

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 3.36 \times 41 + 3.39\sqrt{(41 \times 41.23^2 + 3.36^2 \times 7.21^2)}$$

= 1036 Kg

Re-order point for Finish Coat

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 0.4 \times 41 + 3.39\sqrt{(41 \times 41.23^2 + 0.4^2 \times 7.21^2)}$$

= 911 Kg

Re-order point for Thinner T6

$$= \bar{d} \times \overline{LT} + Z\sqrt{(\overline{LT} \times \sigma_d^2 + \bar{d}^2 \times \sigma_{LT}^2)}$$

$$= 0.19 \times 41 + 3.39\sqrt{(41 \times 41.23^2 + 0.19^2 \times 7.21^2)}$$

= 902 Kg

Safety Stock Calculation

Safety Stock for Red Oxide Primer

$$= Z\sqrt{(\overline{LT} \times \sigma d^2 + \bar{d}^2 \times \sigma LT^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 3.36^2 \times 7.21^2)}$$

$$= 898 Kg$$

Safety Stock for Finish Coat

$$= Z\sqrt{(\overline{LT} \times \sigma d^2 + \bar{d}^2 \times \sigma LT^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 0.4^2 \times 7.21^2)}$$

$$= 895 Kg$$

Safety Stock for Thinner T6

$$= Z\sqrt{(\overline{LT} \times \sigma d^2 + \bar{d}^2 \times \sigma LT^2)}$$

$$= 3.39\sqrt{(41 \times 41.23^2 + 0.19^2 \times 7.21^2)}$$

$$= 994 Kg$$