STEADY *MHD* FREE CONVECTION AND MASS TRANSFER FLOW WITH THERMAL DIFFUSION, HALL AND ION SLIP CURRENTS AND LARGE SUCTION

Abul Kalam Azad

DEPARTMENT OF MATHEMATICS
KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
Khulna-920300, BANGLADESH

STEADY *MHD* FREE CONVECTION AND MASS TRANSFER FLOW WITH THERMAL DIFFUSION, HALL AND ION SLIP CURRENTS AND LARGE SUCTION

A THESIS SUBMITTED FOR PARTIAL FULFILMENT OF THE REQUIREMENT OF MASTER OF PHILOSOPHY IN MATHEMATICS

By

Abul Kalam Azad

Roll No.- 0051513

Session- 2000 – 2001

TO

DEPARTMENT OF MATHEMATICS

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY

Khulna-920300, BANGLADESH

APRIL, 2006

We, the Examination Committee, recommend that the thesis prepared by **Abul Kalam Azad**, Roll No.-**0051513**, Registration No.-118, Session- 2000-2001, titled "**Steady *MHD* Free Convection and Mass Transfer Flow with Thermal Diffusion, Hall and Ion Slip Currents and Large Suction**" be accepted as fulfilling this part of the requirements for the degree of Master of Philosophy in Mathematics from the Department of Mathematics, Khulna University of Engineering and Technology.

Examination Committee

1. Professor Dr, Fouzia Rahman
   Supervisor
   Department of Mathematics
   Khulna University of Engineering & Technology.
   .... Chairman (17-5-06)

2. Dr. Mohammad Arif Hossain
   Co-Supervisor
   Department of Mathematics
   Khulna University of Engineering & Technology.
   .... Member (17.5.06)

3. Head
   Department of Mathematics
   Khulna University of Engineering & Technology.
   .... Member (17.5.06)

4. Md. Abul Kalam Azad
   Examiner
   Department of Mathematics
   Khulna University of Engineering & Technology.
   .... Member

5. Professor Dr. Shamsul Alam Sarkar
   Examiner (External)
   Department of Mathematics
   Rajshahi University.
   .... Member(External) (17.05.06)

DECLARATION

We declare that the thesis presented for the partial fulfillment for the Master of Philosophy in Mathematics is done by the student himself and is not presented any where for any other degree or diploma.

30-4-06

Supervisor
Dr. Fouzia Rahman
Professor
Department of Mathematics

30-4-06

Co-Supervisor
Dr. Mohammad Arif Hossain
Assosiate Professor
Department of Mathematics

Dedicated to

My Beloved Father

Acknowledgement

The author would like to express his deepest gratitude and appreciation to Dr. Fouzia Rahman, Professor and head, Department of Mathematics, Khulna University of Engineering and technology (KUET), Khulna, under whose guidance the work was accomplished. The author would also like to thank Prof. Fouzia for her earnest feelings and help in matters concerning author's research affairs as well as personal affairs.

I express my deepest sense of gratitude to my reverend co-supervisor Dr. Mohammad Arif Hossain, Department of Mathematics, KUET for his willingness to accept me as a research student. He helped me immensely with his supervision, invaluable instructions, encouragement and constructive discussion throughout this research work.

I thank all other teachers of the Department of Mathematics, KUET for their necessary advice and cordial cooperation during the period of study. I thank all the research students of this Department for their help in many respects.

I express my earnest gratitude to Dr. Md. Mahmud Alam, Mathematics Discipline, Khulna University, Khulna, for his personal tribute to my thesis.

I would also like to thank the authority of Ahsanullah College, Khulna and Rayermohal College, Khulna for providing me with an opportunity to undertake this work in the Department of Mathematics, Khulna University of Engineering and Technology, Khulna.

Finally, I would express a special thank to my beloved wife Perveen Sultana for her constant encouragement and generous help. My profound debts to my father and mother are also unlimited.

List of Publications

The paper titled

Steady MHD free convection and mass transfer flow with thermal diffusion, Hall current, ion slip current and large suction.

Authors

[1]Abul Kalam Azad, [1]Mohammad Arif Hossain, [1]Fouzia Rahman and [2]Md. Mahmud Alam

Address

[1]Department of Mathematics
Khulna University of Engineering & Technology, Khulna – 9203

[2]Mathematics Discipline, Khulna University, Khulna – 9208

based on Chapter-2 of this thesis work is accepted for publication (in press) in *Journal of Mathematics & Mathematical Sciences*, Jahangir Nagar University, Bangladesh.

Abstract

The present study is concerned with the study of steady MHD free convection and mass transfer flow past an impulsively started infinite vertical porous plate with thermal diffusion when a strong magnetic field of uniform strength is applied transversely to the direction of flow. The Hall and ion slip currents are taken into account. The effects of rotation on the flow are also considered. Similarity transformations are used to transform the governing partial differential equations into a system of ordinary differential equations. The ordinary differential equations are then solved by perturbation technique based on large suction. The velocity and temperature profiles are shown graphically and results are discussed in terms of Hall parameter β_e, ion slip parameter β_i and other established parameters.

Table of Content

Introduction

The aim of this dissertation is to make some calculations, both analytical and numerical, on Magnetohydrodynamic heat and mass transfer flow that have been of interest to the engineering community and to the investigators dealing with the problem in geophysics and astrophysics. The thermal diffusion effect, which is often neglected in heat and mass transfer processes, has also been included in the analyses for the above mentioned calculations.

The natural convection processes involving the combined mechanism of heat and mass transfer are encountered in many natural processes, in many industrial applications and in many chemical processing systems. In these processes the total energy and material transfer resulting from the combined buoyancy mechanisms are the main features to be determined. In our analyses the combined buoyancy effect arising from the simultaneous diffusion of thermal energy and chemical species are considered on the MHD flow of electrically conducting fluid under the action of a transversely applied magnetic field.

When the magnetic field is very strong the effect of Hall current and ion slip current becomes significant. In our analyses we have therefore taken into account these effects. Further in studying the different aspects of astrophysical and geophysical problems the Coriolis force is necessary to include to the momentum equations. Considering its significance compared to viscous inertia forces, it is generally admitted that the Coriolis force due to Earth's rotation has a strong effect on the hydromagnetic flow in the Earth's liquid core. In our analyses we have therefore taken into account the effect of this Coriolis force on a hydromagnetic heat and mass transfer flow.

The whole thesis consists of 3 chapters. In the first chapter, available information regarding MHD heat and mass transfer flows along with various effects are summarized and discussed. The various non-dimensional parameters occurring in the problem are also discussed. A brief review of the past researches related to the topic has been given. In chapter 2, a specific problem of the MHD free convection and mass transfer flow past an infinite vertical porous plate taking into account the thermal diffusion, Hall and ion slip currents with large suction is considered. The partial differential equations governing the problem under consideration have been transformed by a similarity transformation into a system of ordinary differential equations that are then solved by perturbation technique based on large suction. The influences of the different parameters on the velocity and temperature fields have been discussed with the help of graphs and tables.

The problem considered in the 3 rd chapter is an extension of the problem considered in chapter 2. Here we have taken into account the effect of rotation on the flow.

Chapter 1

Available information regarding MHD heat and mass transfer flows

Chapter 1

Available information regarding MHD heat and mass transfer flows

In this chapter some fundamental topics related to Magnetohydrodynamics (MHD) and mass transfer flows, viz. fundamental equations of fluid dynamics, MHD approximations, MHD equations, dimensionless parameters, free convections, mass transfer, suction etc. have been presented.

1.1 Magnetohydrodynamics

Magnetohydrodynamics is that branch of continuum mechanics that deals with the flow of electrically conducting fluids in presence of electric and magnetic fields. Many natural phenomena and engineering problems are susceptible to MHD analysis.

Faraday (1832) carried out experiments with the flow of mercury in glass tubes placed between poles of a magnet, and discovered that a voltage was induced across the tube due to the motion of the mercury across the magnetic field, perpendicular to the direction of flow and to the magnetic field. He observed that the current generated by this induced voltage interacted with the magnetic field to slow down the motion of the fluid, and this current produced its own magnetic field that obeyed Ampere's right hand rule and thus, in turn distorted the magnetic field.

The first astronomical application of the MHD theory occurred in 1899 when Bigalow suggested that the sun was a gigantic magnetic system. Alfven (1942) discovered MHD waves in the sun. These waves are produced by disturbance that propagates simultaneously in the conducting fluid and the magnetic field.

The current trend for the application of magnetofluiddynamics is toward a strong magnetic field (so that the influence of the electromagnetic force is noticeable) and toward a low density of the gas (such as in space flight and in nuclear fusion research). Under these conditions the Hall current and ion slip current become important.

1.2 Electromagnetic Equations

Magnetohydrodynamic equations are the ordinary electromagnetic and hydrodynamic equations modified to take account of the interaction between the motion of the fluid and electromagnetic field. Formulation of the electromagnetic theory in mathematical form is know as Maxwell's equations. Maxwell's basic equations show the relation of basic field quantities and their production. The basic laws of electromagnetic theory are all contained in special theory of relativity. But here we will always assume that all velocities are small in comparison to the speed of light.

Before writing down the MHD equations we will first of all notice the ordinary electromagnetic equations and hydromagnetic equations (Cramer and Pai, 1973).

First, we give the electromagnetic equations:

Charge continuity	$\nabla.\mathbf{D} = \rho_e$	(1.1)
Current continuity	$\nabla.\mathbf{J} = -\,\partial\rho_e/\partial t$	(1.2)
Magnetic field continuity	$\nabla.\mathbf{B} = 0$	(1.3)
Ampere's law	$\nabla\Lambda\mathbf{H} = \mathbf{J} + \partial D/\partial t$	(1.4)
Faraday's law	$\nabla\Lambda\mathbf{E} = -\,\partial B/\partial t$	(1.5)
Constitutive equations for **D** and **B**	$\mathbf{D} = \epsilon'\mathbf{E}$	(1.6)
	$\mathbf{B} = \mu_e\mathbf{H}$	(1.7)
Lorentz force on a change	$\mathbf{F}_p = q'(\mathbf{E} + \mathbf{q}_p\,\Lambda\mathbf{B})$	(1.8)
Total current density flow	$\mathbf{J} = \sigma\,(\mathbf{E} + \mathbf{q}\Lambda\mathbf{B}) + \rho_e\,\mathbf{q}$	(1.9)

The equations (1.1)- (1.5) are the Maxwell's equations where **D** is the displacement current, ρ_e is the charge density, **J** is the current density, **B** is the magnetic induction, **H** is the magnetic field, **E** is the electric field, ϵ' is the electrical permeability of the medium, μ_e is the magnetic permeability of medium, $\mathbf{q}_p$ velocity of the charge, σ is the electrical conductivity, **q** is the velocity of the fluid and $\rho_e\mathbf{q}$ is the convection current due to charges moving with the fluid.

1.3 Fundamental Equations of Fluid Dynamics of Viscous Fluids

In the study of fluid flow one determines the velocity distribution as well as the state of the fluid over the whole space for all time. There are six unknowns namely, the three components (u, v, w) of velocity **q,** the temperature T, the pressure p and the density ρ of the fluid, which are functions of spatial co-ordinates and time. In order to determine these unknown we have the following equations:

(a) Equation of state, which connects the temperature, the pressure and the density of the fluid.

$$p = \rho RT \tag{1.10}$$

For an incompressible fluid the equation of state is simply

$$\rho = \text{constant} \tag{1.11}$$

(b) Equation of continuity, which gives relation of conservation of mass of the fluid. The equation of continuity for a viscous incompressible fluid is

$$\nabla.\mathbf{q} = 0 \tag{1.12}$$

(c) Equation of motion, also known as the Navier-Stokes equations, which give the relations of the conservation of momentum of the fluid. For a viscous incompressible fluid the equation of motion is

$$\rho D\mathbf{q}/Dt = \mathbf{F} - \nabla p + \mu\nabla^2\mathbf{q} \qquad (1.13)$$

where **F** is the body force per unit volume and the last term on the right hand side represents the force per unit volume due to viscous stresses and p is the pressure . The operator,

$$D/Dt \equiv \partial/\partial t + u\partial/\partial x + v\partial/\partial y + w\partial/\partial z$$

This is known as the material derivative or total derivative with respect to time, and it gives the variation of a certain quantity of the fluid particle with respect to time.

∇^2 is the Laplacian operator.

(d) The equation of energy, which gives the relation of conservation of energy of the fluid. For an incompressible fluid with constant viscosity and heat conductivity the energy equation is

$$\rho C_p DT/Dt = \partial Q/\partial t + k\nabla^2 T + \Phi \qquad (1.14)$$

C_p is the specific heat at constant pressure,

$\partial Q/\partial t$ is the rate of heat produced per unit volume by external agencies,

k is the thermal conductivity of the fluid,

Φ is the viscous dissipation function for an incompressible fluid

$$\Phi = 2\mu\,[(\partial u/\partial x)^2 + (\partial v/\partial y)^2 + (\partial w/\partial z)^2 + 1/2(Y^2_{xy} + Y^2_{yz} + Y^2_{zx})]$$

where

$$Y_{xy} = \partial u/\partial y + \partial v/\partial x$$

$$Y_{yz} = \partial v/\partial z + \partial w/\partial y$$

$$Y_{zx} = \partial w/\partial x + \partial u/\partial z$$

(e) The concentration equation for viscous incompressible fluid is

$$DC/Dt = D_M\nabla^2 C \qquad (1.15)$$

C is the concentration and

D_M is the chemical molecular diffusivity.

1.4 MHD Approximations

The electromagnetic equations as given in 1.1- 1.9 are not usually applied in their present form and require interpretation and several assumptions to provide the set to be used in MHD. In MHD we consider a fluid that is grossly neutral. The charge density ρ_e in Maxwell's equations must then be interpreted as an excess charge density which is generally not large. If we disregard the excess charge density then we must disregard the displacement current. In most problems the displacement current, the excess charge density and the current due to convection of the excess charge are small (Cramer and Pai,1973).

The electromagnetic equations to be used are then the following:

$$\nabla . \mathbf{D} = 0 \tag{1.16}$$

$$\nabla . \mathbf{J} = 0 \tag{1.17}$$

$$\nabla . \mathbf{B} = 0 \tag{1.18}$$

$$\nabla \Lambda \mathbf{H} = \mathbf{J} \tag{1.19}$$

$$\nabla \Lambda \mathbf{E} = - \partial B/\partial t \tag{1.20}$$

$$\mathbf{D} = \epsilon' \mathbf{E} \tag{1.21}$$

$$\mathbf{B} = \mu_e \mathbf{H} \tag{1.22}$$

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{q} \Lambda \mathbf{B}) \tag{1.23}$$

1.5 MHD Equations

We will now modify the equations of fluid dynamics suitably to take account of the electromagnetic phenomena.

(1) The MHD equation of continuity for viscous incompressible electrically conducting fluid remains the same

$$\nabla . \mathbf{q} = 0 \tag{1.24}$$

(2) The MHD momentum equation for a viscous incompressible and electrically conducting fluid is

$$\rho D\mathbf{q}/Dt = \mathbf{F} - \nabla P + \mu \nabla^2 \mathbf{q} + \mathbf{J} \Lambda \mathbf{B} \tag{1.25}$$

where **F** is the body force term per unit volume corresponding to the usual viscous fluid dynamics equations and the new term $\mathbf{J}\Lambda\mathbf{B}$ is the force on the fluid per unit volume produced by the interaction of the current and magnetic field (called a Lorentz force) ;

(3) The MHD energy equation for a viscous incompressible electrically conducting fluid is

$$\rho C_p DT/Dt = \partial Q/\partial t + k\nabla^2 T + \Phi + J^2/\sigma \qquad (1.26)$$

The new term J^2/σ is the Joule heating term and is due to the resistance of the fluid to the flow of current.

(4) The MHD equation of concentration for viscous incompressible electrically conducting fluid remains the same as

$$DC/Dt = D_M \nabla^2 C \qquad (1.27)$$

1.6 Some important dimensionless parameters of Fluid Dynamics and Magnetohydrodynamics

(1) Reynolds number, Re:

It is the most important parameter of the fluid dynamics of a viscous fluid. It represents the ratio of the inertia force to viscous force and is defined as

$$Re = \text{inertia force}/\text{viscous force} = \rho U^2 L^2/\mu UL = UL/\nu$$

where U, L, ρ and μ are the characteristic values of velocity, length, density and coefficient of viscosity of the fluid respectively. When the Reynolds number of the system is small the viscous force is predominant and the effect of viscosity is important in the whole velocity field. When the Reynolds number is large the inertia force is predominant, and the effects of viscosity is important only in a narrow region near the solid wall or other restricted region which is known as boundary layer. If the Reynolds number is enormously large, the flow becomes turbulent.

(2) Prandtl number, Pr:

The Prandtl number is the ratio of kinematic viscosity to thermal diffusivity and may be written as follows

$$Pr = (\text{kinematic viscosity})/(\text{thermal diffusivity}) = \nu / (k / \rho C_p)$$

The value of ν shows the effect of viscosity of the fluid. The smaller the value of ν the narrower is the region which is affected by viscosity and which is known as the boundary layer region when ν is very small. The value of $k/(\rho C_p)$ shows the thermal diffusivity due to heat conduction. The smaller the value of $k/(\rho C_p)$ is the narrower is the region which is affected by the heat conduction and which is known as thermal boundary layer when $k/(\rho C_p)$ is small. Thus the Prandtl number shows the relative importance of heat conduction and viscosity of a fluid. For a gas the Prandtl number is of order of unity.

(3) Schmidt number, Sc:

This is the ratio of the kinematic viscosity to the chemical molecular diffusivity and is defined as

$$Sc = \nu/D_M \quad = \text{(kinematic viscosity)/ (chemical molecular diffusivity)}$$

(4) Grashof number, Gr:

This is defined as

$$Gr = g_o\beta'(\Delta T)L^3/\nu^2$$

and is a measure of the relative importance of the buoyancy force and viscous force. The larger it is the stronger is the convective current. In the above g_o is the acceleration due to gravity, β' is the co-efficient of volume expansion and T is the temperature of the flow field.

(5) Modified Grashof number, Gm:

This is defined as

$$Gm = g_o\beta^*(\Delta C)L^3/\nu^2$$

where β^* is the co-efficient of expansion with concentration and C is the species concentration.

(6) Ekman number, R (rotation parameter):

This is the ratio of the viscous force and the Coriolis force and is defined as

$$R = \nu U / (L^2\,\Omega U)$$

where Ω is the rotational velocity.

(7) Soret number, So:

This is defined as

$$So = D_T\,(T_W - T_\infty) / (\nu(C_w - C_\infty))$$

where D_T is the thermal diffusivity, T_W is the temperature at the plate, T_∞ is the temperature of the fluid at infinity, C_w is the concentration at the plate and C_∞ is the species concentration at infinity.

(8) Magnetic force number, M:

This is obtained from the ratio of the magnetic force to the inertia force and is defined as

$$M = \mu_e\, B_0\, \sigma\, L / (\rho U)$$

where B_o is applied magnetic field.

1.7 Free Convection

In the studies related to heat transfer, considerable effort has been directed towards the convective mode, in which the relative motion of the fluid provides an additional mechanism for the transfer of energy and of material, the latter being a more important consideration in cases where mass transfer, due to a concentration difference, occurs. Convection is inevitably coupled with the conductive mechanisms, since, although the fluid motion modifies the transport process, the eventual transfer of energy from one fluid element to another in its neighborhood is through conduction. Also at the surface, the process is predominantly that of conduction because the relative fluid motion is brought to zero at the surface. A study of the convective heat transfer therefore involves the mechanisms of conduction and solutions, those of radiative processes as well, coupled with those of fluid flow. This makes the study of this mode of heat or mass transfer very complex. The convective mode of heat transfer is divided into two basic processes. If the motion of the fluid is caused by an external agent, such as the externally imposed, flow of a fluid stream over a heated object, the process is termed as forced convection. The fluid flow may be the result of, for instance, a fan, a blower, wind or the motion of the heated object itself. Such problems are very frequently encountered in technology where the heat transfers to or from a body is often due to an imposed flow of a fluid at a different temperature from that of the body. If on the other hand, no such externally induced flow is provided and flow arises naturally simply owing to the effect of a density difference, resulting from a temperature or concentration difference in a body force field, such as the gravitational field, the process is termed as natural convection. The density difference gives rise to buoyancy effects owing to which the flow is generated. A heated body cooling in ambient air generates such a flow in the region surrounding it. Similarly the buoyant flow arising from heat rejection to the atmosphere and to other ambient media, circulations arising in heated rooms, in the atmosphere, and in bodies of water, rise of buoyant flow to cause thermal stratification of the medium, as in temperature inversion, and many other such heat transfer processes in our natural environment, as well as in many technological applications, are included in the area, of natural convection. The flow may also arise owing to concentration differences such as those caused by salinity differences in the sea and by composition differences in chemical processing units, and these cause a natural convection mass transfer. Another classical natural convection problem is the thermal instability that occurs in a liquid heated from below. This subject is of natural interest to geophysicists and astrophysicists, although some applications might arise in boiling heat transfer.

The basic concepts involved in employing the boundary layer approximation to natural convection flows are very similar to those in forced flows. The main difference lies in the fact that the pressure in the region beyond the boundary layer is hydrostatic instead of being imposed by an external flow, and that the velocity outside the layer is zero. However, the basic treatment and analysis remain the same. The book by Schlichting (1968) is an excellent collection of information on the boundary layer analysis. There are several methods for the solution of boundary layer equations, namely the similarity variable method, the perturbation method, analytical method and numerical method etc., details of them are discussed by Rosenberg (1969), Gosman etal. (1969), Patanker and Spalding (1970), Spalding (1977) and Jaluria

(1980). With a parameter associated with the body shapes a similarity on the natural convection flow has also been studied by Pop and Takhar (1993).

In free convection a body forces term viz. $F_e = g_o \rho\beta' (T-T_o)$ appears in the equations of motion where g_o is the acceleration due to gravity, β' is the coefficient of thermal expansion and $T-T_o$ be the excess temperature of the heated parts of the fluid over the parts which remain cold. The nondimensional parameter characterizing free convection is known as Grashoff number and may be defined as

$$Gr = \nu g_o\beta'(T-T_o)/U_o^{\ 3}$$

where T_o is some representative temperature and U_o is some characteristic velocity.

The continuity and energy equations remain the same in cases of free and forced convection. In free convection flow we have a body force term in the momentum equation.

The two dimensional boundary layer momentum equation of MHD steady free convection flow in absence of pressure gradient is

$$u\partial u/\partial x+v\partial u/\partial y = \nu\, \partial^2u/\partial y^2+ g_o\beta(T - T\infty) - \sigma B_0^{\ 2} u/\rho \qquad (1.28)$$

where the flow is in the x direction and magnetic field is acting along y direction.

1.8 Mass Transfer

When a system contains one or more components whose concentration vary from point to point, there is a natural tendency for mass to be transferred, minimizing the concentration differences within the system. The transport of one constituent from a region of higher concentration to that of lower concentration is called mass transfer.

Combined heat and mass transfer problems (Jaluria 1980) are of importance in many processes and have therefore received a considerable amount of attention. In many mass transfer processes heat transfer consideration arise owing to chemical reaction and often due to the nature of the process. In processes such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. In many of these processes the interest lies in the determination of the total energy transfer, although in process such as drying, the interest lies mainly in the overall mass transfer for moisture removal. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, in many industrial applications involving solutions and mixtures in the absence of an externally induced flow and in many chemical processing systems.

Agrawal et al (1977,1980) have studied the combined buoyancy effect on the thermal and mass diffusion on MHD natural convection flows and it is observed that for the fixed Grashof number and Prandtl number the value of dimensionless length parameter decreases as the strength of the magnetic parameter increases. Georgantopoulos et al (1981) discussed the effects of free convection and mass transfer in a conducting liquid, when the fluid is subjected to a transverse magnetic field. Haldavenker and Soundalgekar (1977) studied the effects of mass transfer on free convective flow of an electrically conducting viscous fluid past an infinite porous plate with constant suction and transversely applied magnetic field. An exact analysis was made by Soundalgekar et al. (1979) of the effects of mass transfer and the free convection currents on the MHD Stokes (Rayleigh) problem for the flow of an electrically conducting in compressible viscous fluid past an impulsively started vertical plate under the action of a transversely applied magnetic field. The heat due to viscous dissipation and Joule heating were neglected.

1.9 Thermal Diffusion

Mass fluxes created by temperature gradients are called the Soret or thermal diffusion effect. In general, when the thermal-diffusion effects are of smaller order of magnitude then the effects are described by Fourier's or Fick's laws and is often neglected in heat and mass transfer process. There are, however, exceptions. The thermal-diffusion effect (commonly known as Soret effect), for instance, has been utilized for isotope separation and in mixtures between gases with very light molecular weight (He, H_2) and of medium molecular weight (N_2, air). The diffusion-thermo effect was found to be of such a magnitude that it cannot be neglected (Eckert and Drake, 1972). In view of the importance of this diffusion-thermo effect, Jha and Singh (1990) presented an analytical study for free convection and mass transfer flow for an infinite vertical plate moving impulsively in its own plane taking into account the Soret effect. Kafoussias (1992) studied the MHD free convection and mass transfer flow, past an infinite vertical plate moving on its own plane taking into account the thermal diffusion effect when (1) the boundary surface is impulsively started moving in its own plane (ISP) and (2) it is uniformly accelerated (UAP). For the ISP and UAP cases, it is seen that the effect of magnetic parameter M is to decrease the fluid (water) velocity inside the boundary layer. This influence of the magnetic field on the velocity field is more evident in the presence of thermal diffusion. It is also concluded that the fluid velocity rises due to general thermal diffusion. Hence, the velocity field is considerably affected by the magnetic field and the thermal diffusion. Nanousis (1992) extended the work of Kafoussias (1992) to the case of rotating fluid taken into account the Soret effect.

1.10 Hall Current

The electrical current density **J** represents the relative motion of charged particles in a fluid. The equation of electric current density may be derived in the flow direction and pressure from the diffusion velocities of the charged particles (Cramer and Pai, 1973; Hughes and Young, 1966; Pai, 1962; Shercliff, 1965). The major forces on charged particles are electromagnetic forces. If we consider only the electro-magnetic forces, we may obtain the generalized Ohm's law. However the deduction from the diffusion velocities of charged particles is more complicated than the generalized Ohm's law. When we apply an electric field **E**, there will be an electrical current in the direction of **E**. If the magnetic field **B** is perpendicular to **E**, there will be an electro magnetic force $\mathbf{J} \wedge \mathbf{B}$ which is perpendicular to both **E** and **B**. Such a force will cause the charged particles to move in the direction perpendicular to both **E** and **B**. We have a new component of electric current density in the direction perpendicular to both **E** and **B**, which is known as Hall current and which was first discovered by

Hall. Due to the presence of this current the efficiency of the MHD generator or accelerator is reduced. Cowling (1957) has discussed the generalized Ohm's law taking Hall current into account in the absence of electric field and is of the form

$$\mathbf{J} + \omega_e T_e (\mathbf{J} \wedge \mathbf{B})/B_o = \sigma(\mathbf{q} \wedge \mathbf{B} + \nabla p_e / en_e) \tag{1.29}$$

The generalized Ohm's law taking Hall current into account in presence of electric field should be written in the following form.

$$\mathbf{J} + \omega_e T_e (\mathbf{J} \wedge \mathbf{B})/B_o = \sigma(\mathbf{E} + \mathbf{q} \wedge \mathbf{B} + \nabla p_e / en_e) \tag{1.30}$$

where

- $\omega_e T_e$ = Hall parameter
- $\mathbf{E}$ = $(E_x, E_y, E_z,)$ the electric field,
- ω_e = the cyclotron frequencies of electron,
- T_e = the collision frequencies of electrons with ions,
- σ = the electrical conductivity of the fluid,
- $\mathbf{J}$ = (J_x, J_y, J_z) the current density,
- $\mathbf{B}$ = the magnetic field,
- B_o = the applied magnetic field,
- $\mathbf{q}$ = (u,v,w) velocity of the fluid,
- e = the electric charge,
- n_e = the number density of electrons,
- p_e = the electron pressure.

1.11 Ion slip current

The masses of ions and electrons are different. For the same electro magnetic force, the motion of ions is different from that of electrons. Usually the diffusion velocity of electrons is much larger than that of ions. As a first approximation, we may consider that the electric current density is determined mainly by the diffusion velocity of the electrons in a plasma. However when the electro magnetic force is very large (such as in the case of very strong magnetic field), the diffusion velocity of ions may not be negligible. If we include the diffusion velocity of ions as well as that of electrons, we have the phenomena of ion slip. When we include the Hall current, ion slip, and collisions between electrons and neutral particles, the generalized Ohm's law should be written in the following form;

$$\mathbf{J} + \beta_e (\mathbf{J} \wedge \mathbf{B})/B_o - \beta_e \beta_i (\mathbf{J} \wedge \mathbf{B}) \wedge \mathbf{B}/B_o^2 = \sigma(\mathbf{E} + \mathbf{q} \wedge \mathbf{B}) \tag{1.31}$$

where

- β_e = $\omega_e T_e$
- β_i = $\omega_i T_i$
- $\omega_i T_i$ = ion slip parameter,
- ω_i = the ion cyclotron frequencies,
- T_i = the collision frequencies of ion,
- ω_e = the cyclotron frequencies of electron,
- T_e = the collision frequencies of electrons with ions,

Let the induced magnetic field be negligible and $\mathbf{B} = (0, B_o, 0)$. The equation of conservation of electric charge $\nabla.\mathbf{J} = 0$ gives J_y =constant, where $\mathbf{J} = (J_x, J_y, J_z)$. If the plate is electrically non conducting this constant is zero and hence $J_y = 0$ every where within the flow.

Spliting (1.31) in its component form and then solving we get

$$J_z = \alpha [E_z + B_o u] \sigma - \beta [E_x - B_o w] \sigma \qquad (1.32)$$

$$J_x = \alpha [E_x - B_o w] \sigma + \beta [E_z + B_o u] \qquad (1.33)$$

where $$\alpha = (1+\beta_i\beta_e)/ ((1+\beta_i\beta_e)^2 + \beta_e^2) \qquad (1.34)$$

and $$\beta = \beta_e/((1+\beta_i\beta_e)^2 + \beta_e^2) \qquad (1.35)$$

1.12 Suction and Large Suction

For ordinary boundary layer flows with adverse pressure gradients, the flow will eventually separate from the surface. Separation of the flow causes many undesirable features over the whole field; for instance if separation occurs on the surface of an airfoil, the lift of the airfoil will decrease and the drag will enormously increase. In some problem we wish to maintain laminar flow without separation. Various means have been proposed to prevent the separation of boundary layer flows, suction is one of them. The retarded fluid in the boundary layer is sucked into the body.

The stabilizing effect of the boundary layer development has been well known for several years and till to date it is still the most of efficient, simple and common method of boundarycontrol. Hence the effect of suction on hydromagnetic boundary layer is of great interest in astrophysics. It is often necessary to prevent separation of the boundary layer to reduce the drag and attain high lift values.

Many authors have made mathematical studies on this problem, especially in the case of steady flow. Among them the name of Cobble (1977) may be cited who obtained the conditions under which similarity solution exists for hydromagnetic boundary layer flow past a semi-infinite flat plate with or without suction. Following this, Sundalgekar and Ramanamurthy(1980) analyzed the thermal boundary layer. For large values of suction velocity employing asymptotic analysis, in the spirit of Nanbu(1971), Bestman (1990) studied the boundary layer flow past a semi-infinite heated porous plate for two component plasma. Suction or blowing causes a double effect with respect to the heat transfer. The boundary layer suction was first applied by Prandtl (1904) in his fundamental works on boundary layer on a circular cylinder.

When the rate of suction is very high then it is called large suction. Singh (1985) studied the problem of Soundelgeker and Ramanamurthy (1980) for large value of suction parameter by making use of the perturbation technique, as has been done by Nanbu (1971). Later Singh and Dikshit (1988) studied the hydromagnetic flow past a continuously moving semi-infinite porous plate employing the same perturbation technique. They also derived similarity solutions for large suction. The large suction in fact enabled them to obtain analytical solutions and indeed these analytical solutions are of immense value that compliments various numerical solutions.

1.13 Effect of Rotation and Equation of Motion in a Rotating Co-ordinate system

In the last decade, considerable progress has been made in the general theory of rotating fluids because of its application in cosmic and geophysical sciences. The steady and unsteady Ekman layers of an incompressible fluid have been investigated as basic boundary layers in a rotating fluid appearing in the oceanic, atmospheric, cosmic fluid dynamics and solar physics or geophysical problems. It is well known that in a rotating fluid near a flat plate, an Ekman layer exists where the viscous and coriolis force are of the same order of magnitude. The Ekman layer flow on a horizontal plate has been studied by Batchelor (1970). The effect of a uniform transverse magnetic field on such a layer has been investigated by Gupta (1972). Mazumder et al. (1976a, b) have studied the flow and heat transfer in a hydromagnetic Ekman layer on a porous plate with Hall effects. Debnath (1974) has investigated the unsteady boundary layer flow in the semi-infinite expanse of an electrically conducting rotating viscous fluid bounded by an infinite non-conducting porous plate with uniform suction or blowing in the presence of a transverse uniform magnetic field.

If one takes a body of fluid and rotates its boundaries at a constant angular velocity $\mathbf{\Omega}$ then at any time sufficiently long after starting the rotation, the whole body will rotate with this angular velocity, moving as if it were a rigid body. There are no viscous stresses acting within the fluid. Any disturbance i.e. any thing that would produce a motion in a non-rotating system, will produce motion relative to this rigid body rotation. This relative motion can be considered as the flow pattern; it is the pattern that will be observed by an observer fixed to the rotating boundaries.

The effect of using a rotating frame of reference is well known form the mechanics of solid systems, where there are accelerations associated with the use of a non-inertial frame that can be taken into account by introducing centrifugal and Coriolis force. The statement may be expressed in a form appropriate to fluid system by

$$(D\mathbf{q}/Dt)_I = (D\mathbf{q}/Dt)_R + \mathbf{\Omega}\wedge(\mathbf{\Omega}\wedge\mathbf{r}) + 2\mathbf{\Omega}\wedge\mathbf{q}_R = -(1/\rho)\,\nabla P + \nu\nabla^2\mathbf{q} \qquad (1.36)$$

The subscripts I and R refer to inertial and rotating frames of reference. $(D\mathbf{q}/Dt\,)_I$ is thus the acceleration that the fluid particle is experiencing and so $\rho(D\mathbf{q}/Dt)_I$ is the quantity to be equated with the sum of the various forces acting on the fluid particle. $(D\mathbf{q}/Dt)_R$ is the acceleration relative to the rotating frame and can thus be expanded in the usual way

$$(D\mathbf{q}/Dt)_R = \partial\mathbf{q}_R/\partial t + (\mathbf{q}.\nabla)\mathbf{q}_R \qquad (1.37)$$

Dropping the subscript R as all velocities will be referred to the rotating frame the equation of motion is

$$\partial\mathbf{q}/\partial t + (\mathbf{q}.\nabla)\mathbf{q} = -(1/\rho)(\nabla p) - \mathbf{\Omega}\wedge(\mathbf{\Omega}\wedge\mathbf{r}) - 2\mathbf{\Omega}\wedge\mathbf{q} + \nu\nabla^2\mathbf{q} \qquad (1.38)$$

The second and third terms on the right hand side of equation (1.38) are respectively the centrifugal and Coriolis forces. In many problems the centrifugal force is unimportant. This is because it can be expressed as gradient of scalar quantity.

$$\mathbf{\Omega}\wedge(\mathbf{\Omega}\wedge\mathbf{r}) = -\nabla(1/2\Omega^2 r^2) \tag{1.39}$$

where r is the distance from the axis of rotation. Hence replacing pressure P considering

$$p - 1/2\,(\rho\,\Omega^2 r^2) = P \text{ (say)} \tag{1.40}$$

in the equation (1.38). Then the equation (1.38) reduces to

$$Dq / Dt = -(1/\rho)(\nabla P) - 2\mathbf{\Omega}\wedge \mathbf{q} + \nu\nabla^2\mathbf{q} \tag{1.41}$$

Two important dimensionless parameters appearing in rotating fluid are the Ekman number $E = \nu / (\Omega L^2)$ and the Rossby number $\varepsilon = U/ \Omega L$ where L is some characteristic length and U is some characteristic velocity.

1.14 : Relevant equations

We will consider the following equations to solve the relevant problem:

The continuity equation :

$$\partial u/\partial x+\partial v/\partial y = 0 \tag{1.42}$$

The momentum equation : Taking convection as a result of combined buoyancy effects of thermal and mass diffusion the equations of motion reduce to

$$u\partial u/\partial x+v\partial u/\partial x = \nu\partial^2 u/\partial y^2 + g_o\,\beta'(T-T\infty)+g_o\,\beta^*(C - C\infty) - B_o\,J_z/\rho \tag{1.43}$$

and $$u\,\partial w/\partial x+v\partial w/\partial y = \nu\,\partial^2 w/\partial y^2 + 1/\rho B_o\,J_x \tag{1.44}$$

with the values of J_x and J_z as follows

$$J_z = \alpha\,[E_z + B_o\,u]\,\sigma - \beta\,[E_x - B_o\,w]\,\sigma \tag{1.45}$$

$$J_x = \alpha\,[E_x - B_o\,w]\,\sigma + \beta\,[E_z + B_o\,u] \tag{1.46}$$

where $$\alpha = (1+\beta_i\beta_e)/ ((1+\beta_i\beta_e)^2 + \beta_e^2) \tag{1.47}$$

and $$\beta = \beta_e/((1+\beta_i\beta_e)^2 + \beta_e^2) \tag{1.48}$$

The energy equation :

$$u\partial T/\partial x + v\partial T/\partial y = k/(\rho C_p)\partial^2 T/\partial y^2 \tag{1.49}$$

The concentration equation : Taking thermal diffusion into account the concentration equation reduces to

$$u\,\partial C/\partial x + v\,\partial C/\partial y = D_M\,\partial^2 C/\partial y^2 + D_T\,\partial^2 T/\partial y^2 \tag{1.50}$$

Chapter 2

Steady *MHD* free convection and mass transfer flow with thermal diffusion, Hall current, ion slip current and large suction.

Chapter 2

Steady *MHD* free convection and mass transfer flow with thermal diffusion, Hall current, ion slip current and large suction.

Hall effect on unsteady MHD free convection and mass transfer flow near an infinite vertical porous plate with variable suction has been studied by Sattar and Alam (1995). When the conducting fluid is a partially ionized gas e.g. water gas seeded with potassium the effect of Hall and ion slip currents are also significant. Ram and Thakar (1993) studied the effect of Hall and ion slip currents on MHD free convection flow in a rotating fluid. They used the finite difference method to solve the problem. Steady MHD free convection and mass transfer flow with thermal diffusion and large suction, both in fixed and rotating system, was demonstrated by Alam and Sattar (1999, 2001). They have employed the perturbation technique based on large suction, as has been demonstrated by Singh and Dikshit (1988) and Bestman (1990), to obtain the similarity solutions of the governing equations of their problem. It may be mentioned that Alam and Satter have neglected the effect of Hall current and ion slip current.

In view of the above studies in the present work we have studied the MHD free convection and mass transfer flow with thermal diffusion, Hall and ion slip currents and large suction of a viscous incompressible electrically conducting partially ionized fluid past an impulsively started infinite vertical plate.

2.1. Governing Equations

For this problem let us consider a steady free convection and mass transfer flow of a viscous incompressible and electrically conducting partially ionized fluid past an impulsively started semi-infinite vertical porous plate with large suction under the influence of a transversely applied magnetic field. Let the x and y-axis be along and normal to the plate respectively. Let u and v be the velocity components along x and y axis respectively. Initially the plate as well as the fluid was at rest and the temperature of the fluid and the plate were also same. The plate temperature and the fluid concentration are instantly raised from T_∞ and C_∞ to T_w and $C_w(x)$ respectively, where T_∞ and C_∞ are the temperature and concentration of the uniform flow. A uniform strong magnetic field of magnitude B_o is taken to be acting along the y axis as shown in Fig 1. The induced magnetic field is assumed to be negligible so that $\mathbf{B} = (0, B_o, 0)$ where B_o is the constant transversely applied magnetic field. If $\mathbf{J} = (J_x, J_y, J_z)$ is the current density, the equation of conservation of electric charge $\nabla.\mathbf{J}= 0$ gives J_y =constant. Since the plate is electrically non conducting, this constant is zero and hence, J_y =0 every where with in the flow.

When the electromagnetic force is very large (such as in the case of very strong magnetic field), the diffusion velocity of ions may not be negligible. If we include the diffusion velocity of ions as well as that of electrons, we have the phenomenon of ion

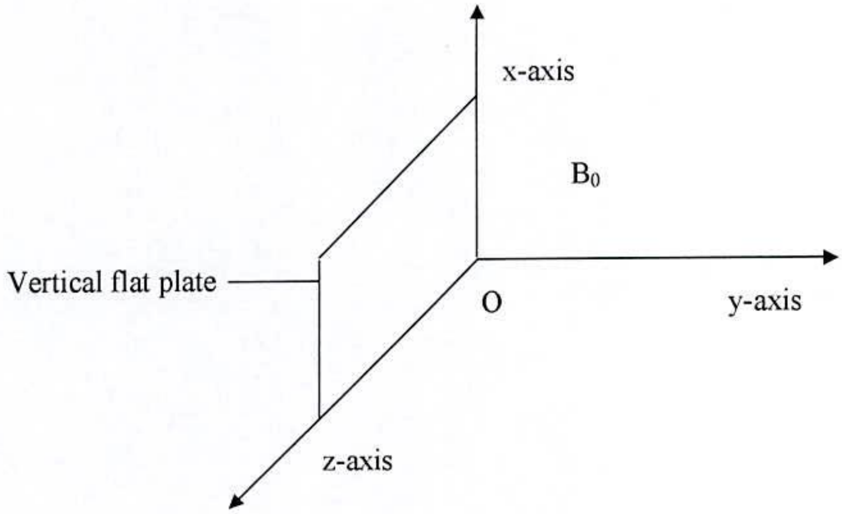

Fig 2. 1 The flow configuration with the coordinate

slip. When we include the Hall current, ion slip current and collisions between electrons and neutral particles, the generalized Ohm's law should be written in the following form

$$\mathbf{J} + \beta_e (\mathbf{J} \wedge \mathbf{B})/B_o - \beta_e \beta_i (\mathbf{J} \wedge \mathbf{B}) \wedge \mathbf{B}/B_o^2 = \sigma(\mathbf{E} + \mathbf{q} \wedge \mathbf{B}) \tag{2.1}$$

where $\beta_e = \omega_e T_e$ and $\beta_i = \omega_i T_i$

$\omega_e T_e$ = Hall parameter
$\omega_i T_i$ = ion slip parameter
$\mathbf{E}$ = $(E_x, E_y, E_z,)$ the electric field,
ω_e = the electron cyclotron frequencies,
ω_i = the ion cyclotron frequencies,
T_e = the collision frequencies of electron,
T_i = the collision frequencies of ion,
σ = the electrical conductivity of the fluid,
q = (u,v,w) fluid velocity,
$\mathbf{J}$ = (J_x, J_y, J_z) the current density,
$\mathbf{B}$ = the magnetic induction,
B_o = the applied magnetic field.

Spliting (2.1) in its component form and then solving we get

$$J_z = \alpha [E_z + B_o u] \sigma - \beta [E_x - B_o w] \sigma \tag{2.2}$$

$$J_x = \alpha [E_x - B_o w] \sigma + \beta [E_z + B_o u] \tag{2.3}$$

where $$\alpha = (1+\beta_i\beta_e)/ ((1+\beta_i\beta_e)^2 + \beta_e^2) \tag{2.4}$$

and $$\beta = \beta_e/((1+\beta_i\beta_e)^2 + \beta_e^2) \tag{2.5}$$

The basic equations relevant to the problem are:

continuity equation

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{2.6}$$

momentum equations

$$u\partial u/\partial x + v\partial u/\partial y = \nu\partial^2 u/\partial y^2 + g_o\, \beta'(T-T_\infty) + g_o\, \beta^* (C - C_\infty) - B_o J_z/\rho \tag{2.7}$$

$$u\partial w/\partial x + v\partial w/\partial y = \nu\partial^2 w/\partial y^2 + B_o J_x/\rho \tag{2.8}$$

energy equation

$$u\partial T/\partial x + v\partial T/\partial y = (k/\rho C_p)\, \partial^2 T/\partial y^2 \tag{2.9}$$

concentration equation

$$u\partial C/\partial x + v\partial C/\partial y = D_M\, \partial^2 C/\partial y^2 + D_T\, \partial^2 T/\partial y^2 \tag{2.10}$$

where g_o is the acceleration due to gravity, β' is the co-efficient of volume expansion, β^* is the co efficient of expansion with concentration, ρ is the density, T is the temperature of the flow field, T_∞ is the temperature of the fluid at infinity, C is the species concentration, C_∞ is the species concentration at infinity, D_M is the molecular diffusivity and D_T is the thermal diffusivity.

The boundary conditions for the problem are:

$$\begin{aligned} &u = U_o,\ v = v_o(x),\ w = 0,\ T = T_w,\ C = C_w(x) \text{ at } y = 0 \\ &u = 0,\ v = 0,\ w = 0,\ T = T_\infty,\ C = C_\infty \text{ as } y \to \infty \end{aligned} \tag{2.11}$$

where U_o is the uniform velocity and $v_o(x)$ is the velocity of suction at the plate and $C_w(x)$ is the variable concentration at the plate. Now for reasons of similarity the plate concentration $C_w(x)$ is taken to be

$$C_w(x) = C_\infty + (C_o - C_\infty)\ \bar{x} \tag{2.12}$$

where $\bar{x} = xU_o/\nu$ and C_o is the mean concentration.

2.2. Mathematical formulation

Since our main goal is to attain similarity solutions and to achieve that we have introduced the following similarity variables:

$$\left.\begin{aligned} &\eta = y\sqrt{(U_o/(2\nu x))} \\ &f'(\eta) = u/U_o \\ &g(\eta) = w/U_o \\ &\theta(\eta) = (T-T_\infty)/(T_w-T_\infty) \\ &\phi(\eta) = (C-C_\infty)/(\bar{x}(C_o - C_\infty)) \end{aligned}\right| \tag{2.13}$$

Now in terms of (2.13) the equation (2.6) can be integrated to

$$v = \sqrt{(\nu U_o / (2x))}\,(\eta f' - f) \tag{2.14}$$

With the aid of relations in (2.12), (2.13) and (2.14) the following dimensionless parameters are defined

$Gr = g_o\, \beta'(T_w - T_\infty)\, 2x/U_0^{\,2}$, $\quad Gm = g_o\, \beta^*\,(C_o - C_\infty)\, 2x^2/(\nu U_o)$,

$N_1' = 2x\,(B_o\, \alpha\sigma E_z)/(U_o^{\,2}\rho)$, $\quad N_2' = 2x\,(B_o\, \sigma\beta E_x)/(U_o^{\,2}\rho)$

$M = 2xB_o^2\sigma/(\rho U_o)$, $\quad M_1 = \alpha M$, $\quad M_2 = \beta M$,

$N_1 = 2x\,(B_o\, \alpha\sigma E_x)/(U_o^2\rho)$, $\quad N_2 = 2x\,(B_o\, \beta\sigma E_z)/(U_o^2\rho)$

$Pr = \rho\nu C_p/k$, $Sc = \nu/D_M$, $\quad So = (D_T\,(T_w - T_\infty))/(xU_o\,(C_w - C_\infty))$

The equations (2.7) and (2.8), thus reduce to the following ordinary differential equations

$$f''' + ff'' - M_1 f' + Gr\theta + Gm\phi - M_2 g - N_1' + N_2' = 0 \tag{2.15}$$

$$g'' + g'f - M_1 g + M_2 f' + N_1 + N_2 = 0 \tag{2.16}$$

where Gr is the local Grashof number, Gm is the local modified Grashof number, M is the local magnetic force number, Pr is the Prandtl number, Sc is the Schmidt number and So is the local Soret number.

We now consider further the case of short circuit problem in which the applied electric field $\mathbf{E} = \mathbf{0}$. Now for this value $N_1 = 0$, $N_2 = 0$, $N_1' = 0$ and $N_2' = 0$. Then the equation (2.15) and (2.16) reduces to

$$f''' + ff'' - M_1 f' - M_2 g + Gr\theta + Gm\phi = 0 \tag{2.17}$$

$$g'' + g'f + f'M_2 - M_1 g = 0 \tag{2.18}$$

Using the similarity variables and dimensionless parameters the equations (2.9) and (2.10), reduce to the following differential equations

$$\theta'' + Prf\theta' = 0 \tag{2.19}$$

$$\phi'' - 2Scf'\phi + Scf\phi' + So\,Sc\,\theta'' = 0 \tag{2.20}$$

Subject to the above formulations the boundary conditions (2.11) now transform to

$$\begin{aligned} &f = f_w,\ f' = 1,\ g = 0,\ \theta = 1,\ \phi = 1,\ \text{at}\ \eta = 0 \\ &f' = 0,\ g = 0,\ \theta = 0,\ \phi = 0\ \text{as}\ \eta \to \infty \end{aligned} \tag{2.21}$$

where $f_w = -\, v_o\,(x)\sqrt{(2x/(\nu U_o))}$ is the transpiration parameter and primes denote derivative with respect to η. Here $f_w > 0$ indicates the suction and $f_w < 0$ the injection. The solution of the equations (2.17) – (2.20) subject to the boundary conditions (2.21) are now sought and are presented in the following sub section.

2. 3. Solution

Since the solutions are sought for large suction, the further transformations are made as

$$\left.\begin{aligned} &\zeta = \eta f_w, \qquad f(\eta) = f_w\, F(\zeta), \qquad g(\eta) = f_w^2 G(\zeta), \\ &\theta(\eta) = f_w^{\,2}\, H(\zeta), \qquad \phi(\eta) = f_w^{\,2}\, P(\zeta) \end{aligned}\right| \tag{2.22}$$

Substituting (2.22) in equations (2.17) – (2.20) we have

$$F''' + FF'' + \epsilon\,\{-\,M_1F' - M_2G + Gr\,H + Gm\,P\} = 0 \tag{2.23}$$

$$G'' + G'F + \epsilon\,\{F'M_2 - M_1G\} = 0 \tag{2.24}$$

$$H'' + Pr\,FH' = 0 \tag{2.25}$$

$$P'' - 2\,Sc\,F'P + Sc\,F\,P' + So\,Sc\,H'' = 0 \tag{2.26}$$

where $\epsilon = 1/f_w^{\,2}$

Also the boundary conditions (2.21) transform to

$$\left.\begin{aligned} &F = 1,\ F' = \epsilon,\ G = 0,\ H = \epsilon,\ P = \epsilon \quad \text{at} \quad \zeta = 0 \\ &F' = 0,\ G = 0,\ H = 0,\ P = 0 \quad \text{as} \quad \zeta \to \infty \end{aligned}\right| \tag{2.27}$$

Now for large suction $f_w >> 1$, so that ϵ is very small, therefore, following Bestman (1990) F, G, H and P can be expanded in terms of small perturbation quantity ϵ as

$$F(\zeta) = 1 + \epsilon F_1(\zeta) + \epsilon^2 F_2(\zeta) + \epsilon^3 F_3(\zeta) + \dots\dots\dots\dots \tag{2.28}$$

$$G(\zeta) = \epsilon\, G_1(\zeta) + \epsilon^2 G_2(\zeta) + \epsilon^3 G_3(\zeta) + \dots\dots\dots\dots \tag{2.29}$$

$$H(\zeta) = \epsilon H_1(\zeta) + \epsilon^2 H_2(\zeta) + \epsilon^3 H_3(\zeta) + \dots\dots\dots\dots \tag{2.30}$$

$$P(\zeta) = \epsilon P_1(\zeta) + \epsilon^2 P_2(\zeta) + \epsilon^3 P_3(\zeta) + \dots\dots\dots\dots \tag{2.31}$$

Then substituting $F(\zeta)$, $G(\zeta)$, $H(\zeta)$ and $P(\zeta)$ from (2.28) – (2.31) in the equations(2.23) – (2.26), we have the following set of ordinary differential equations with respective boundary conditions for $F_i(\zeta)$, $G_i(\zeta)$, $H_i(\zeta)$ and $P_i(\zeta)$ (i= 1,2,3,--------) as:

For the first order i.e. O (ϵ):

$$F_1'''+F_1''=0 \tag{2.32}$$

$$G_1''+G_1'=0 \tag{2.33}$$

$$H_1''+Pr\, H_1'=0 \tag{2.34}$$

$$P_1''+Sc\, P_1'+So\, Sc\, H_1''=0 \tag{2.35}$$

$$\left.\begin{array}{l} F_1=0,\ F_1'=1,\ G_1=0,\ H_1=1,\ P_1=1,\ \text{at}\ \zeta=0 \\ F_1'=0,\ G_1=0,\ H_1=0,\ P_1=0\ \text{as}\ \zeta\to\infty \end{array}\right| \tag{2.36}$$

For the second order i. e. O (ϵ^2):

$$F_2'''+F_2''+F_1F_1''-M_1F_1'-M_2G_1+Gr\,H_1+Gm\,P_1=0 \tag{2.37}$$

$$G_2''+G_2'+F_1G_1'+M_2F_1'-M_1G_1=0 \tag{2.38}$$

$$H_2''+PrH_2'+PrF_1H_1'=0 \tag{2.39}$$

$$P_2''-2Sc\,P_1F_1'+ScP_2'+ScF_1P_1'+So\,Sc\,H_2''=0 \tag{2.40}$$

$$\left.\begin{array}{l} F_2=0,\ F_2'=0,\ G_2=0,\ H_2=0,\ P_2=0\ \text{at}\ \zeta=0 \\ F_2'=0,\ G_2=0,\ H_2=0,\ P_2=0\quad \text{as}\ \zeta\to\infty \end{array}\right| \tag{2.41}$$

For the third order i. e. O (ϵ^3);

$$F_3'''+F_3''+F_1F_2''+F_2F_1''-M_1F_2'-M_2G_2+GrH_2+GmP_2=0 \tag{2.42}$$

$$G_3''+G_3'+F_1G_2'+F_2G_1'+M_2F_2'-M_1G_2=0 \tag{2.43}$$

$$H_3''+PrF_1H_2'+P_rH_3'+PrF_2H_1'=0 \tag{2.44}$$

$$P_3''-2ScP_2F_1'-2ScP_1F_2'+ScP_3'+ScF_1P_2'+ScF_2P_1'+ScSoH_3''=0 \tag{2.45}$$

$$\left.\begin{array}{l} F_3=0,\ F_3'=0,\ G_3=0,\ H_3=0,\ P_3=0,\ \text{at}\ \zeta=0 \\ F_3'=0,\ G_3=0,\ H_3=0,\ P_3=0\ \text{as}\ \zeta\to\infty \end{array}\right| \tag{2.46}$$

etc.

The solutions of the above equations up to order 3 under the prescribed boundary conditions are obtained in a straightforward manner and are

$$F_1=1-e^{-\zeta} \tag{2.47}$$

$$G_1 = 0 \tag{2.48}$$

$$H_1 = e^{-Pr\,\zeta} \tag{2.49}$$

$$P_1 = A_1\, e^{-Pr\,\zeta} + A_2 e^{-Sc\,\zeta} \tag{2.50}$$

$$F_2 = A_7 + A_6 e^{-\zeta} + A_3 \zeta\, e^{-\zeta} + 0.25 e^{-2\zeta} + A_4\, e^{-Pr\,\zeta} + A_5\, e^{-Sc\,\zeta} \tag{2.51}$$

$$G_2 = M_2 \zeta\, e^{-\zeta} \tag{2.52}$$

$$H_2 = A_8\, e^{-Pr\,\zeta} - \zeta Pr\, e^{-Pr\,\zeta} - A_8 e^{-\zeta(1+Pr)} \tag{2.53}$$

$$P_2 = A_9\, e^{-\zeta(1+Pr)} + A_{10}\, e^{-\zeta(1+Sc)} + A_{11}\, e^{-Pr\,\zeta} - A_{12}\, \zeta e^{-Sc\,\zeta} + A_{13} \zeta e^{-Pr\,\zeta}$$
$$+ A_{14}\, e^{-Sc\,\zeta} \tag{2.54}$$

$$F_3 = -\,0.07 e^{-3\zeta} - 0.5 A_3\, \zeta\, e^{-2\zeta} - A_3\, e^{-2\zeta} - 0.5 A_{15} \zeta^2\, e^{-\zeta} + 0.25 A_{16}\, e^{-2\zeta} + A_{17} \zeta\; e^{-Sc\,\zeta}$$
$$+ A_{18} \zeta\, e^{-Pr\,\zeta} - A_{19}\, e^{-\zeta(1+Sc)} - A_{20}\, e^{-\zeta(1+Pr)} + A_{21}\, e^{-\zeta} + A_{22} + A_{23}\, e^{-Sc\,\zeta}$$
$$+ A_{24} e^{-Pr\,\zeta} + A_{25} \zeta\, e^{-\zeta} \tag{2.55}$$

$$G_3 = -\,0.5 M_2 \zeta\, e^{-2\zeta} + 0.5 A_{26} \zeta^2\, e^{-\zeta} + A_{27} \zeta\, e^{-\zeta} + A_{28}\, e^{-Pr\,\zeta}$$
$$+ A_{29}\, e^{-Sc\,\zeta} + A_{30} e^{-\zeta} \tag{2.56}$$

$$H_3 = (A_{31} + Pr) \zeta\, e^{-Pr\,\zeta} + A_{32}\, e^{-\zeta(2+Pr)} - A_{34}\, e^{-2Pr\zeta} + A_{35}\, \zeta e^{-\zeta(1+Pr)} + A_{36}\, e^{-\zeta(1+Pr)}$$
$$+ A_{37}\, e^{-Pr\,\zeta} + 0.5 Pr^2 \zeta^2\, e^{-Pr\zeta} + A_5 Pr^2\, e^{-\zeta(Sc+Pr)}/(Sc^2 + ScPr) \tag{2.57}$$

$$P_3 = A_{38} e^{-\zeta(2+Pr)} + A_{39} e^{-\zeta(2+Sc)} + A_{40} e^{-2Pr\zeta} + A_{41} e^{-\zeta(Pr+Sc)} + A_{42} e^{-2Sc\zeta} + A_{43} \zeta e^{-\zeta(1+Sc)}$$
$$+ A_{44}\, \zeta e^{-\zeta(1+Pr)} + A_{45} \zeta^2 e^{-Sc\zeta} + A_{46}\, \zeta^2 e^{-Pr\zeta} + A_{47} e^{-\zeta(1+Pr)} + A_{48} e^{-\zeta(1+Sc)} + A_{49} e^{-Pr\,\zeta}$$
$$+ A_{50} \zeta e^{-Sc\,\zeta} + A_{51} \zeta e^{-Pr\,\zeta} + A_{52} e^{-Sc\,\zeta} \tag{2.58}$$

where the constants A_i (where i=1, 2, 3 ------- 52) are shown in appendix 2.A. The velocity, the temperature and the concentration fields are thus obtained from (2.28) to (2.31) as

$$u/U_o = f'(\eta) = F_1' + \boldsymbol{\epsilon}\, F_2' + \boldsymbol{\epsilon}^2\, F_3' \tag{2.59}$$

$$w/U_o = g(\eta) = G_1 + \boldsymbol{\epsilon}\, G_2 + \boldsymbol{\epsilon}^2 G_3 \tag{2.60}$$

$$\theta(\eta) = H_1 + \boldsymbol{\epsilon} H_2 + \boldsymbol{\epsilon}^2 H_3 \tag{2.61}$$

$$\phi(\eta) = P_1 + \boldsymbol{\epsilon}\, P_2 + \boldsymbol{\epsilon}^2 P_3 \tag{2.62}$$

Thus with the help of the solutions (2.47) – (2.58) the velocity, temperature and concentration distributions can be calculated out from (2.59) – (2.62). The velocity and temperature distributions are shown in Figs 2.2 – 2.14.

2.4. Skin – friction coefficient, Nusselt number and Sherwood number

The quantities of chief physical interest are the local skin friction coefficient, local Nusselt number and local Sherwood number.
The equations defining the wall skin friction are

$$\tau_x = \mu(\partial u/\partial y)_{y=0}$$

$$\tau_z = \mu(\partial w/\partial y)_{y=0}$$

Thus from equations (2.59) and (2.60) we have

$$\tau_x \propto f''(0)$$

$$\tau_z \propto g'(0)$$

and in turn we have

$$f''(0) = -1 + \boldsymbol{\epsilon}\,[1 - 2A_3 + A_4 Pr^2 + A_5 Sc^2 + A_6] + \boldsymbol{\epsilon}^2\,[-5/8 - 2A_3 - A_{15} + A_{16} - 2A_{17}Sc - 2A_{18}Pr - A_{19}(Sc+1)^2 - A_{20}(Pr+1)^2 + A_{21} + A_{23}Sc^2 + A_{24}Pr^2 - 2A_{25}] \quad (2.63)$$

and

$$g'(0) = \boldsymbol{\epsilon} M_2 + \boldsymbol{\epsilon}^2[-0.5M_2 + A_{27} - A_{28}Pr - A_{29}Sc - A_{30}] \quad (2.64)$$

The local Nusselt number denoted by Nu is proportional to $-(\partial T/\partial y)_{y=0}$, hence we have from (2.61)

$$Nu \propto -\theta'(0)$$

and in turn we have

$$\theta'(0) = -Pr + \boldsymbol{\epsilon}[-A_8 Pr - Pr + A_8(Pr+1)] \quad (2.65)$$

The local Sherwood number denoted by Sh is proportional to $-(\partial C/\partial y)_{y=0}$, hence we have from (2.62)

$$Sh \propto -\phi'(0)$$

and in turn we have

$$\phi'(0) = -A_1 Pr - A_2 Sc + \epsilon[-A_9(Pr+1) - A_{10}(Sc+1) - A_{11}Pr - A_{12} + A_{13} - A_{14}Sc] \quad (2.66)$$

The values proportional to the skin friction coefficient, local Nusselt number and local Sherwood number are respectively obtained from (2.63) - (2.66). These values are sorted in tables 2. 1 – 2.4.

2.5. Results and Discussions

For the purpose of discussing the results some numerical calculations are carried out for non-dimensional primary ($f'(\eta)$) and secondary ($g(\eta)$) velocities. The velocity profiles for the x and z components of velocity are shown in Figs2.2 – 2.14 for different values of β_i, β_e, Gr, Gm, So and f_w for fixed values of M, Pr and Sc. The value of M is taken to be large which corresponds to a strong magnetic field, that is generally encountered in nuclear engineering in connection with the cooling of reactors. Negative values of Gr which indicates the heating of the plate are also taken into account. The value of Sc is taken to be 0.6 which corresponds to water vapor, Pr is taken equal to 0.71 which corresponds to its value in case of air. The magnetic force number M is chosen arbitrarily being equal to 5.0 which imply a strong magnetic field. $Gr < 0$ corresponds to heating of the plate by free convection currents. We have presented the non dimensional velocity components represented by $f'$and g for an impulsive flow in figures 2.2 -2.13 respectively.

Figure 2.2 represents the velocity profiles of primary velocity f' for different β_e with fixed Pr, Sc, So, β_i, f_w and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increase in β_e produced rapid increase in f'. For negative Gr and Gm, f' is positive and with the increase of β_e, f' decreases rapidly.

Figure 2.3 represents the velocity profiles of secondary velocity g for different β_e with fixed Pr, Sc, So, f_w, β_i and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increase in β_e produced rapid increase in g . For negative Gr and Gm, g is negative and with the increase of β_e, g decreases rapidly.

Figure 2.4 represents the velocity profiles of primary velocity f' for different β_i with fixed Pr, Sc, So, β_e, f_w and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increase in β_i produced slow increase in f'. For negative Gr and Gm, f' is positive and with the increase of β_e, f' slowly decreases. It may be noted that the sensitivity of f' is more with respect to β_e than β_i.

Figure 2.5 represents the velocity profiles of secondary velocity g for different β_i with fixed Pr, Sc, So, f_w, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increase in β_i produced rapid decrease in g. For negative of Gr and Gm, g is negative and with the increase of β_i, g increases rapidly.

Figure 2.6 represents the velocity profiles of primary velocity f' for different So with fixed Pr, Sc, β_i, β_e, f_w and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increase in So produced slow increase in f'. For negative Gr and Gm, f' is positive and with the increase of So, f' slowly decreases.

Figure 2.7 represents the velocity profiles of secondary velocity g for different So with fixed Pr, Sc, f_w, β_i, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increase in So produced rapid increases in g. For negative Gr and Gm, g is negative and with the increase of So, g decreases rapidly.

Figure 2.8 represents the velocity profiles of primary velocity f' for different f_w with fixed Pr, Sc, So, β_i, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increase in f_w produced rapid increase in f'. For negative Gr and Gm, f' is positive and with the increase of f_w, f' decreases rapidly. It may be noted that the sensitivity of f' is more with respect to f_w than So.

Figure 2.9 represents the velocity profiles of secondary velocity g for different f_w with fixed Pr, Sc, β_i, So, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increases in f_w produced rapid decrease in g. For negative Gr and Gm, g is negative and with the increase of f_w, g increases rapidly.

Figure 2.10 represents the velocity profiles of primary velocity f' for different values of Gr with fixed Pr, Sc, f_w, M, So, β_i and β_e along with fixed Gm (either positive or negative). For positive Gm, considering Gr as positive it is found that f' is negative and with the increase in Gr, f' decreases. It is also observed that for negative Gm and considering Gr as negative, f' is positive and with the decrease in Gr, f' increases.

Figure 2.11 represents the velocity profiles of secondary velocity g for different values of Gr with fixed Pr, Sc, f_w, M, So, β_i and β_e along with fixed Gm (either positive or negative). For positive Gm, considering Gr as positive it is found that g is pogitive and with the increase in Gr, g increases. It is also observe that for negative Gm and considering Gr as negative g is negative and with the decrease in Gr, g decreases.

Figure 2.12 represents the velocity profiles of primary velocity f' for different values of Gm with fixed Pr, Sc, f_w, M, So, β_i and β_e along with fixed Gr (either positive or negative). For positive Gr, considering Gm as positive it is found that f' is negative and with the increase in Gm, f' decreases. It is also observe that for negative Gr and considering Gm as negative f' is positive and with the decreases in Gm, f' increases.

Figure 2.13 represents the velocity profiles of secondary velocity g for different values of Gm with fixed Pr, Sc, f_w, M, So, β_i and β_e along with fixed Gr (either positive or negative). For positive Gr, considering Gm as positive it is found that g is positive and with the increase in Gm, g increases rapidly. It is also observed that for negative Gr and considering Gm as negative g is negative and with the decrease in Gm, g decreases rapidly. It may be noted that the sensitivity of g is more with respect to Gm than Gr.

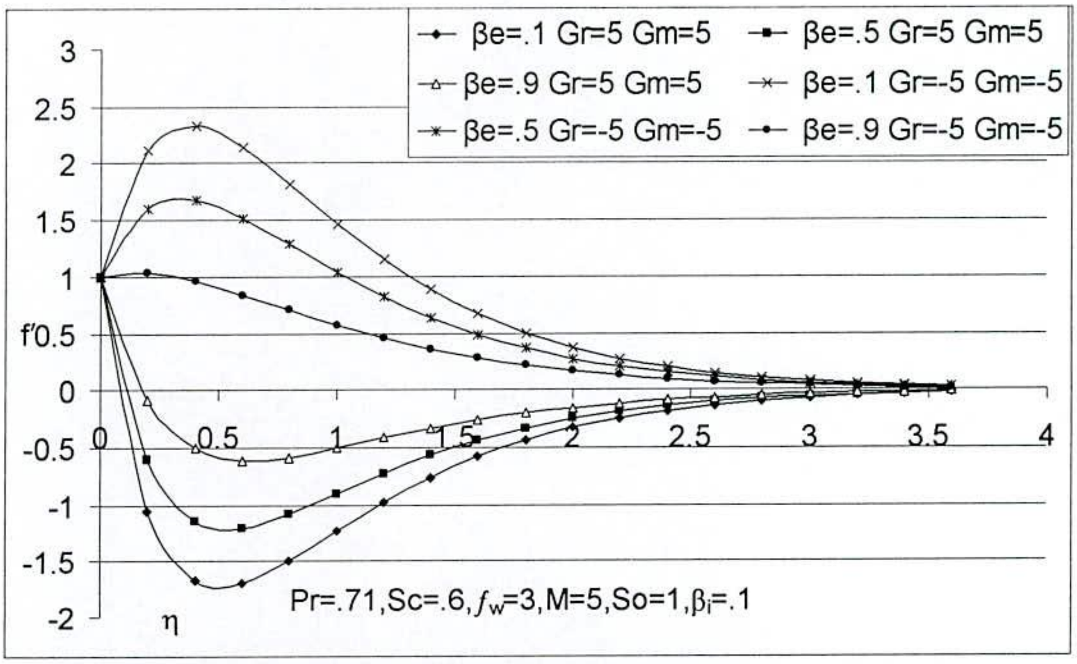

Fig.2.2.Primary velocity profiles due to cooling and heating of the plate for different values of βe

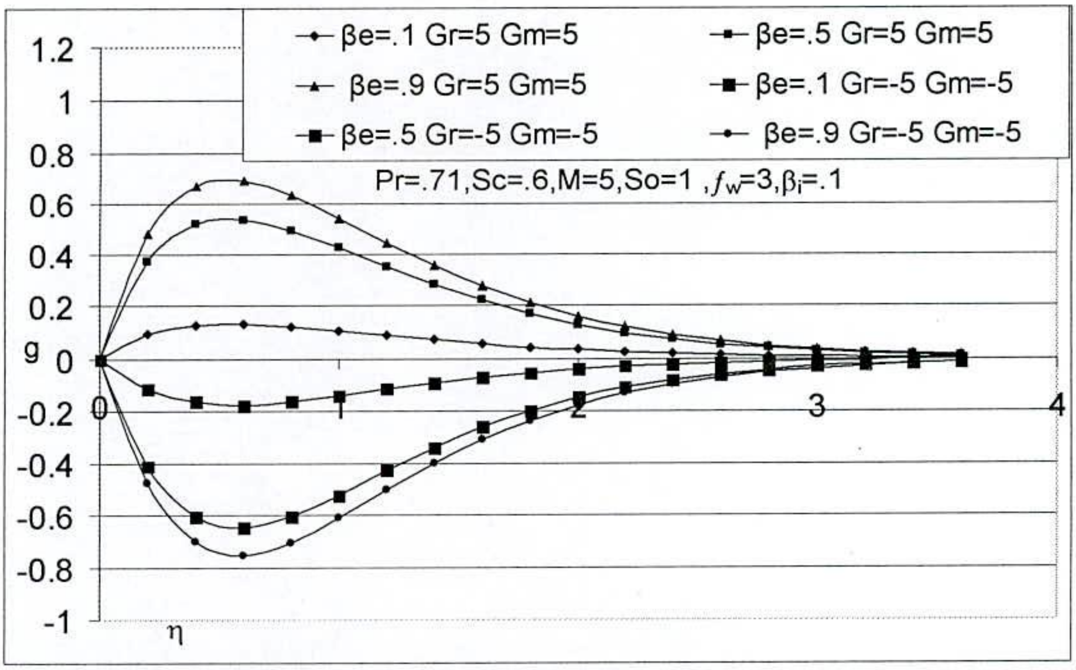

Fig.2.3.Secondary velocity profiles due to cooling and heating of the plate for different values of βe

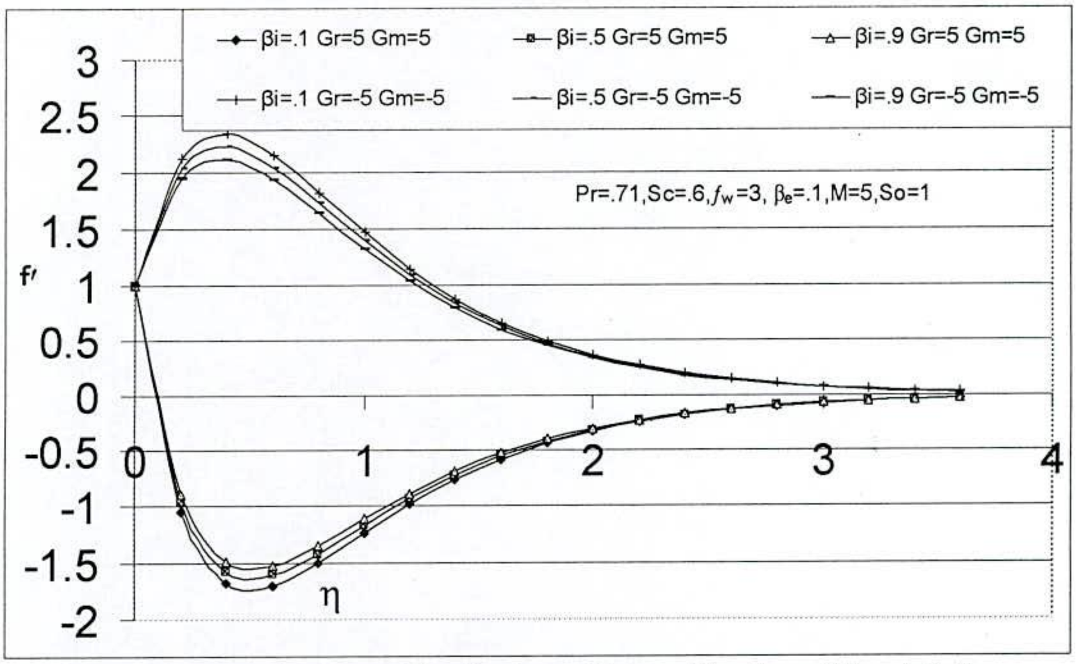

Fig.2.4.Primary velocity profiles due to cooling and heating of the plate for different values of βi

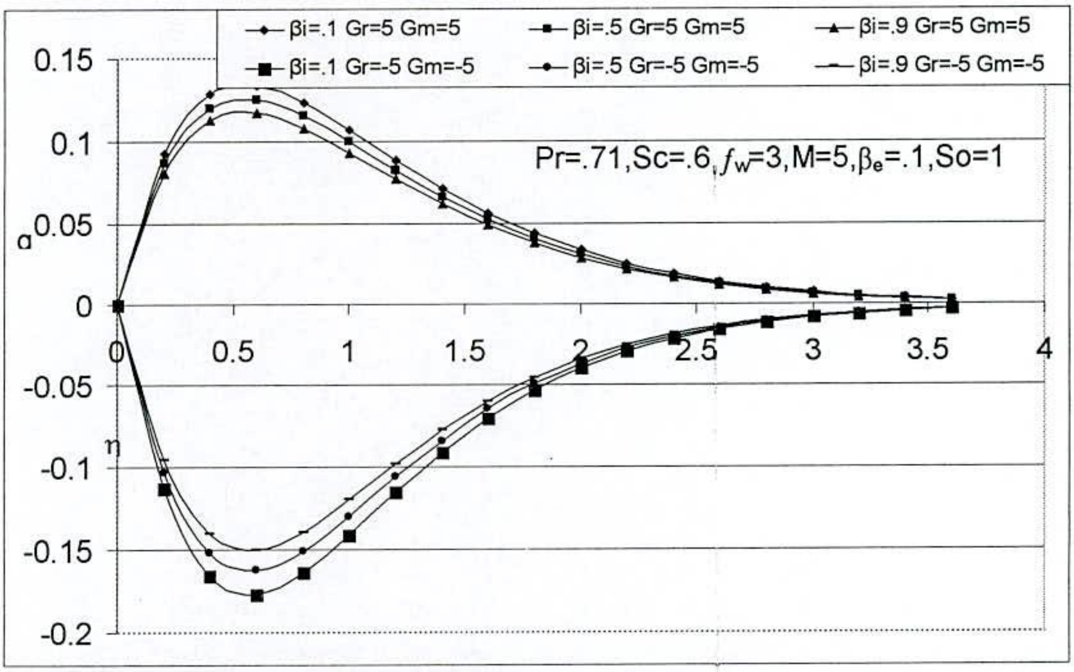

Fig.2.5.Secondary velocity profiles due to cooling and heating of the plate for different values of βi

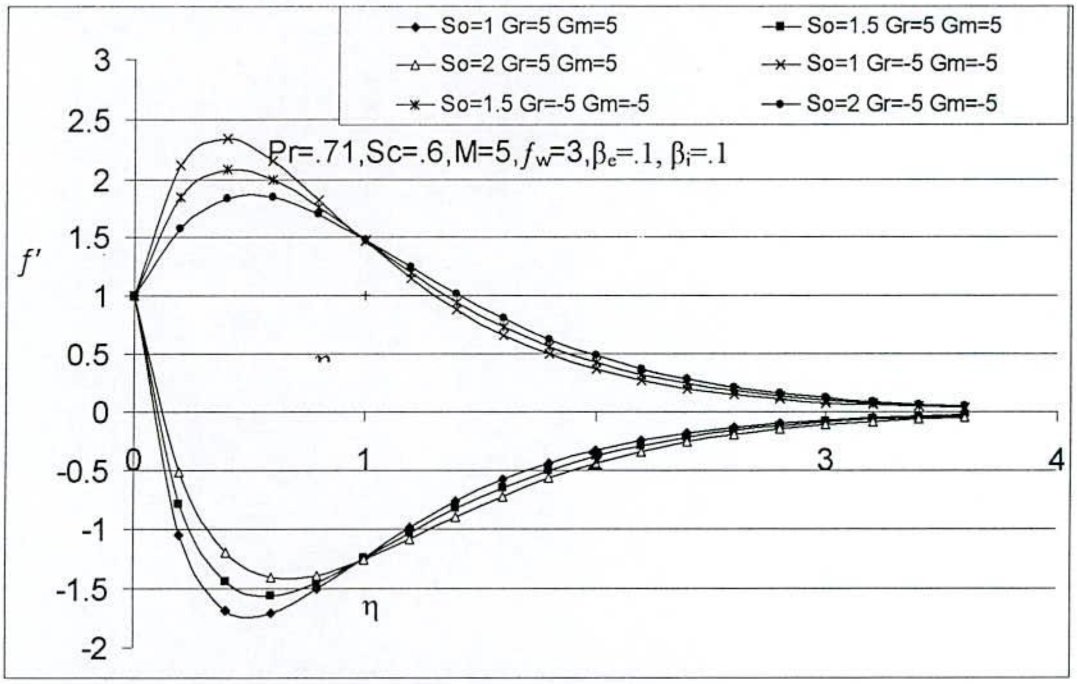

Fig.2.6.primary velocity profiles due to cooling and heating of the plate for different values of So

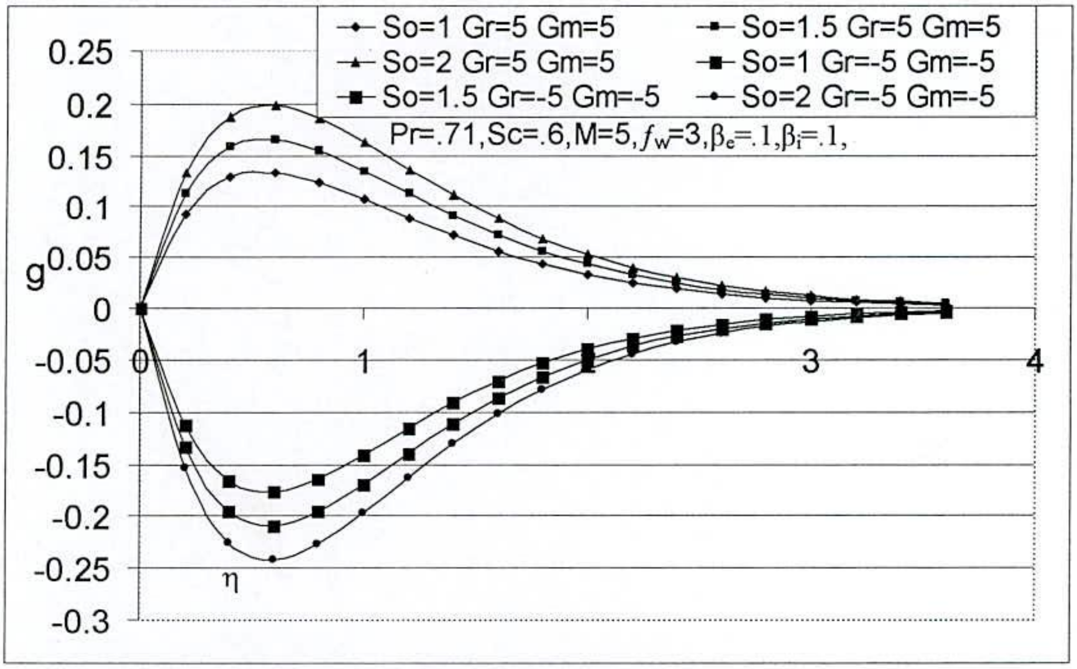

Fig.2.7.Secondery velocity profiles due to cooling and heating of the plate fror different values of So.

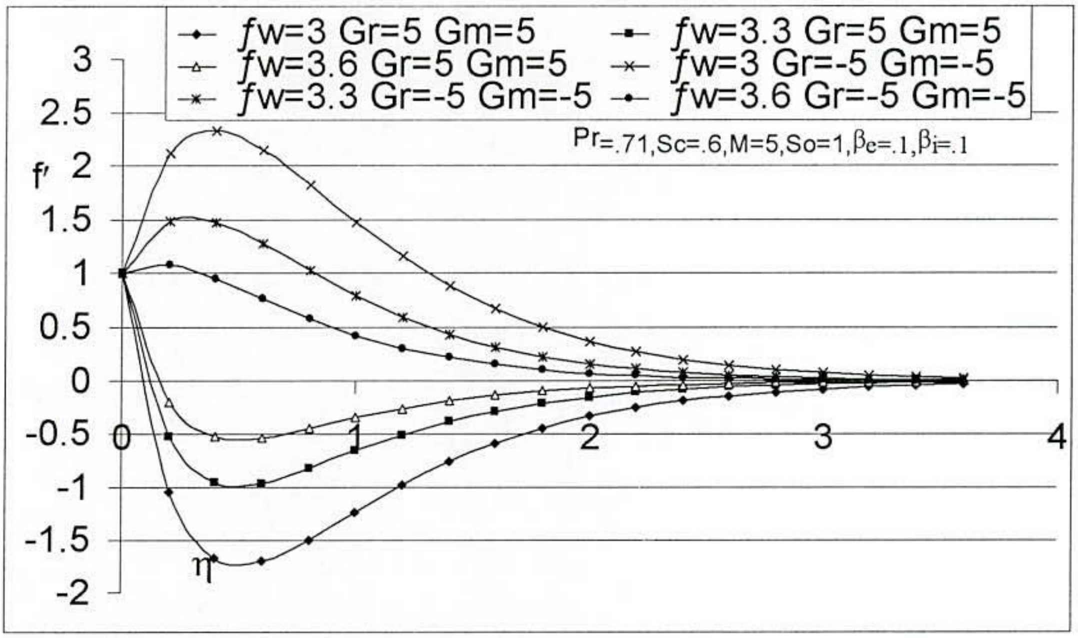

Fig.2.8. Primary velocity profiles due to cooling and heating of the plate for different values of *f*w

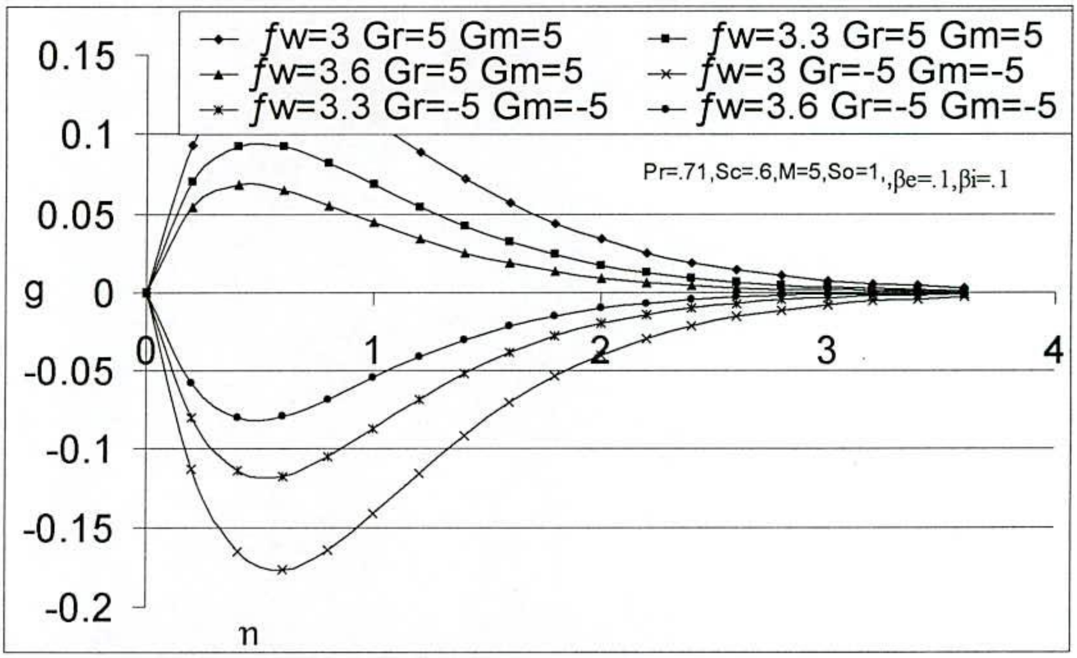

Fig.2.9.Secondary velocity profiles due to cooling and heating of the plate for different values of *f*w

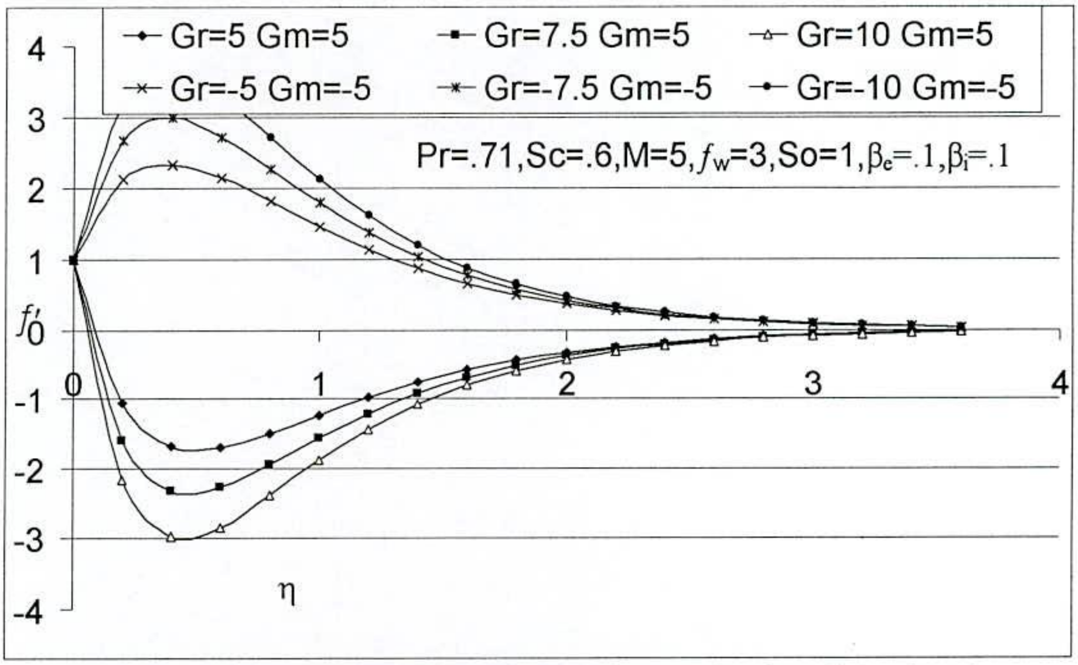

Fig 2.10.Primary velocity profiles due to cooling and heating of the plate for different values of Gr

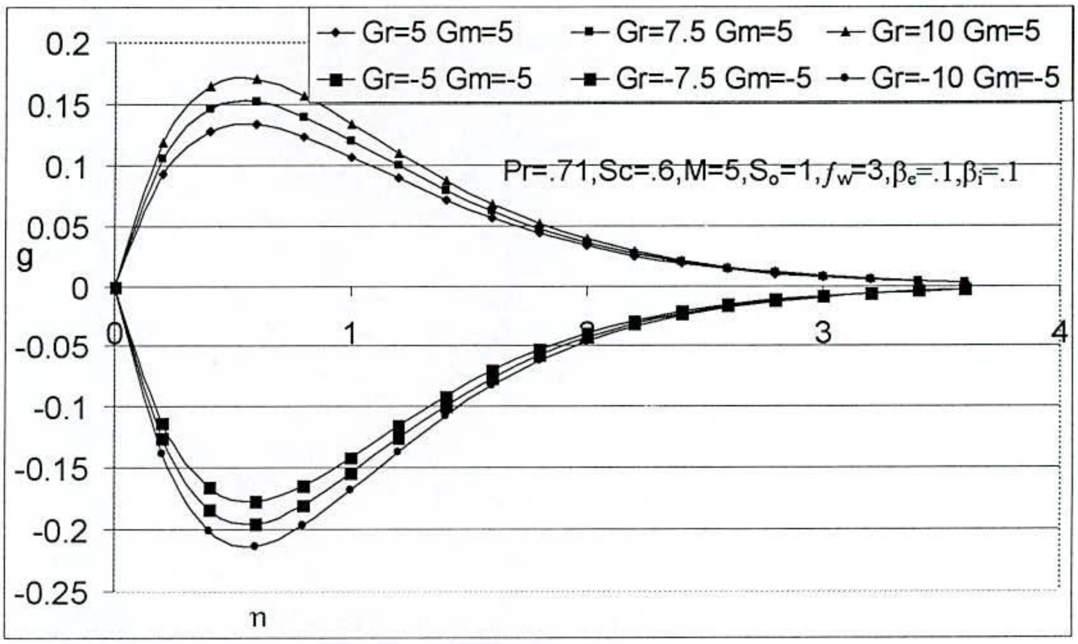

Fig.2.11.Secondary velocity profiles due to cooling and heating of the plate for the different values of Gr

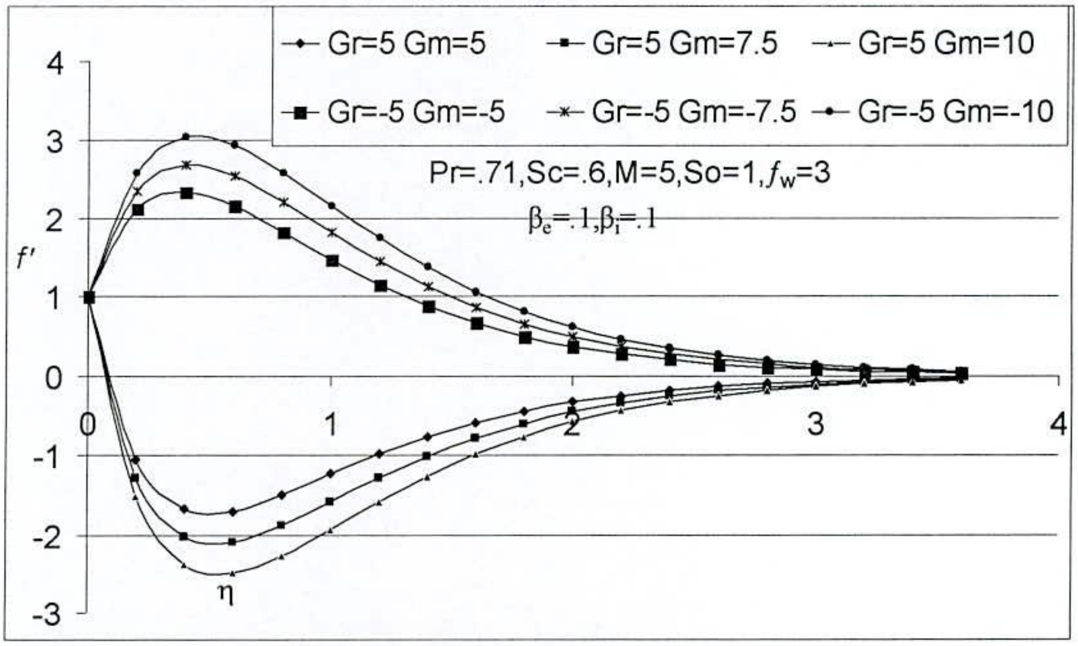

Fig.2.12.Primary velocity profiles due to cooling and heating of the plate for different values of Gm

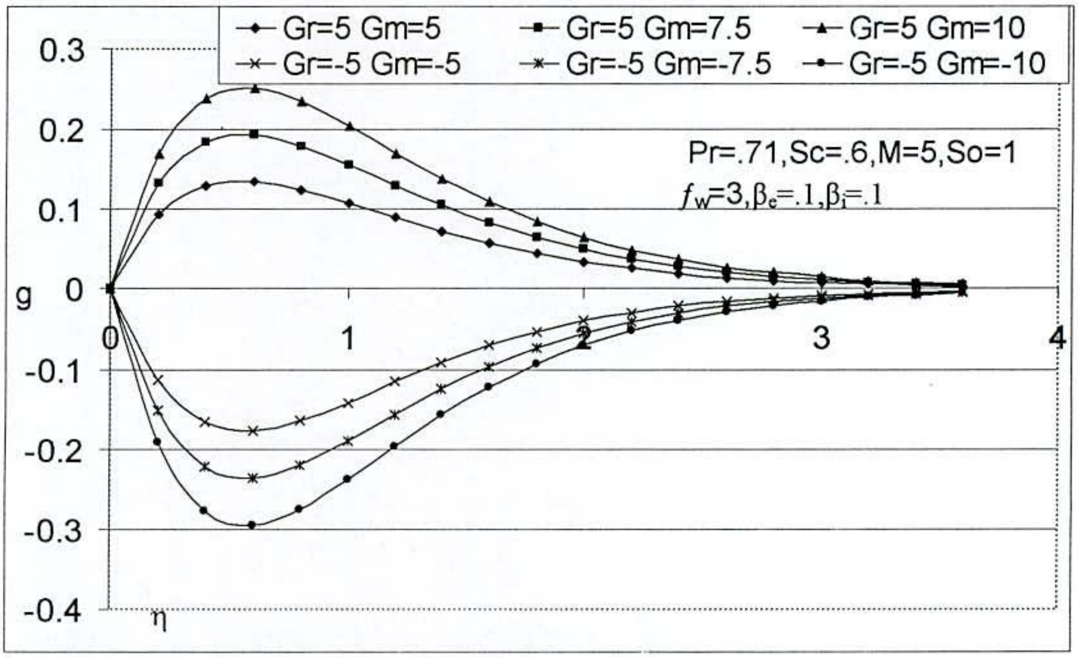

Fig.2.13.Secondary velocity profiles due to cooling and heating of the plate for different values of Gm

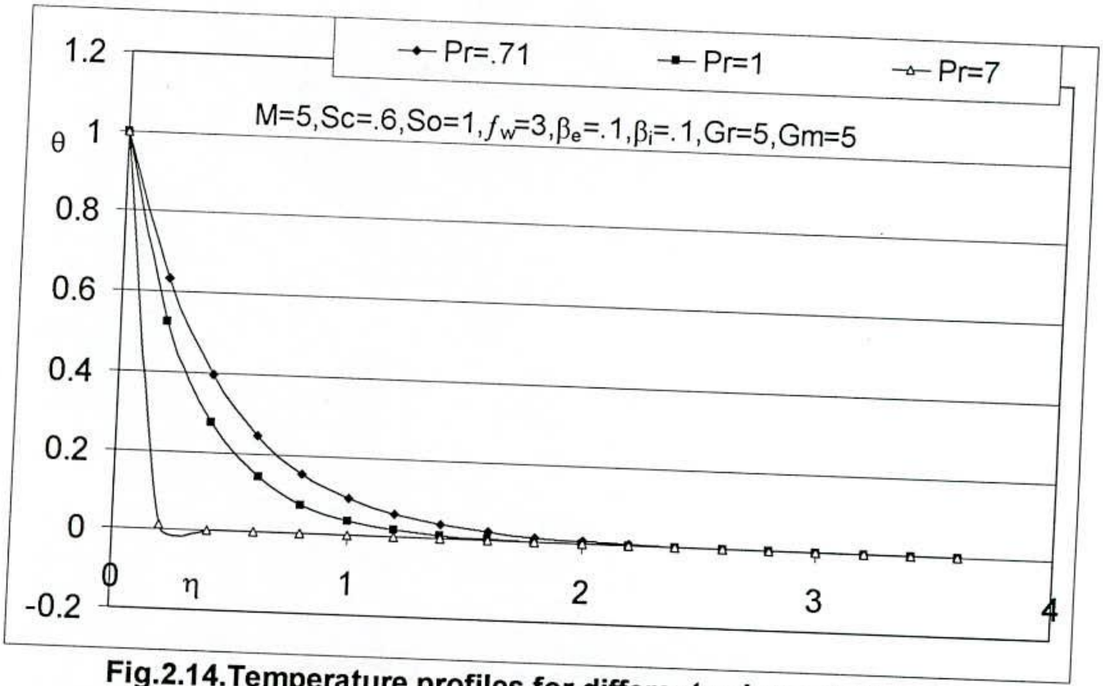

Fig.2.14.Temperature profiles for different values of Pr

Figure 2.14 represent the temperature profiles for different values of Pr with fixed Sc, So, M, f_w, β_i, β_e, Gr and Gm. From the figure it is observed that with the increase in Pr, θ decrease. This decrease is very large in case of water (Pr = 7.0). We also observe that for Pr=7.0 the field temperature remain less than the uniform flow temperature for most part of the boundary layer.

Finally the effect of various parameters on the components of the skin fiction coefficient τ_x and τ_z are shown in Table 2.1 -2.4. From table 2.1, we observe that the component τ_x increase with the increase of f_w but, τ_z decreases in the case of cooling of the plate. It is further observed that, in the case of heating of the plate, with the increase of suction parameter f_w τ_x decreases where as τ_z increases i. e. then reverse effect between τ_x and τ_z is observed. From table 2.2, we observe that the τ_x and τ_z both increases with the increase of Hall parameter (β_e). Where as for fixed β_e with the increase in β_i,τ_x and τ_z both decreases. In the equations (2.65) and (2.66) we observe that the parameters $M_1$and M_2 are absent. For this cause we avoid the analysis of Nu and Sh with the increase of β_i and β_e. From table 2.3, in the case of cooling of the plate we observe that τ_x and τ_z decreases but Sh increases with the increase of Sc. Again we observe that τ_x and Nu both increase but τ_z and Sh both decrease with the increase of Pr. Further we observe from table 2.4, that the Skin friction coefficient τ_x increase and τ_z decreases with the decrease of magnetic parameter (M).But Nu and Sh does not change with the decrease of M. Because in the equations (2.65) and (2.66) we observe that the parameters $M_1$and M_2 are absent. Again we observe from table 2.4, that τ_x decreases but τ_z and Sh both increase with the increase of So.

Table 2.1

Numerical values proportional to τ_x, τ_z, Nu and Sh for $\beta_e = 0.6$, $\beta_i = 0.1$, M=10, Pr=0.71, Sc=0.6 and So=1

f_w	Gr	Gm	τ_x	τ_z	Nu	Sh
3	5	5	-8.27376	1.64233	0.756134	0.30246
3.5	5	5	-4.65048	0.974077	0.743894	0.268379
4	5	5	-2.95336	0.630228	0.73595	0.246259
3	-5	-5	5.873719	-2.27039	0.756134	0.30246
3.5	-5	-5	2.103407	-1.13791	0.743894	0.268379
4	-5	-5	0.408706	-0.60778	0.73595	0.246259

Table 2.2

Numerical values proportional to τ_x, τ_z, Nu and Sh for f_w=3, M=10, Gr=5, Gm=5, Pr=0.71, Sc=0.6 and So=1

β_e	β_i	τ_x	τ_z	Nu	Sh
0.1	0.1	-11.121	0.330465	0.756134	0.30246
0.3	0.1	-10.2356	0.94239	0.756134	0.30246
0.5	0.1	-8.96255	1.444802	0.756134	0.30246
0.6	0.1	-8.27376	1.64233	0.756134	0.30246
0.6	0.3	-7.73148	1.425173	0.756134	0.30246
0.6	0.5	-7.22594	1.24662	0.756134	0.30246

Table 2.3

Numerical values proportional to τ_x, τ_z, Nu and Sh for $\beta_e = 0.6$, $\beta_i = 0.1$, Gr=5, Gm= 5, f_w=3, M=10 and So=1

Sc	Pr	τ_x	τ_z	Nu	Sh
0.6	0.71	-8.27376	1.64233	0.756134	0.30246
0.6	7.00	-8.36608	0.836253	7.097222	-3.46771
0.22	0.71	-18.3145	6.82546	0.756134	0.119251
0.6	0.71	-8.27376	1.64233	0.756134	0.30246
0.75	0.71	-5.46997	1.309473	0.756134	0.370243

Table 2.4

Numerical values proportional to τ_x, τ_z, Nu and Sh for $\beta_e = 0.6$, $\beta_i = 0.1$, f_w=3, Gr=5, Gm=5, Pr=0.71 and Sc=0.6

So	M	τ_x	τ_z	Nu	Sh
1	5	-3.76816	0.99953	0.756134	0.30246
2	5	-2.14164	1.38337	0.756134	- 0.12008
3	5	-0.51511	1.767211	0.756134	- 0.54262
1	5	-3.76816	0.99953	0.756134	0.30246
1	6	-4.68641	1.156628	0.756134	0.30246
1	7	-5.5961	1.299457	0.756134	0.30246

Appendix 2. A

$A_1 = (So\ Sc\ Pr)/(Sc-Pr)$; $A_2 = 1- A_1$; $A_3 = 1 + M_1$;

$A_4 = (Gr + A_1Gr)/(Pr^3 - Pr^2)$; $A_5 = GmA_2/(Sc^3 - Sc^2)$;

$A_6 = A_3 - 0.5 - A_4Pr - A_5Sc$; $A_7 = A_4Pr + A_5Sc + 0.25 - A_3 - A_4 - A_5$;

$A_8 = Pr^2/(1+Pr)$; $A_9 = B_1/((Pr+1)(Pr-Sc+1))$; $A_{10} = B_2/(Sc+1)$;

$A_{11} = B_3/(Pr(Pr - Sc)) + B_5(2Pr - Sc)/(Pr^2(Pr - Sc)^2)$; $A_{12} = B_4/Sc$;

$A_{13} = B_5/(Pr(Pr - Sc))$; $A_{14} = -A_9 - A_{10} - A_{11}$; $A_{15} = A_3 + M_1A_3 - M_2^2$;

$A_{16} = 1 + 2A_3 + 0.5M - 2A_6$; $A_{17} = B_{12}/(Sc^2 - Sc^3)$; $A_{18} = B_{11}/(Pr^2 - Pr^3)$;

$A_{19} = B_{10}/(Sc(Sc+1)^2)$; $A_{20} = B_9/(Pr(Pr+1)^2)$;

$A_{21} = A_{17} - B_{13}Sc + A_{18} - B_{14}Pr + A_{19}(Sc+1) + A_{20}(Pr+1)$
$+5/24 + 1.5A_3 - B_{15}Sc - B_{16}Pr + B_6 - 2A_{15} - 0.5A_{16}$;

$A_{22} = A_{20} + 5/72 + A_3 - B_{13} - B_{14} + A_{19} - B_{15} - B_{16} - A_{21} - 0.25A_{16}$;

$A_{23} = B_{13} + B_{15}$; $A_{24} = B_{14} + B_{16}$; $A_{25} = B_6 - 2A_{15}$;

$A_{26} = -M_1M_2 - M_2 - M_2A_3$; $A_{27} = A_{26} + B_{17}$; $A_{28} = A_4M_2/(Pr-1)$;

$A_{29} = A_5M_2/(Sc - 1)$; $A_{30} = -A_{28} - A_{29}$; $A_{31} = -(A_8 + 1 + A_7)Pr$;

$A_{32} = 0.25Pr^2/(4+2Pr)$; $A_{33} = (Pr^2 + 2A_8Pr^2 - A_6Pr^2 + A_8Pr)/(1+Pr)$;

$A_{34} = -0.5A_4/Pr$; $A_{35} = B_{18}/(Pr+1)$; $A_{36} = (2+Pr)B_{18}/(Pr+1)^2 - A_{33}$;

$A_{37} = -A_{32} + A_{34} - A_{36} - A_5Pr^2/(Sc^2 + ScPr)$ $A_{38} = B_{19}/(4+4Pr+Pr^2-2Sc-ScPr)$;

$A_{39} = B_{20}/(4+2Sc)$; $A_{40} = B_{25}/(4Pr^2-2ScPr)$; $A_{41} = B_{26}/(Pr^2+ScPr)$;

$A_{42} = B_{27}/(2Sc^2)$; $A_{43} = B_{27}/(1+Sc)$; $A_{44} = B_{23}/(1+2Pr+Pr^2-Sc-ScPr)$;

$A_{45} = B_{29}/(2Sc)$; $A_{46} = B_{32}/(Pr^2-ScPr)$; $A_{47} = B_{33} + B_{38}$;

$A_{48} = B_{34} + B_{37}$; $A_{49} = B_{35} + B_{41} + B_{43}$; $A_{50} = B_{36} + B_{39}$; $A_{51} = B_{40} + B_{42}$;

$A_{52} = -(A_{38} + A_{39} + A_{40} + A_{41} + A_{42} + A_{47} + A_{48} + A_{49})$

$B_1 = 2ScA_1 - A_1ScPr + ScSoA_8(1+pr)^2$; $B_2 = 2\ Sc\ A_2 - A_2Sc^2$;

$B_3 = ScA_1Pr - SoScA_8Pr^2 - Sc\ So\ 2\ Pr^2$; $B_4 = Sc^2A_2$; $B_5 = ScSoPr^3$

$B_6 = A_6 - 2A_3 - A_7 + A_6M_1 - M_1A_3$; $B_7 = Pr_2A_4 + M_1A_4Pr + GrA_8 + GmA_{11}$;

$B_8 = A_5Sc^2 + M_1ScA_5 + GmA_{14}$; $B_9 = A_4Pr^2 + A_4 + A_8Gr - A_9Gm$;

$B_{10} = A_5Sc^2 + A_5 - A_{10}Gm$; $B_{11} = GrPr - GmA_{13}$; $B_{12} = GmA_{12}$;

$B_{13} = B_{12}(2Sc-3Sc^2)/(Sc^2-Sc^3)^2$; $B_{14} = B_{11}(2Pr - 3Pr^2)/(Pr^2 - Pr^3)^2$;

$B_{15} = B_8/(Sc^3-Sc^2)$; $B_{16} = B_7/(Pr^3 - Pr^2)$; $B_{17} = -M_2A_6 + M_2 + A_3M_2$;

$B_{18} = Pr^3 + A_3Pr^2$;

$B_{19} = 2Sc\ A_9 - ScA_1 - ScA_9(1+Pr) + 0.25ScPrA_1 + 0.25\ A_2Sc^2 - ScSoA_{32}(2+pr)^2$;

$B_{20} = 2Sc\ A_{10} - ScA_2 - ScA_{10}(1+Sc)$;

$B_{21} = 2Sc\, A_{11} - 2ScA_1A_6 - 2ScA_1A_3 - ScA_9(1+Pr) - ScPrA_{11} + A_{13}Sc$

$\quad +PrScA_1A_6 + ScSo\,(A_{33}-A_{36})(1+Pr)^2 + 2SoScA_{35}(1+Pr);$

$B_{22} = -2Sc\, A_{12} - 2ScA_2A_3 + A_{12}\,Sc^2 + A_3A_2\,Sc^2;$

$B_{23} = 2ScA_{13} - PrScA_{13} + PrScA_1A_3 + 2ScA_1A_3 + ScSo\, A_{35}(1+Pr)^2;$

$B_{24} = 2ScA_{14} - 2ScA_2A_6 + 2ScA_2A_3 - ScA_{12} + ScA_{10}(1+Sc) + A_{14}\,Sc^2 - A_6A_2\,Sc^2;$

$B_{25} = -4ScSoA_{34}Pr^2 - PrScA_4A_1;$ $B_{26} = 2A_5A_1\,Sc^2 - 2PrScA_2A_4 + PrScA_1A_5 + A_2A_4\,Sc^2;$

$B_{27} = -A_2A_5\,Sc^2;$

$B_{28} = -ScSoA_{37}Pr^2 + PrScA_7A_1 + PrScA_{11} - ScA_{13} + 2PrScSoA_{31} - ScSo + 2ScSoPr\,;$

$B_{29} = -Sc^2A_{12};$ $B_{30} = A_2A_7\,Sc^2 + A_{14}\,Sc^2 + ScA_{12};$

$B_{31} = -ScSoA_{31}Pr^2 + PrScA_{13} - ScSoPr^2 + 2ScSoPr;$ $B_{32} = -0.5ScSoPr^2;$

$B_{33} = B_{21}/(1+2Pr+Pr^2-Sc-ScPr);$ $B_{34} = B_{24}/(1+Sc);$ $B_{35} = B_{28}/(Pr^2-ScPr);$

$B_{36} = -B_{30}/Sc;$ $B_{37} = B_{22}(2+Sc)/(1+Sc)^2;$

$B_{38} = B_{23}(2+2Pr-Sc)/(1+2Pr+Pr^2-Sc-ScPr)^2;$ $B_{39} = -B_{29}/Sc^2\,;$

$B_{40} = B_{31}/(Pr^2-ScPr);$ $B_{41} = B_{31}(2Pr-Sc)/(Pr^2-ScPr)^2;$

$B_{42} = 2B_{32}(2Pr-Sc)/(Pr^2-ScPr)^2;$ $B_{43} = B_{32}(2Pr-Sc)^2/(Pr^2-ScPr)^3 - 2B_{32}/(Pr^2-ScPr)^2$

References

Bestman, A. R. (1990). International J. Energy Resch. 14, 384.

Ram, P. C. and Takhar, H. S. (1993). The Japan Society of Fluid Machanics 5983.

Satter, S. A.and Alam, M. M. (1995). Indian Jr. of Pure and Appli. Math. 26(2), 157.

Singh, A. K. and Dikshit, C. K. (1988). Astrophys, Space Science, 148, 249

Chapter 3

Steady *MHD* free convection and mass transfer flow with thermal diffusion, Hall current, ion slip current and large suction in a rotating system.

Chapter 3

Steady *MHD* free convection and mass transfer flow with thermal diffusion, Hall current, ion slip current and large suction in a rotating system.

An important type of rotating boundary layer flow is the flow over rotating blades, occurring in turbines, helicopters and propellers. Alam and Satter(2001) studied the steady two-dimensional MHD free convective and mass transfer flow with thermal diffusion and large suction past an infinite vertical porous plate in rotating system. Ram and Takhar(1993) studied the MHD free convection flow past an infinite vertical plate with Hall and ionslip currents when the fluid and the plate are in a state of rigid rotation. In view of the above investigations we aim to study the MHD free convection and mass transfer flow with thermal diffusion, Hall current and ionslip current and large suction of a viscous incompressible electrically conducting partially ionized fluid past an impulsively started infinite vertical plate. The whole system is assumed to be in a state of rigid body rotation. In fact the problem considered in this chapter is an extension of that of chapter 2. In this case we have taken into account the effect of rotation on the flow considered in the previous chapter.

3.1. Governing Equations

The flow model considered is the steady MHD free convection flow of a viscous, incompressible and electrically conducting partially ionized fluid past an infinite vertical porous plate with thermal diffusion, Hall current, ion slip current and large suction in a rotating system under the influence of a transversely applied magnetic field. In fact this model is an extension of the previous model considered in section 2.1, through the introduction of a rotating system. In addition to the assumptions made in section 2.1, in the present model we consider that the fluid and the plate are in a state of solid body rotation with a constant angular velocity Ω about y-axis, which is taken to be perpendicular to the plate. With the same assumptions and approximations as considered in section 2.1, the flow configuration in a rotating system is shown in Fig 3.1

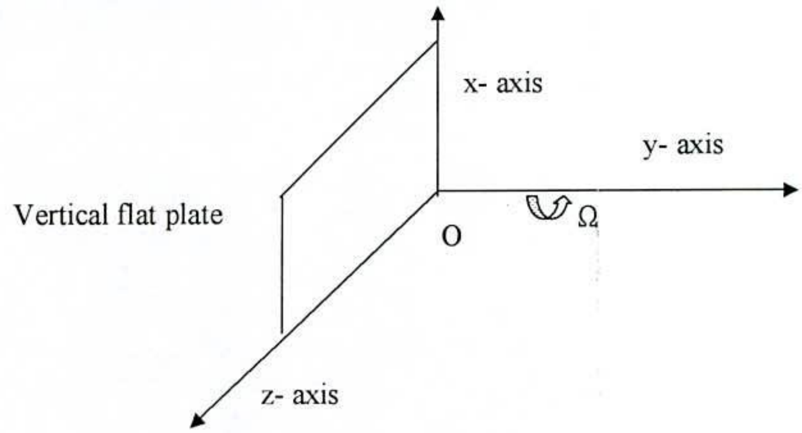

Fig 3.1 The flow configuration with the coordinate system.

The basic equations relevant to the problem are:

continuity equation

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{3.1}$$

momentum equation

$$u\partial u/\partial x + v\partial u/\partial y - 2w\Omega = \nu\partial^2 u/\partial y^2 + g_o \beta'(T-T\infty) + g_o \beta^* (C- C\infty) - B_o J_z/\rho \tag{3.2}$$

$$u\partial w/\partial x + v\partial w/\partial y + 2u\Omega = \nu\partial^2 w/\partial y^2 + B_o J_x/\rho \tag{3.3}$$

energy equation

$$u\partial T/\partial x + v\partial T/\partial y = (k/\rho C_p)\, \partial^2 T/\partial y^2 \tag{3.4}$$

concentration equation

$$u\partial C/\partial x + v\partial C/\partial y = D_M\, \partial^2 C/\partial y^2 + D_T\, \partial^2 T/\partial y^2 \tag{3.5}$$

where Ω is the angular velocity, g_o is the acceleration due to gravity, β' is the co-efficient of volume expansion, β^* is the co efficient of expansion with concentration, ρ is the density, T is the temperature of the flow field, T_∞ is the temperature of the fluid at infinity, C is the species concentration, C_∞ is the species concentration at infinity, D_M is the molecular diffusivity and D_T is the thermal diffusivity.

The boundary conditions for the problem are

$$\left.\begin{array}{l} u = U_o,\ v = v_o(x),\ w = 0,\ T = T_w,\ C = C_w(x) \text{ at } y = 0 \\ u = 0,\ v = 0,\ w = 0,\ T = T_\infty,\ C = C_\infty \quad \text{as } y \to \infty \end{array}\right| \tag{3.6}$$

where U_o is the uniform velocity and v_o (x) is the velocity of suction at the plate and $C_w(x)$ is the variable concentration at the plate. Now for reasons of similarity the plate concentration C_w (x) is taken to be

$$C_w(x) = C_\infty + (C_o - C_\infty)\, \bar{x} \tag{3.7}$$

where $\bar{x} = xU_o/\nu$ and C_o is considered to be the mean concentration.

3.2. Mathematical Formulations

As in section **2.2** we introduce the following similarity variables:

$$\left.\begin{array}{l} \eta = y\sqrt{(U_o/(2\nu x))} \\ f'(\eta) = u/U_o \\ g(\eta) = w/U_o \\ \theta(\eta) = (T-T_\infty)/(T_w-T_\infty) \\ \phi(\eta) = (C- C_\infty)/(\bar{x}\,(C_o - C_\infty)) \end{array}\right| \tag{3.8}$$

Now in terms of (3.8) the equation (3.1) can be integrated to

$$v = \sqrt{(\nu U_o/(2x))}\,(\eta f' - f) \tag{3.9}$$

With the aid of relations in (3.7), (3.8) and (3.9) the following dimensionless parameters are defined

$Gr = g_o \beta'(T_w - T_\infty)2x/U_0^{\,2}$, $Gm = g_o \beta^* (C_o - C_\infty)2x^2/(\nu U_o)$,

$N_1' = 2x\,(B_o \alpha\sigma E_z)/(U_o^{\,2}\rho)$, $N_2' = 2x(B_o \sigma\beta E_x)/(U_o^{\,2}\rho)$

$M = 2xB_o^2\sigma/(\rho U_o^2)$, $M_1 = \alpha M$, $M_2 = \beta M$, $R = x\Omega/U_o$

$N_1 = 2x\,(B_o \alpha\sigma E_x)/(U_o^2\rho)$, $N_2 = 2x(B_o \beta\sigma E_z)/(U_o^2\rho)$

$Pr = \rho\nu C_p/k$, $Sc = \nu/D_M$, $So = (D_T(T_w - T_\infty))/(xU_o(C_w - C_\infty))$

The equations (3.2) and (3.3), thus reduce to the following dimensionless differential equations

$$f''' + ff'' + 4Rg - M_1 f' - M_2 g + Gr\theta + Gm\phi - N_1' + N_2' = 0 \tag{3.10}$$

$$g'' + g'f - 4Rf' + f'M_2 - M_1 g + N_1 + N_2 = 0 \tag{3.11}$$

where Gr is the local Grashof number, Gm is the local modified Grashof number, T_w is the temperature at the plate, M is the local magnetic force number, Pr is the Prandtl number, Sc is the Schmidt number, So is the local Soret number and R is the rotational parameter.

We now consider further the case of short circuit problem in which the applied electric field $\mathbf{E} = \mathbf{0}$. Now for this we have $N_1 = 0$, $N_2 = 0$, $N_1' = 0$ and $N_2' = 0$. Then the above equations (3.10) and (3.11) reduces to

$$f''' + ff'' + 4Rg - M_1 f' - M_2 g + Gr\theta + Gm\phi = 0 \tag{3.12}$$

$$g'' + g'f - 4Rf' + f'M_2 - M_1 g = 0 \tag{3.13}$$

The equations (3.4) and (3.5), using the similarity variables and nondimentional parameters reduce to the following dimensionless differential equations

$$\theta'' + Prf\theta' = 0 \tag{3.14}$$

$$\phi'' - 2Scf'\phi + Scf\phi' + So\,Sc\,\theta'' = 0 \tag{3.15}$$

Subject to the above formulations the boundary conditions (3.6) now transform to

$$\left.\begin{aligned} &f = f_w,\ f' = 1,\ g = 0,\ \theta = 1,\ \phi = 1 \text{ at } \eta = 0 \\ &f' = 0,\ g = 0,\ \theta = 0,\ \phi = 0 \text{ as } \eta \to \infty \end{aligned}\right| \tag{3.16}$$

where $f_w = - v_o (x)\sqrt{(2x/(\nu U_o))}$ is the transpiration parameter and primes denote derivative with respect to η. Here $f_w > 0$ indicates the suction and $f_w < 0$ the injection. The solutions of the equations (3.12) – (3.15) subject to the boundary conditions (3.16) are now sought and are presented in the following section.

3.3. Solution

As has been done in section **2.3,** the following further transformations are made

$$\zeta = \eta f_w, \quad f(\eta) = f_w F(\zeta), \quad g(\eta) = f_w^2 G(\zeta), \quad \theta(\eta) = f_w^2 H(\zeta), \quad \phi(\eta) = f_w^2 P(\bar{\zeta}) \tag{3.17}$$

Substituting (3.17) in equations (3.12) – (3.15) we have

$$F''' + FF'' + \epsilon \{- M_1F' + (4R - M_2)G + Gr H + GmP\} = 0 \tag{3.18}$$

$$G'' + G'F + \epsilon \{F'(M_2 - 4R) - M_1G\} = 0 \tag{3.19}$$

$$H'' + PrFH' = 0 \tag{3.20}$$

$$P'' - 2 Sc F'P + Sc F P' + SoSc H'' = 0 \tag{3.21}$$

where $\epsilon = 1/f_w^2$

Also the boundary conditions (3.16) transform to

$$F = 1, F' = \epsilon, G = 0, H = \epsilon, P = \epsilon \text{ at } \zeta = 0; \quad F' = 0, G = 0, H = 0, P = 0 \text{ as } \zeta \to \infty \tag{3.22}$$

Now for large suction $f_w >> 1$, so that ϵ is very small, therefore, following Singh and Dikshit (1988) and Bestman (1990), F, G, H and P can be expanded in terms of small perturbation quantity ϵ as

$$F(\zeta) = 1 + \epsilon F_1(\zeta) + \epsilon^2 F_2(\zeta) + \epsilon^3 F_3(\zeta) + \ldots \tag{3.23}$$

$$G(\zeta) = \epsilon G_1(\zeta) + \epsilon^2 G_2(\zeta) + \epsilon^3 G_3(\zeta) + \ldots \tag{3.24}$$

$$H(\zeta) = \epsilon H_1(\zeta) + \epsilon^2 H_2(\zeta) + \epsilon^3 H_3(\zeta) +- \ldots \tag{3.25}$$

$$P(\zeta) = \epsilon P_1(\zeta) + \epsilon^2 P_2(\zeta) + \epsilon^3 P_3(\zeta) + \ldots \tag{3.26}$$

Then substituting $F(\zeta)$, $G(\zeta)$, $H(\zeta)$ and $P(\zeta)$ from (3.23) – (3.26) in the equations(3.18) – (3.21), we have the following set of ordinary differential equations and the boundary conditions for $F_i(\zeta)$, $G_i(\zeta)$, $H_i(\zeta)$ and $P_i(\zeta)$ (i= 1,2,3,.........);

For the first order i. e. $0(\epsilon)$:

$$F_1''' + F_1'' = 0 \tag{3.27}$$

$$G_1'' + G_1' = 0 \tag{3.28}$$

$$H_1'' + Pr\, H_1' = 0 \tag{3.29}$$

$$P_1'' + Sc\, P_1' + So\, Sc\, H_1'' = 0 \tag{3.30}$$

$$\left.\begin{array}{l} F_1 = 0,\ F_1' = 1,\ G_1 = 0,\ H_1 = 1,\ P_1 = 1, \text{ at } \zeta = 0 \\ F_1' = 0,\ G_1 = 0,\ H_1 = 0,\ P_1 = 0 \text{ as } \zeta \to \infty \end{array}\right| \tag{3.31}$$

For the second order i. e. $0(\epsilon^2)$:

$$F_2''' + F_2'' + F_1 F_1'' - M_1 F_1' + (4R - M_2) G_1 + Gr\, H_1 + Gm\, P_1 = 0 \tag{3.32}$$

$$G_2'' + G_2' + F_1 G_1' + (M_2 - 4R) F_1' - M_1 G_1 = 0 \tag{3.33}$$

$$H_2'' + Pr H_2' + Pr F_1 H_1' = 0 \tag{3.34}$$

$$P_2'' - 2Sc\, P_1 F_1' + Sc P_2' + Sc F_1 P_1' + So\, Sc\, H_2'' = 0 \tag{3.35}$$

$$\left.\begin{array}{l} F_2 = 0,\ F_2' = 0,\ G_2 = 0,\ H_2 = 0,\ P_2 = 0 \text{ at } \zeta = 0 \\ F_2' = 0,\ G_2 = 0,\ H_2 = 0,\ P_2 = 0 \quad \text{as } \zeta \to \infty \end{array}\right| \tag{3.36}$$

For the third order i. e. $0(\epsilon^3)$;

$$F_3''' + F_3'' + F_1 F_2'' + F_2 F_1'' - M_1 F_2' + (4R - M_2) G_2 + Gr H_2 + Gm P_2 = 0 \tag{3.37}$$

$$G_3'' + G_3' + F_1 G_2' + F_2 G_1' + (M_2 - 4R) F_2' - M_1 G_2 = 0 \tag{3.38}$$

$$H_3'' + Pr F_1 H_2' + Pr H_3' + Pr F_2 H_1' = 0 \tag{3.39}$$

$$P_3'' - 2Sc P_2 F_1' - 2Sc P_1 F_2' + Sc P_3' + Sc F_1 P_2' + Sc F_2 P_1' + Sc So H_3'' = 0 \tag{3.40}$$

$$\left.\begin{array}{l} F_3 = 0,\ F_3' = 0,\ G_3 = 0,\ H_3 = 0,\ P_3 = 0, \text{ at } \zeta = 0 \\ F_3' = 0,\ G_3 = 0,\ H_3 = 0,\ P_3 = 0 \text{ as } \zeta \to \infty \end{array}\right| \tag{3.41}$$

etc.

The solutions of the above equations up to order 3 under the prescribed boundary conditions are obtained in a straightforward manner and are

$$F_1 = 1 - e^{-\zeta} \tag{3.42}$$

$$G_1 = 0 \quad (3.43)$$

$$H_1 = e^{-Pr\zeta} \quad (3.44)$$

$$P_1 = A_1 e^{-Pr\zeta} + A_2 e^{-Sc\zeta} \quad (3.45)$$

$$F_2 = 0.25e^{-2\zeta} + A_3\zeta e^{-\zeta} + A_4 e^{-Pr\zeta} + A_5 e^{-Sc\zeta} + A_6 e^{-\zeta} + A_7 \quad (3.46)$$

$$G_2 = (M_2 - 4R)\zeta e^{-\zeta} \quad (3.47)$$

$$H_2 = A_8 e^{-Pr\zeta} - \zeta Pr e^{-Pr\zeta} - A_8 e^{-\zeta(1+Pr)} \quad (3.48)$$

$$P_2 = A_9 e^{-\zeta(1+Pr)} + A_{10} e^{-\zeta(1+Sc)} + A_{11} e^{-Pr\zeta} - \zeta A_{12} e^{-Sc\zeta} + \zeta A_{13} e^{-Pr\zeta} + A_{14} e^{-Sc\zeta} \quad (3.49)$$

$$F_3 = -0.07e^{-3\zeta} - 0.5A_3\zeta e^{-2\zeta} - A_3 e^{-2\zeta} - 0.5A_{15}\zeta^2 e^{-\zeta} + 0.25A_{16}e^{-2\zeta} + A_{17}\zeta e^{-Sc\zeta} + A_{18}\zeta e^{-Pr\zeta}$$
$$- A_{19} e^{-\zeta(1+Sc)} - A_{20} e^{-\zeta(1+Pr)} + A_{21} e^{-\zeta} + A_{22} + A_{23} e^{-Sc\zeta} + A_{24} e^{-Pr\zeta} + A_{25}\zeta e^{-\zeta} \quad (3.50)$$

$$G_3 = -0.5(M_2 - 4R)\zeta e^{-2\zeta} + 0.5A_{26}\zeta^2 e^{-\zeta} + A_{27}\zeta e^{-\zeta} + A_{28}e^{-Pr\zeta} + A_{29}e^{-Sc\zeta} + A_{30}e^{-\zeta} \quad (3.51)$$

$$H_3 = (A_{31}+1)\zeta e^{-Pr\zeta} + A_{32} e^{-\zeta(2+Pr)} + A_{36} e^{-\zeta(1+Pr)} - A_{34} e^{-2Pr\zeta} + A_{35}\zeta e^{-\zeta(1+Pr)} + A_{37} e^{-Pr\zeta}$$
$$+ 0.5\zeta^2 e^{-Pr\zeta} \quad (3.52)$$

$$P_3 = A_{38}e^{-\zeta(2+Pr)} + A_{39}e^{-\zeta(2+Sc)} + A_{40}e^{-2Pr\zeta} + A_{41}e^{-\zeta(Pr+Sc)} + A_{42}e^{-2Sc\zeta} + A_{43}\zeta e^{-\zeta(1+Sc)}$$
$$+ A_{44}\zeta e^{-\zeta(1+Pr)} + A_{45}\zeta^2 e^{-Sc\zeta} + A_{46}\zeta^2 e^{-Pr\zeta} + A_{47}e^{-\zeta(1+Pr)} + A_{48}e^{-\zeta(1+Sc)} + A_{49}e^{-Pr\zeta}$$
$$+ A_{50}\zeta e^{-Sc\zeta} + A_{51}\zeta e^{-Pr\zeta} + A_{52}e^{-Sc\zeta} \quad (3.53)$$

where the constants A_i (where i=1, 2, 3……..) are shown in appendix 3.A. The velocity, the temperature and the concentration fields are thus obtained from (3.23) to (3.26) as

$$u/U_o = f'(\eta) = F_1' + \epsilon F_2' + \epsilon^2 F_3' \quad (3.54)$$

$$w/U_o = g(\eta) = G_1 + \epsilon G_2 + \epsilon^2 G_3 \quad (3.55)$$

$$\theta(\eta) = H_1 + \epsilon H_2 + \epsilon^2 H_3 \quad (3.56)$$

$$\phi(\eta) = P_1 + \epsilon P_2 + \epsilon^2 P_3 \quad (3.57)$$

Thus with the help of the solutions (3.42) – (3.53) the velocity, temperature and concentration distributions can be calculated from (3.54) – (3.57). The velocity and distributions are shown in Figs 3.2- 3.15.

3.4. Skin – friction coefficient, Nusselt number and Sherwood number

The quantities of chief physical interest are the local skin friction coefficients, Nusselt number and Sherwood number.
The equations defining the wall skin friction are

$$\tau_x = \mu(\partial u/\partial y)_{y=0}$$

$$\tau_z = \mu(\partial w/\partial y)_{y=0}$$

Thus from equations (3.54) and (3.55) we have

$$\tau_x \propto f''(0)$$

$$\tau_z \propto g'(0)$$

and in turn we have

$$f''(0) = -1 + \epsilon\,[A_6 - 2A_3 + 1 + A_4Pr^2 + A_5Sc^2] + \epsilon^2\,[A_{23}Sc^2 + A_{24}Pr^2 - 2A_{25} + A_{21} - 2A_{17}Sc - 2A_{18}Pr - A_{19}(Sc+1)^2 - A_{20}(Pr+1)^2 - 5/8 - 2A_3 - A_{15} + A_{16}] \quad (3.58)$$

and

$$g'(0) = \epsilon(M_2 - 4R) + \epsilon^2[-A_{30} + A_{27} - 0.5(M_2 - 4R) - A_{28}Pr - A_{29}Sc] \quad (3.59)$$

The local Nusselt number denoted by Nu is proportional to $-(\partial T/\partial y)_{y=o}$, hence we have from (3.56)

$$Nu \propto -\theta'(0)$$

and in turn we have

$$\theta'(0) = -Pr + \epsilon[-A_8Pr - Pr + A_8(Pr+1)] \quad (3.60)$$

The local Sherwood number denoted by Sh is proportional to $-(\partial C/\partial y)_{y=o}$, hence we have from (3.57)

$$Sh \propto -\phi'(0)$$

and in turn we have

$$\phi'(0) = -A_1Pr - A_2Sc + \epsilon[-A_{14}Sc - A_9(Pr+1) - A_{10}(Sc+1) - A_{11}Pr - A_{12} + A_{13}] \quad (3.61)$$

Thus the values proportional to the skin friction coefficients, Nusselt number and Sherwood number are respectively obtained from (3.58) - (3.61). These values are sorted in tables 3.1 - 3. 3.

3.5. Results and Discussions

For the purpose of discussing the results some numerical calculations are carried out for non-dimensional primary ($f'(\eta)$) and secondary ($g(\eta)$) velocities. Before discussion it may be pointed out that in this problem we have added the effect of rotation with the case of previous problem. As a result we have additional term of the order of **R** in the results (in eqs. 3.47 and 3.51). If **R** is chosen as zero these equations transform to 2.52 and 2.56 respectively, which confirms the result obtained here. The velocity profiles for the x and z components of velocity are shown in Figs 3.2 – 3.15 for different values of β_i, β_e, Gr, Gm, So, R and f_w for fixed values of M, Pr and Sc. The value of M is taken to be large which corresponds to a strong magnetic field, which is generally encountered in nuclear engineering in connection with the cooling of reactors. Negative values of Gr, which indicates the heating of the plate by free convection currents, are also taken into account. The value of Sc is taken to be 0.6

which corresponds to water vapor, Pr is taken equal to 0.71 which corresponds to its value in case of air. The magnetic force number M is chosen arbitrarily being equal to 5.0 which imply a strong magnetic field.

Figure 3.2 represents the velocity profiles of primary velocity f' for different β_e with fixed Pr, Sc, So, R, β_i, f_w and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increases in β_e produced rapid increases in f'. For negative Gr and Gm, f' is positive and with the increase of β_e, f' decreases rapidly.

Figure 3.3 represents the velocity profiles of secondary velocity g for different β_e with fixed Pr, Sc, So, R, f_w, β_i and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is negative and increases in β_e produces rapid decreases in g for $\beta_e > 0.17$. When $\beta_e = 0.17$ then g = 0,and it becomes positive if $\beta_e <$ 0.17. Again for negative Gr and Gm, g is positive for $\beta_e > 0.17$and increases rapidly with the increase of β_e. At the lower value of $\beta_e < 0.17$ g becomes negative. It is also found that all curves intersect g=o at the point $\eta = 1.9$ When $1.9 < \eta < 4.0$ the values of g is just reverse in sign, though the values are very small.

Figure 3.4 represents the velocity profiles of primary velocity f' for different β_i with fixed Pr, Sc, So, R, β_e, f_w and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increases in β_i produced slow increases in f'. For negative Gr and Gm, f' is positive and with the increase of β_i, f' slowly decreases. It may be noted that the sensitivity of f' is more with respect to β_e than β_i.

Figure 3.5 represent the velocity profiles of secondary velocity g for different β_i with fixed Pr, Sc, So, R, f_w, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increases in β_i produced increase in g. For negative Gr and Gm, g is negative and with the increase of β_i, g decreases gradually. It is also found that when $\eta = 1.95$ then all curves intersect. It is also found that when $\eta > 1.95$ then the values of g reverses its signs. It may be noted that g is less sensitive on β_i than β_e.

Figure 3.6 represent the velocity profiles of primary velocity f' for different So with fixed Pr, Sc, R, β_i, β_e, f_w and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative. For $0 < \eta < 0.65$ with the increases of So, f' increases slowly. But when $0.65 < \eta < 4$ the effect is reversed i.e. f' decreases with the increases of So. For negative Gr and Gm, f' is positive when η lies between 0 to 0.65, f' decreases slowly with the increases in So. But after that point the effect is reversed as in the cease with positive Gr and Gm.

Figure 3.7 represent the velocity profiles of secondary velocity g for different So with fixed Pr, Sc, R, f_w, β_i, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increase in So produced rapid increase in g. For negative Gr and Gm, g is negative and with the increase of So, g decreases rapidly. It is found that in the case of the cooling of the plate g becomes zero at $\eta = 2.05$ with So = 1. The same is happened at $\eta = 2.35$ and $\eta = 2.55$ when So= 1.5 and So = 2 respectively. It is further noticed that in the case of the heating of the plate g becomes zero at η =1.9, 2.3, and 2.5 with So= 1.0, 1.5 and 2.0 respectively.

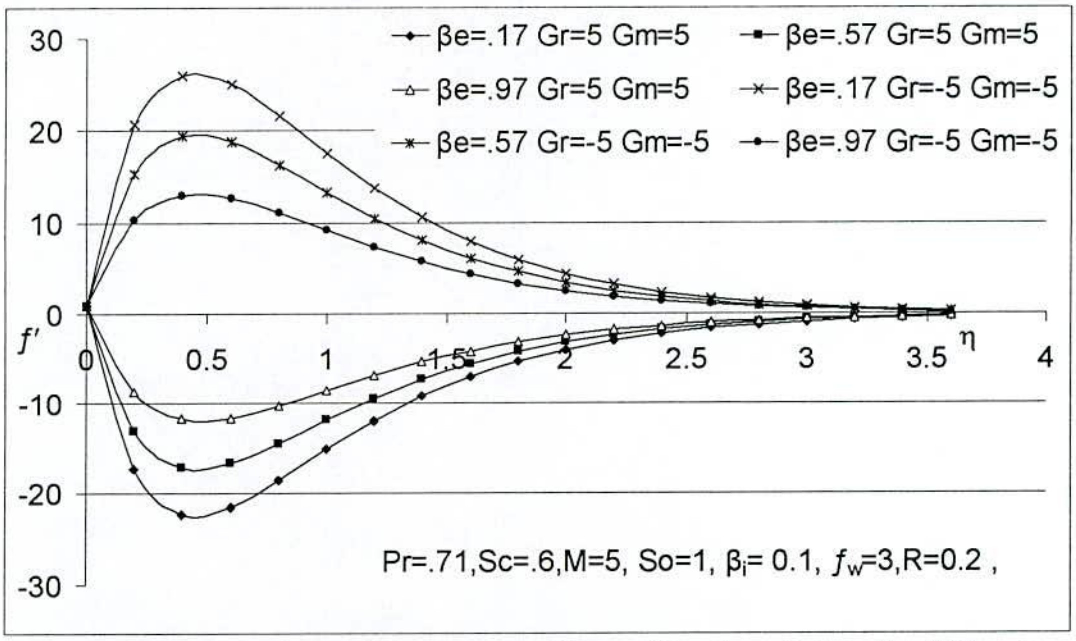

Fig. 3.2 Primary velocity profiles due to cooling and heating of plate for different values of βe

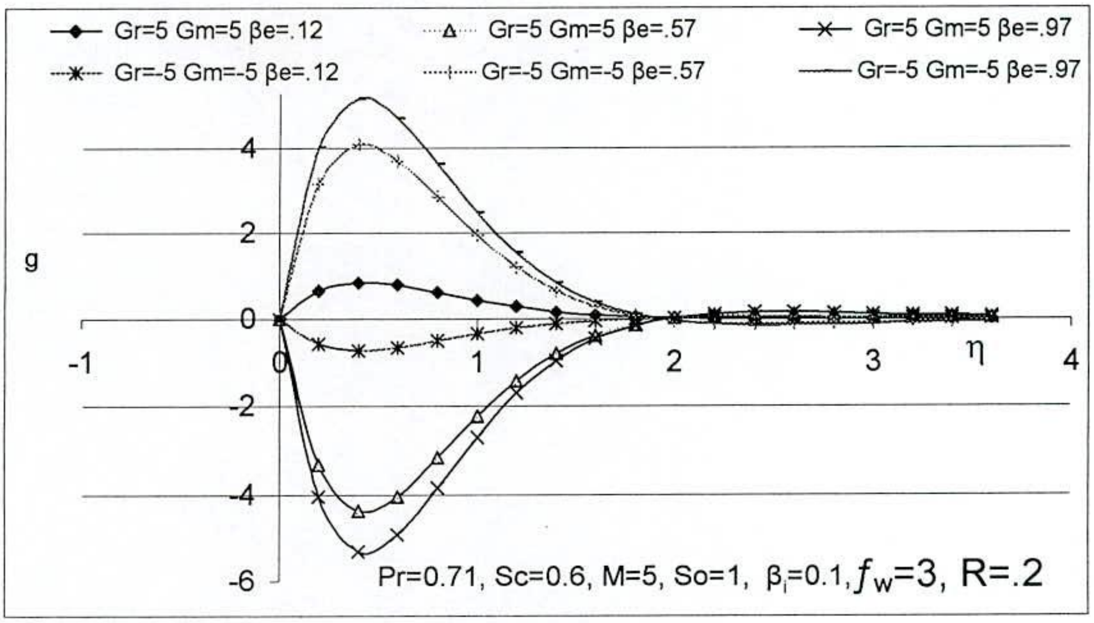

Fig.3.3. Secondary velocity profiles due to cooling and heating of the plate for different values of βe.

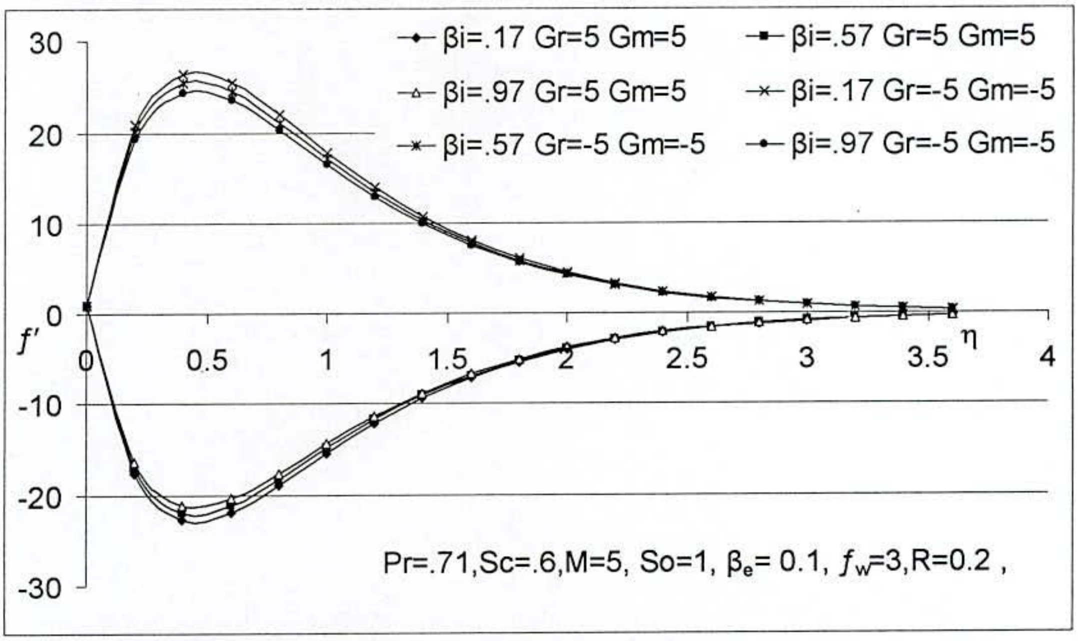

Fig. 3.4 Primary velocity profiles due to cooling and heating of plate for different values of βi.

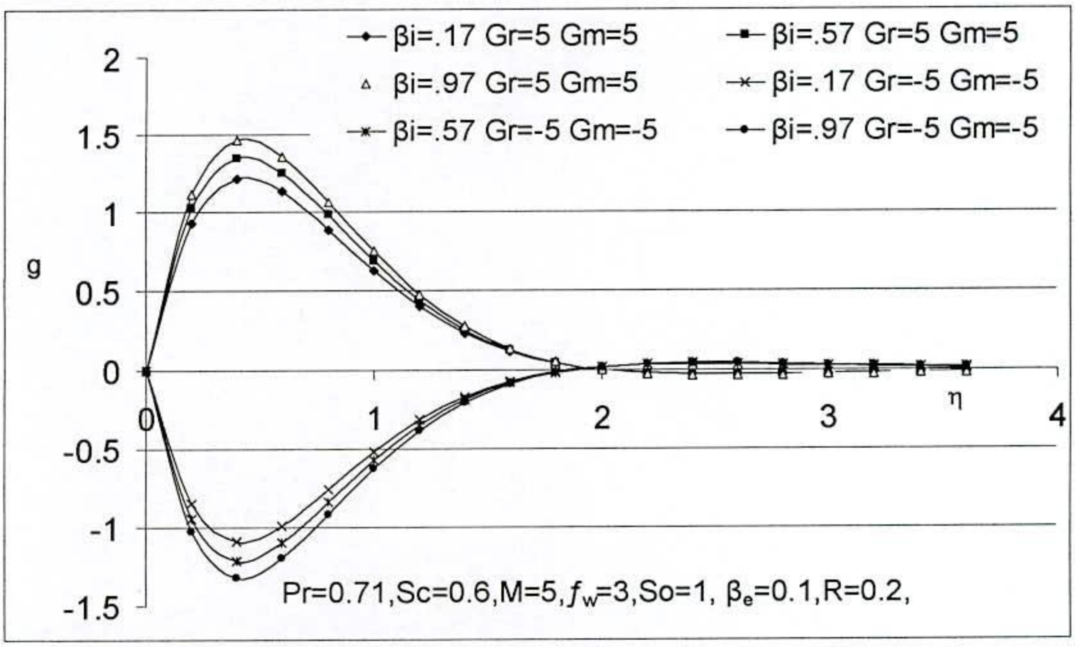

Fig.3.5. Secondary velocity profiles due to cooling and heating of the plate for different values of βi

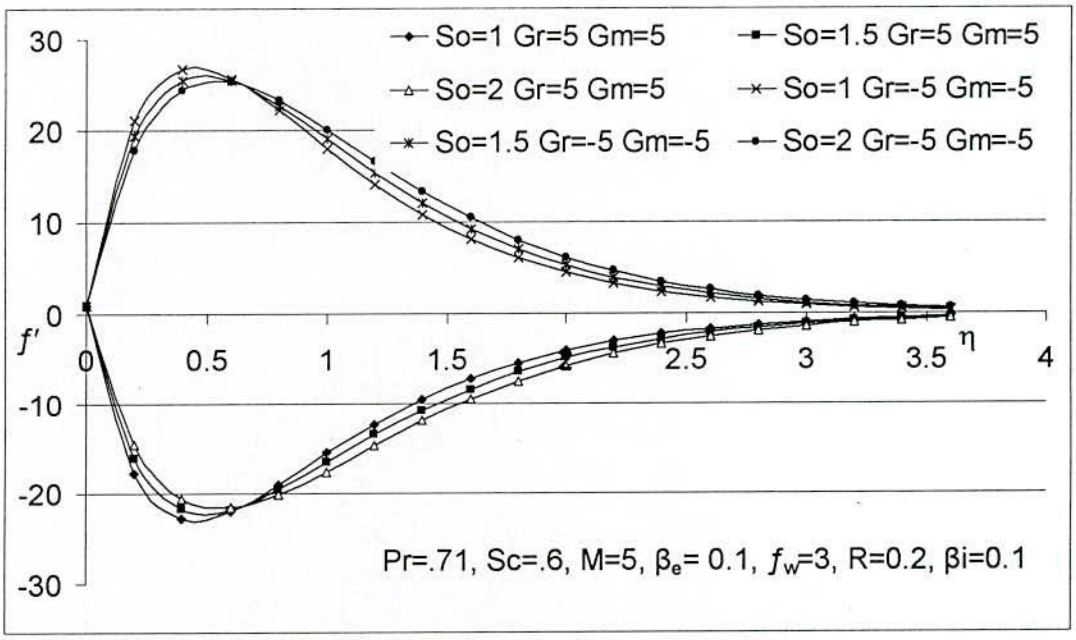

Fig. 3.6 Primary velocity profiles due to cooling and heating of plate for different values of So.

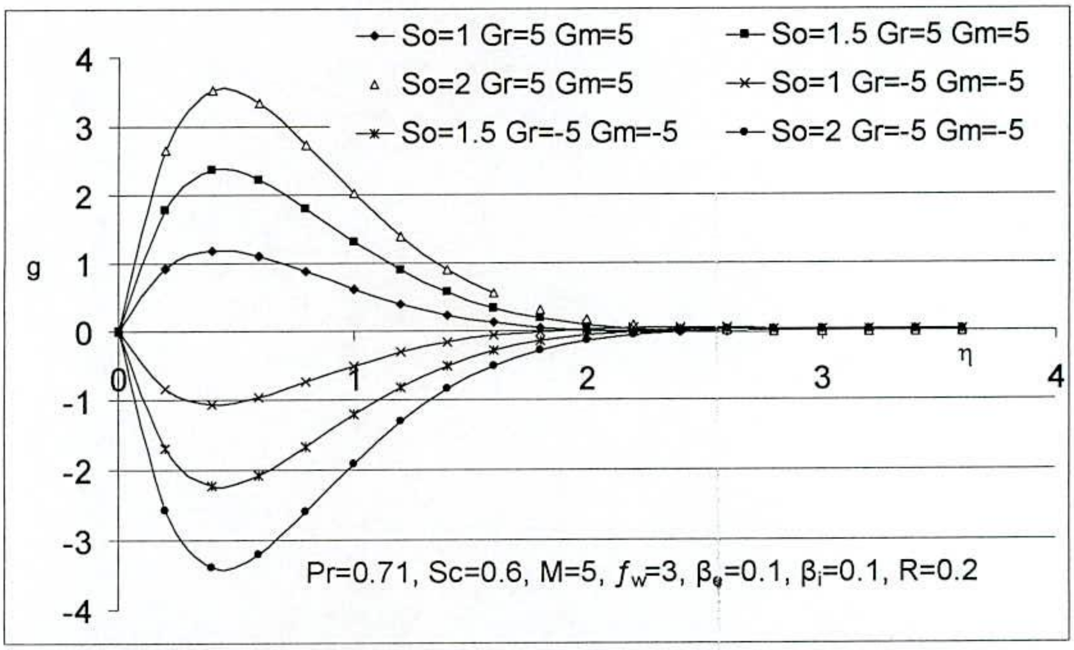

Fig.3.7. Secondary velocity profiles due to cooling and heating of the plate for different values of So.

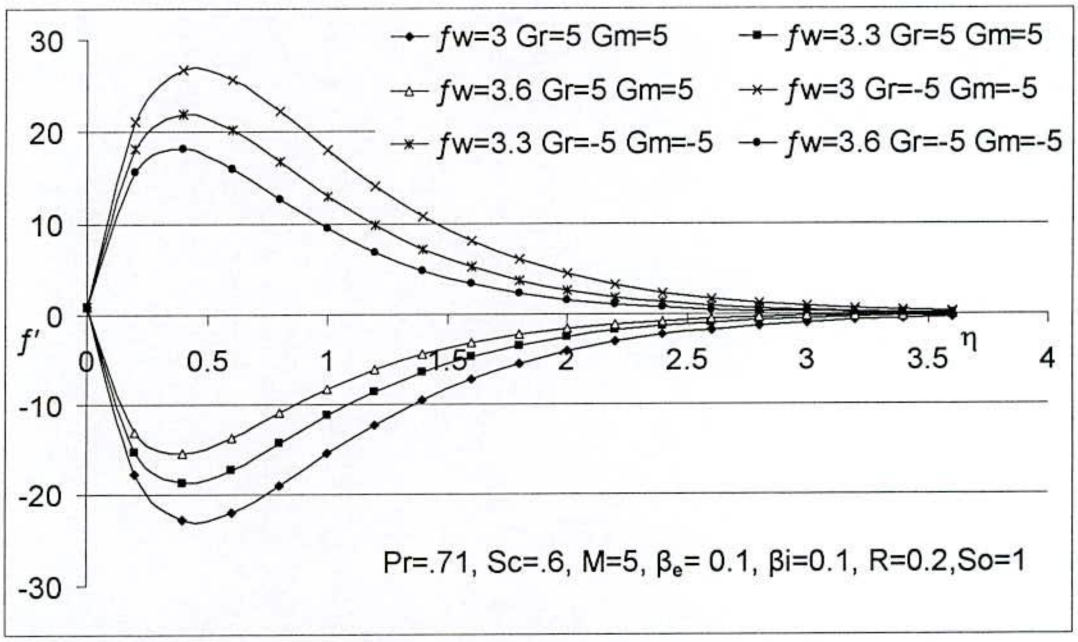

Fig. 3.8 Primary velocity profiles due to cooling and heating of plate for different values of *f*w.

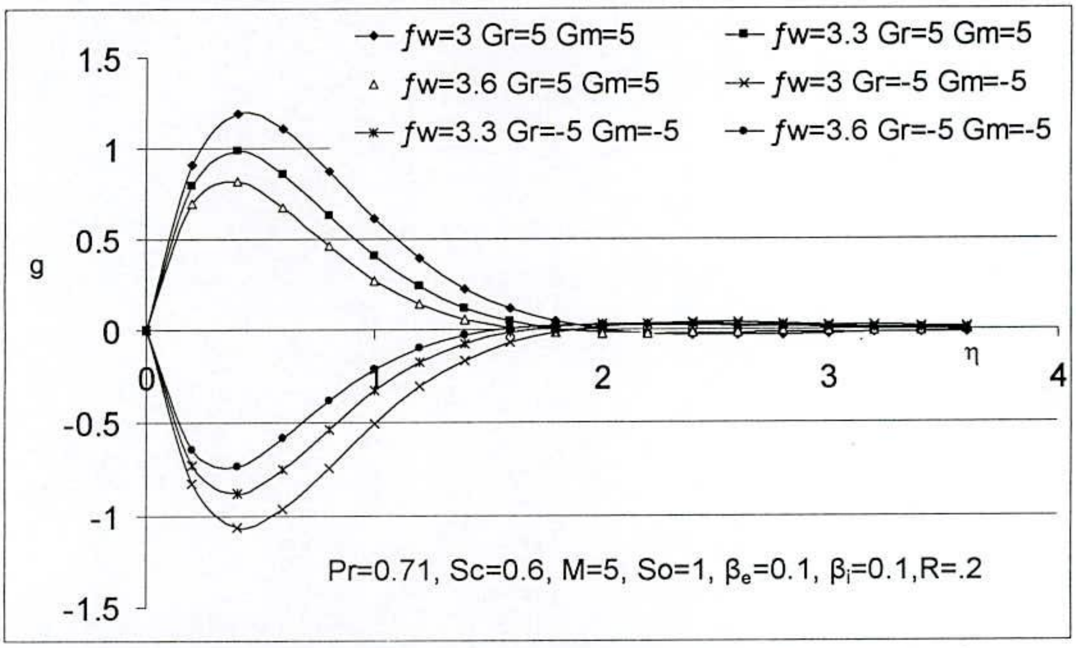

Fig.3.9. Secondary velocity profiles due to cooling and heating of the plate for different values of *f*w.

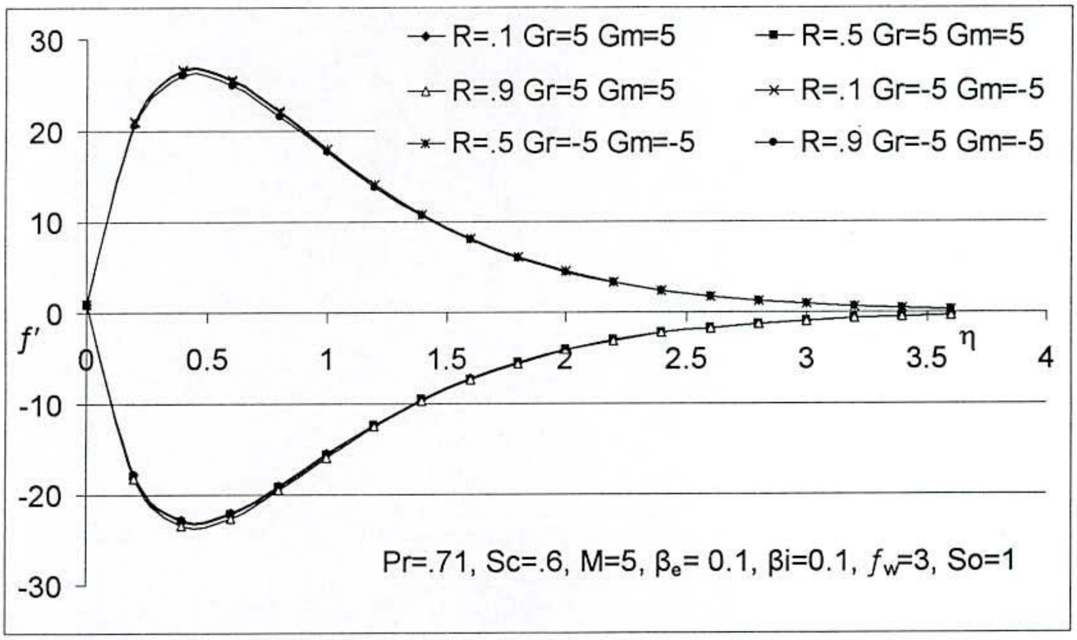

Fig. 3.10 Primary velocity profiles due to cooling and heating of plate for different values of R

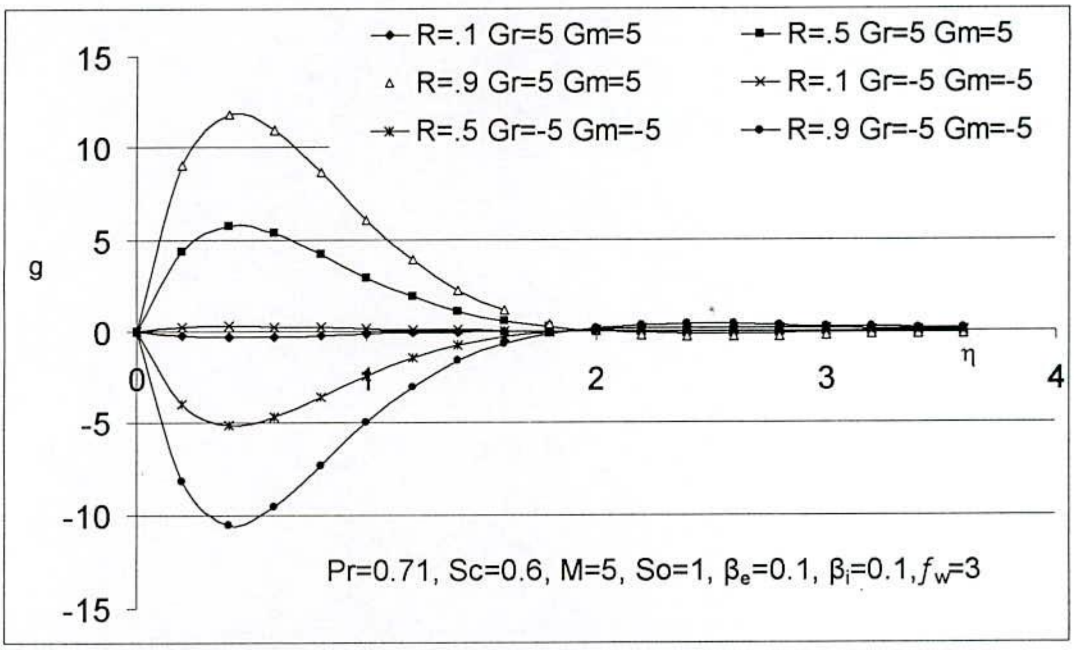

Fig.3.11. Secondary velocity profiles due to cooling and heating of the plate for different values of R.

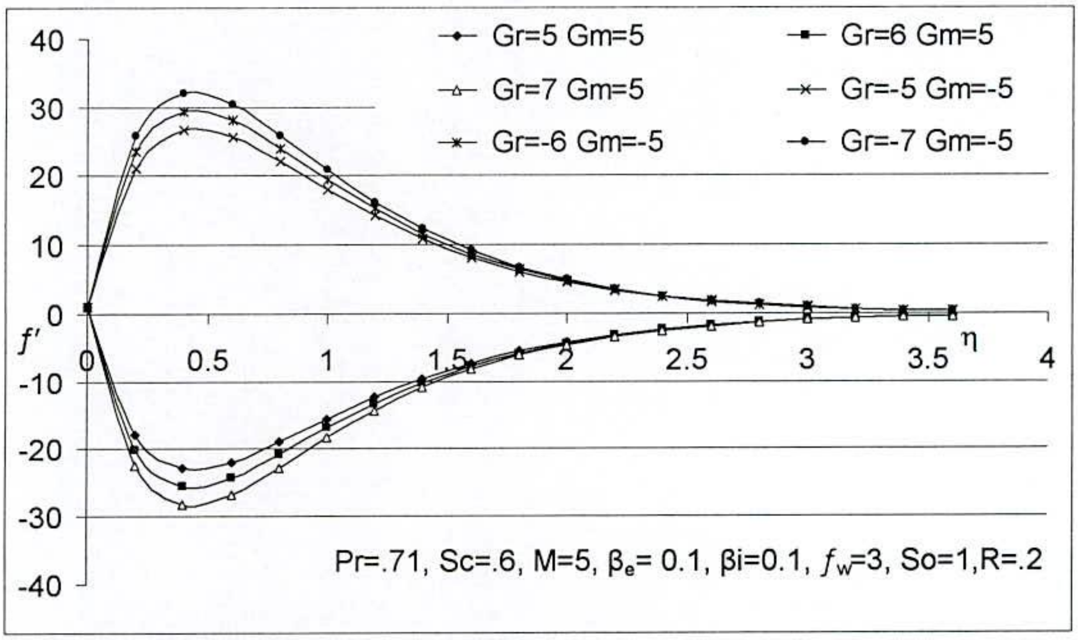

Fig. 3.12 Primary velocity profiles due to cooling and heating of plate for different values of Gr.

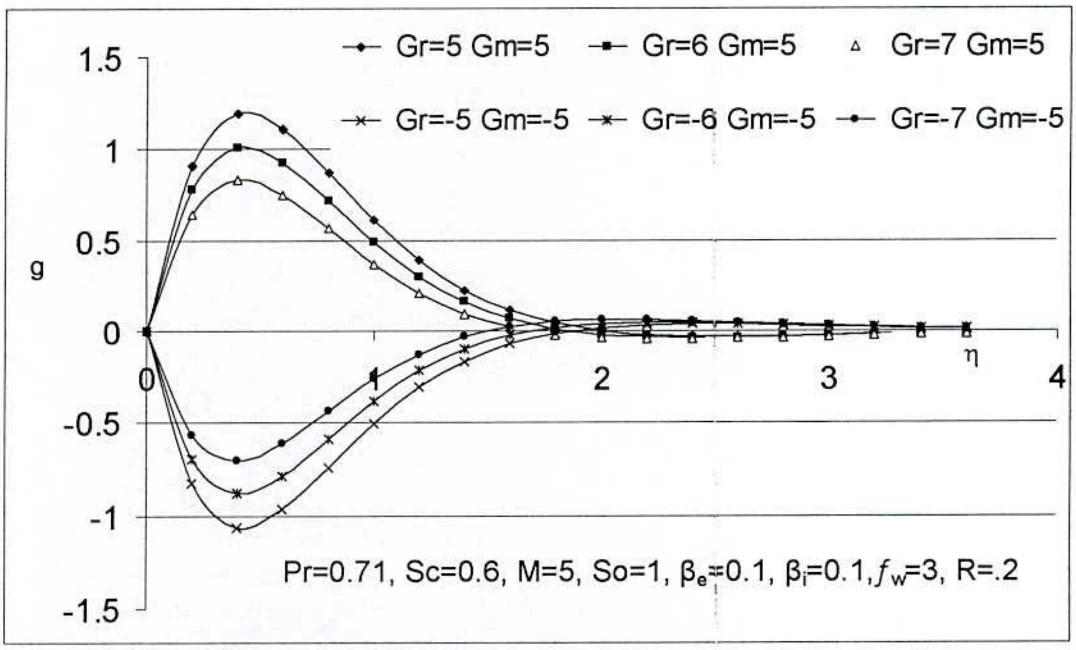

Fig.3.13. Secondary velocity profiles due to cooling and heating of the plate for different values of Gr.

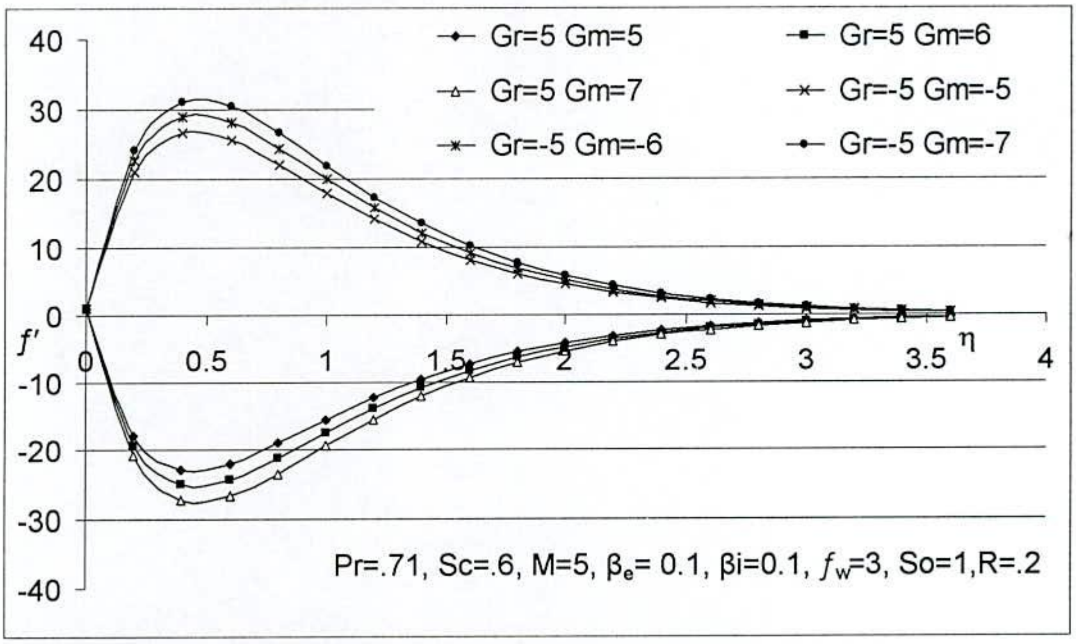

Fig. 3.14 Primary velocity profiles due to cooling and heating of plate for different values of Gm.

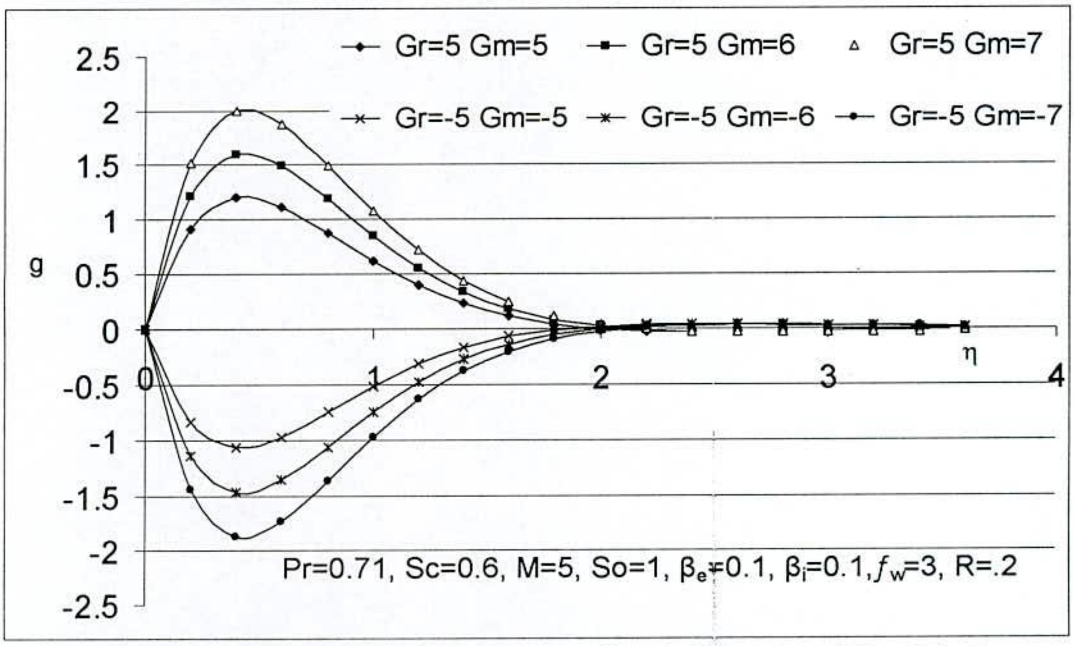

Fig.3.15. Secondary velocity profiles due to cooling and heating of the plate for different values of Gm.

Figure 3.8 represent the velocity profiles of primary velocity f' for different f_w with fixed Pr, Sc, So, R, β_i, β_e, R, and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increase in f_w produced steady increase in f'. For negative Gr and Gm, f' is positive and with the increase of f_w, f' decreases steadily. It may be noted that the sensitivity of f' is more with respect to f_w than So.

Figure 3.9 represent the velocity profiles of secondary velocity g for different f_w with fixed Pr, Sc, R, β_i, So, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increase in f_w produced steady decrease in g. For negative Gr and Gm, g is negative and with the increase of f_w, g increases steadily. It is observed that for the both cooling and heating of the plate g becomes zero with $f_w = 3$ at $\eta = 1.65$,but if $\eta > 1.65$ then g changes its sign with very small value. Similar situations took place with $f_w = 3.3$ and 3.6 at $\eta = 1.85$ and 2.05 respectively.

Figure 3.10 represent the velocity profiles of primary velocity f' for different R with fixed Pr, Sc, So, β_i, β_e, f_w, and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, f' is negative and increase in R produced slowly decreases in f'. For negative Gr and Gm, f' is positive and with the increase of R, f' slowly decreases.

Figure 3.11 represent the velocity profiles of secondary velocity g for different R with fixed Pr, Sc, f_w, β_i, So, β_e and Gr and Gm (both positive or both negative). It is found that for positive values of Gr and Gm, g is positive and increases rapidly for $R > 0.12$, when $R = 0.12$ then $g = 0$, but if $R < 0.12$ then g is negative. For negative Gr and Gm, g is negative and decreases rapidly for $R > 0.12$, when $R = 0.12$ then $g = 0$, but if $R < 0.12$ then g is positive. It is also found that all curves intersect at the point $\eta = 1.9$ where $g = 0$. When $\eta > 1.9$ then reverse effect of g is observed.

Figure 3.12 represents the velocity profiles of primary velocity f' for different values of Gr with fixed Pr, Sc, f_w, M, So, R, β_i and β_e along with fixed Gm (either positive or negative). For positive Gm, considering Gr as positive it is found that f' is negative and with the increase in Gr, f' decreases slowly. It is also observed that for negative Gm and considering Gr as negative f' is positive and with the decreases in Gr f' increases slowly.

Figure 3.13 represents the velocity profiles of secondary velocity g for different values of Gr with fixed Pr, Sc, f_w, M, So, R, β_i and β_e along with fixed Gm (either positive or negative). For positive Gm, considering Gr as positive it is found that g is positive and with the increase in Gr, g decreases slowly. It is also observed that for negative Gm and considering Gr as negative g is negative and with the decreases in Gr, g increases slowly. It is also found that when $Gr = 5$, $\eta = 1.7$ then $g = 0$, if $Gr = 5$ and $\eta > 1.7$ then reverse effect of g is observed. When $Gr = 6$, $\eta = 1.9$ then $g = 0$, if $Gr = 6$ and $\eta > 1.9$ then reverse effect of g is observed. Again when $Gr = 7$, $\eta = 2.05$ then $g = 0$, if $Gr = 7$ and $\eta > 2.05$ then reverse effect of g is observed. Further for the heating of the plate when $Gr = 5$, $\eta = 1.5$ then $g = 0$, if $Gr = 5$ and $\eta > 1.5$ then reverse effect of g is observed. When $Gr = 6$, $\eta = 1.75$ then $g = 0$, if $Gr = 6$ and

$\eta > 1.75$ then reverse effect of g is observed. Again when Gr = 7, $\eta = 1.9$ then g = 0, if Gr = 7 and $\eta > 1.9$ then reverse effect of g is observed.

Figure 3.14 represents the velocity profiles of primary velocity f' for different values of Gm with fixed Pr, Sc, f_w, M, So, R, β_i and β_e along with fixed Gr (either positive or negative). For positive Gr, considering Gm as positive it is found that f' is negative and with the increase in Gm, f' slowly decreases. It is also observed that for negative Gr and considering Gm as negative f' is positive and with the decreases in Gm, f'slowly increases.

Figure 3.15 represents the velocity profiles of secondary velocity g for different values of Gm with fixed Pr, Sc, f_w, M, So, R, β_i and β_e along with fixed Gr (either positive or negative). For positive Gr, considering Gm as positive it is found that g is positive and with the increase in Gm, g increases rapidly. It is also observe that for negative Gr and considering Gm as negative g is negative and with the decreases in Gm, g decreases rapidly. It may be noted that the sensitivity of g is more with respect to Gm than Gr. It is further observed that for the cooling of the plate when Gm = 5, $\eta = 2.05$ then g = 0, if Gm = 5 and $\eta > 2.05$ then reverse effect of g is observed. When Gm = 6, $\eta = 2.1$ then g = 0, if Gm = 6 and $\eta > 2.1$ then reverse effect of g is observed. Again when Gm = 7, $\eta = 2.15$ then g = 0, if Gm = 7 and $\eta > 2.15$ then reverse effect of g is observed. Further for the heating of the plate when Gm = 5, $\eta = 1.9$ then g = 0, if Gm = 5 and $\eta > 1.9$ then reverse effect of g is observed. When Gm = 6, $\eta = 2.05$ then g = 0, if Gm = 6 and $\eta > 2.05$ then reverse effect of g is observed. Again when Gm = 7, $\eta = 2.1$ then g = 0, if Gm = 7 and $\eta > 2.1$ then reverse effect of g is observed.

Finally the effect of various parameters on the components of the skin friction coefficient τ_x and τ_z are shown in table 3.1 -3.3. From table 3.1, we observe that the component τ_x increase with the increase of f_w but, τ_z and Sh decreases in the case of cooling of the plate. It is further observed that, in the case of heating of the plate, with the increase of suction parameter f_w, τ_x and Sh decreases where as τ_z increases. It is farther observed that the skin friction coefficient τ_x and τ_z both decreases when the rotation parameter R increases. Again for the heating of the plate with the increase of R, τ_x and τ_z both increases. From table 3.2 in the case of cooling of the plate we observe that the τ_x and τ_z both increases with the increase of Hall parameter β_e, where as for fixed β_e with the increase in β_i, τ_x increases but τ_z decreases. Again for the heating of the plate with the increase of β_e, τ_x and τ_z both decreases, where as for the heating of the plate with the increase in β_i, τ_x decreases but τ_z increases. From equations (3.60) and (3.61) we observe that Nu and Sh do not depend on the parameters $M_1$and M_2 and thus not on β_i and β_e. From table 3.3 in the case of cooling of the plate we observe that the skin friction coefficients τ_x and τ_z decreases but the Sherwood number Sh increases with the increase of Sc. Further we observe that τ_x and τ_z both increases but Nu and Sh both decreases with the increase of Pr. Again in the case of heating of the plate τ_x, τ_z and Sh increases with the increase of Sc. Further we observe that τ_x, τ_z,Nu and Sh all decreases with the increase of Pr.

Table 3.1

Numerical values proportional to τ_x, τ_z, Nu and Sh for β_e= 0.6, β_i =0.1, Pr =0.71, Sc=0.6, M=5 and So=1

Gr	Gm	R	f_w	τ_x	τ_z	Nu	Sh
10	4	0.2	3	-371.746	0.694725	0.708865	0.55938
10	4	0.3	3	-371.655	0.467339	0.708865	0.55938
10	4	0.2	3.3	-306.832	0.60782	0.708968	0.524346
-10	-4	0.2	3	371.7283	-0.85017	0.708865	0.55938
-10	-4	0.3	3	371.8191	-0.57191	0.708865	0.55938
-10	-4	0.2	3.3	306.2459	-0.66896	0.708968	0.524346

Table 3.2

Numerical values proportional to τ_x, τ_z, Nu and Sh for R=0.2 f_w=3, Pr =0.71, Sc=0.6,M=5 and So=1

Gr	Gm	β_e	β_i	τ_x	τ_z
10	4	0.3	0.1	-454.005	0.235659
10	4	0.6	0.1	-371.746	0.694725
10	4	0.6	0.3	-352.875	0.539235
-10	-4	0.3	0.1	455.7263	-0.40061
-10	-4	0.6	0.1	371.7283	-0.85017
-10	-4	0.6	0.3	352.7218	-0.61357

Table 3.3

Numerical values proportional to τ_x, τ_z, Nu and Sh for β_e= 0.6, β_i =0.1, R=0.2, f_w=3, M=5 and So=1

Gr	Gm	Pr	Sc	τ_x	τ_z	Nu	Sh
10	4	0.71	0.22	-254.77	25.17645	0.708865	0.230152
10	4	0.71	0.60	-371.746	0.694725	0.708865	0.55938
10	4	7	0.60	-138.719	2.438216	-21.2917	-3.20313
-10	-4	0.71	0.22	254.7525	-25.3319	0.708865	0.230152
-10	-4	0.71	0.60	371.7283	-0.85017	0.708865	0.55938
-10	-4	7	0.60	138.7017	-2.59367	-21.2917	-3.20313

Appendix 3.A

$A_1 = (So\ Sc\ Pr)/(Sc-Pr)$; $A_2 = 1- A_1$; $A_3 = 1 + M_1$;

$A_4 =(Gr + A_1Gr)/(Pr^3 - Pr^2)$; $A_5 = GmA_2/(Sc^3 - Sc^2)$;

$A_6 = A_3 - 0.5 - A_4Pr - A_5 Sc$; $A_7 = A_4Pr + A_5Sc + 0.25 - A_3 - A_4 - A_5$;

$A_8 = Pr^2 /(1+Pr)$; $A_9 = B_1/((Pr+1)(Pr-Sc+1))$; $A_{10} = B_2/(Sc+1)$;

$A_{11} = B_3/(Pr(Pr - Sc))+B_5(2Pr - Sc)/(Pr^2(Pr - Sc)^2)$:

$A_{12} = B_4/Sc$; $A_{13}= B_5 / (Pr (Pr - Sc))$; $A_{14} = -A_9 - A_{10} - A_{11}$;

$A_{15} = A_3 + M_1A_3 -(M_2 - 4R)^2$; $A_{16} = 1 + 2A_3 + 0.5M - 2A_6$;

$A_{17} = B_{12}/(Sc^2 - Sc^3)$; $A_{18} = B_{11}/ (Pr^2 -Pr^3)$; $A_{19} = B_{10}/(Sc(Sc +1)^2)$;

$A_{20} = B_9/(Pr(Pr +1)^2)$;

$A_{21} = A_{17} - B_{13}Sc + A_{18} - B_{14} Pr + A_{19}(Sc +1) +A_{20}(Pr +1) +5/24 +1.5A_3$
$\quad - B_{15}Sc -B_{16}Pr +B_6 - 2A_{15} - 0.5A_{16}$;

$A_{22}= A_{20}+ 5/72 + A_3 - B_{13} - B_{14}+ A_{19}-B_{15} - B_{16} - A_{21} - 0.25A_{16}$;

$A_{23} = B_{13} + B_{15}$; $A_{24} = B_{14} + B_{16}$; $A_{25} = B_6 - 2A_{15}$;

$A_{26} = (M_2 - 4R)(-1-A_3 -M_1)$; $A_{27} = A_{26} + B_{17}$;

$A_{28} = A_4 (M_2 - 4R)/(Pr- 1)$; $A_{29} = A_5(M_2 -4R)/(Sc - 1)$; $A_{30}= -A_{28} - A_{29}$;

$A_{31} = -A_8Pr -Pr+A_7$; $A_{32} =(A_8Pr^2 +A_8Pr-.25Pr)/(4+2Pr)$;

$A_{33} =((1+2A_8)Pr^2 +(A_5+A_6+A_8)Pr)/(1+Pr)$;

$A_{34} =0.5A_4/Pr$; $A_{35} = B_{18}/(Pr+1)$; $A_{36}= (2+Pr)B_{18}/(Pr+1)^2 -A_{33}$;

$A_{37} = -A_{32}+ A_{33} +A_{34} - A_{36}$; $A_{38} = B_{19}/(4+4Pr +Pr^2-2Sc-ScPr)$;

$A_{39} = B_{20}/(4 +2Sc)$; $A_{40} = B_{25}/(4Pr^2-2ScPr)$; $A_{41}= B_{26}/(Pr^2+ScPr)$;

$A_{42} = B_{27}/ (2Sc^2)$; $A_{43} = B_{27}/(1+Sc)$; $A_{44} = B_{23}/(1+2Pr +Pr^2-Sc-ScPr)$;

$A_{45} = - B_{29}/(2Sc)$; $A_{46} = B_{32}/(Pr^2-ScPr)$; $A_{47} =B_{33} +B_{38}$;

$A_{48} =B_{34} +B_{37}$; $A_{49} =B_{35} +B_{41}+B_{43}$; $A_{50} =B_{36} +B_{39}$;

$A_{51} =B_{40} +B_{42}$; $A_{52} = -(A_{38} +A_{39} +A_{40} +A_{41} +A_{42}+A_{47} +A_{48} +A_{49})$

$B_1 = 2ScA_1 - A_1ScPr + ScSoA_8(1 + pr)^2$; $B_2 = 2\ Sc\ A_2 -A_2\ Sc^2$;

$B_3 = ScA_1Pr -SoScA_8Pr^2 -Sc\ So\ 2\ Pr^2$; $B_4 = Sc^2 A_2$; $B_5 = Sc\ SoPr^3$;

$B_6 = A_6 - 2A_3 - A_7 + A_6 M_1 - M_1 A_3$; $B_7 = Pr_2A_4 + M_1A_4Pr + GrA_8 +GmA_{11}$;

$B_8 = A_5Sc^2 + M_1ScA_5 + Gm\ A_{14}$; $B_9 = A_4Pr^2+A_4+ A_8Gr-A_9Gm$;

$B_{10} = A_5 Sc^2 +A_5 - A_{10} Gm$; $B_{11} = Gr\ Pr -GmA_{13}$; $B_{12} = GmA_{12}$;

$B_{13} = B_{12} (2Sc-3Sc^2)/(Sc^2- Sc^3)^2$; $B_{14} = B_{11}(2Pr - 3\ Pr^2)/(Pr^2 - Pr^3)^2$;

$B_{15} = B_8/(Sc^3- Sc^2)$; $B_{16} = B_7/(Pr^3 - Pr^2)$; $B_{17} = (M_2 - 4R)(1-A_6+A_3)$;

$B_{18}= Pr^3-A_3Pr$;

$B_{19} = 2Sc\,A_9 - ScA_1 - ScA_9(1+Pr) + 0.25ScPrA_1 + 0.25\,A_2Sc^2 - ScSoA_{32}(2+pr)^2;$

$B_{20} = 2Sc\,A_{10} - ScA_2 - ScA_{10}(1+Sc);$

$B_{21} = 2Sc\,A_{11} - 2ScA_1A_6 - 2ScA_1A_3 - ScA_9(1+Pr) - ScPrA_{11}$

$+ A_{13}Sc + PrScA_1A_6 + 2SoScA_{35}(1+Pr) - ScSo\,A_{36}(1+Pr)^2;$

$B_{22} = -2Sc\,A_{12} - 2ScA_2A_3 + A_{12}Sc^2 + A_3A_2Sc^2;$

$B_{23} = 2ScA_{13} - PrScA_{13} + PrScA_1A_3 + 2ScA_1A_3 + ScSo\,A_{35}(1+Pr)^2;$

$B_{24} = 2ScA_{14} - 2ScA_2A_6 + 2ScA_2A_3 - ScA_{12} + ScA_{10}(1+Sc) + A_{14}Sc^2 - A_6A_2Sc^2;$

$B_{25} = -4ScSoA_{34}Pr^2 - PrScA_4A_1;$

$B_{26} = -2A_5A_1Sc^2 - 2PrScA_2A_4 + PrScA_1A_5 + A_2A_4Sc^2 - A_5Pr^2(Sc+Pr)/Sc;$

$B_{27} = -A_2A_5Sc^2;$

$B_{28} = -ScSoA_{37}Pr^2 + PrScA_7A_1 + PrScA_{11} - ScA_{13} + 2PrScSoA_{31} + ScSoPr^2;$

$B_{29} = -Sc^2A_{12};$ $B_{30} = A_2A_7Sc^2 + A_{14}Sc^2 + ScA_{12};$

$B_{31} = PrScA_{13} - ScSoA_{31}Pr^2 + ScSoPr^3;$ $B_{32} = -0.5ScSoPr^4;$

$B_{33} = B_{21}/(1+2Pr+Pr^2-Sc-ScPr);$ $B_{34} = B_{24}/(1+Sc);$ $B_{35} = B_{28}/(Pr^2-ScPr);$

$B_{36} = -B_{30}/Sc;$ $B_{37} = B_{22}(2+Sc)/(1+Sc)^2;$

$B_{38} = B_{23}(2+2Pr-Sc)/(1+2Pr+Pr^2-Sc-ScPr)^2;$ $B_{39} = -B_{29}/Sc^2;$ $B_{40} = B_{31}/(Pr^2-ScPr);$

$B_{41} = B_{31}(2Pr-Sc)/(Pr^2-ScPr)^2;$ $B_{42} = 2B_{32}(2Pr-Sc)/(Pr^2-ScPr)^2;$

$B_{43} = 2B_{32}(2Pr-Sc)^2/(Pr^2-ScPr)^3 - 2B_{32}/(Pr^2-ScPr)^2$

References

Alam, M. M. and Satter, M. A. (2001) Dhaka University Jr. of Sc.,49(2), 147-153.

Bestman, A. R. (1990). International J. Energy Resch. 14, 384.

Ram, P. C. and Takhar, H. S. (1993). The Japan Society of Fluid Machanics 5983.

Singh, A. K. and Dikshit, C. K. (1988). Astrophys, Space Science, 148, 249.

References

References

Alam, M. M. and Sattar, M. A. (1999). Ind. Jr. of Theoretical Phys., **47(1)**, 19.

Alam, M. M. and Satter, M. A.(2000). J. of Energy Heat and mass transfer., **22**, 31.

Alam, M. M. and Sattar, M. A. (2001).Dhaka Uni. Jr. of Sc., **49(2)**,147.

Alfaven, H. (1942). Arkiv F. Mat. Astro. O. Fysik., 295(**2**).

Agrawal, H. L. . Ram, P. C. and Singh, S. S. (1977). Acta, Phys. Acad. Sci. Hung., **42**, 49.

Agrawal, H. L. . Ram, P. C. and Singh, S. S. (1980). Can. J. Chem. Engg., **58**, 131.

Batchelor, G. K. (1970). An Introduction to fluid dynamics, Cambridge University press.

Bastman, A. R. (1990). Astrophys. Space Sci., **173**,93.

Cobble, M. H. (1977). J. Engg. Maths.,**11**, 249.

Cowling, T. G. (1957). Magnetohydrodynamics, Interscience Publ. Inc., New York.

Cramer, K. R. and Pai, S. I. (1973). Magnetofluid Dynamics for Engineers and applied physicists, McGraw Hill Book Company, New York.

Debnath, L. (1974). Phys. Fluids, **17(9)**, 1704.

Eckert, E. R.G. and Drake, R. M. (1972). Analysis of Heat and Mass Transfer, McGraw-Hill Book Co., New York.

Farady, M. (1832). Elec. Phill. Trans., **15**, 175.

Gergantopoulos, G.A., Koullias, J., Gousdas, C. L. and Couragenis, C. (1981). Astrophys. Space Sci., **74**, 359.

Gosman, A. D., Pun, W. M., Runchal, A. K., Spalding, D. B. and Walfshtein, M., (1969). Heat and Mass transfer in recirculating flows, Academic Press, London.

Gupta, A. S. (1972). J. Phys. Fluids, **15**, 930.

Haldavneker, D. D. and Soundalgeker, V. M. (1977). Acta. Phys. Acad. Sci. Hung., **43**,(3/4),243.

Hughes, W. F. and Young, F. J. (1966). The electromagnetodynamics of fluids, Jhon Wiley, New York.

Jaluria, Y. (1980). Natural convection heat and mass transfer vol. 5, Pergamon Press, Oxford, UK.

Jha, B. K. and Singh, A.K. (1990). Astrophys. Space Sci. **173**, 251.

Kafoussias, N. G. (1992). Astrophys. Space Sci. **192**,11.

Mazumder, B. A. Gupta, A. S. and Datta, N. (1976a). Int. J. Heat Mass Transfer, **19**, 523.

Mazumder, B. S. Gupta, A. S. and Datta, N. (1976b). Int. J. Eng. Sci., **14**, 258.

Nanbu, K. (1971). AIAA, J. **9**, 1642.

Nanousis, N. (1992). Astrophys. Space Sci.,**191**, 313.

Patankar, S. V. and Spalding, D. B. (1970). Heat and Mass Transfer in Boundary Layers, Intertext Books, London.

Pai, S. I. (1962). Magnetogasdynamics and Plasma dynamics, Springier Verlag, New York.

Pop, I. and Takhar, H. S. (1993).ZAMM, **73**, T534

Prandtl, L. (1904). Proc. Third Internal Math. Cong. Heidelberg [English translation in NACA Technical Memo. 452]

Ram, P. C. and Takhar, H. S. (1993). Fluid Dy. Res., **11**, 99.

Rosenberg, D. U. V. (1969). Method for numerical solutions of partial differential equations , Elsevier, New York.

Satter, S. A. and Alam, M. M. (1995). Indian Jr. of Pure and Applied Math **26(2)**, 157.

Schlichting, H. (1968). Boundary Layer theory, McGraw Hill, New York.

Shercliff, J. A.(1965). A textbook of Magnetohydrodynamics, Pergamon Press, UK.

Singh, A. K. (1985). Astrophys, Space Sci., **115**,387.

Singh, A. K. and Dikshit, C. K. (1988). Astrophys, Space Sci., **148**, 249.

Soundalgeker, V. M. Gupta, S. K. and Birajdar, N. S. (1979). Nucl. Eng. Dec., **53(3)**, 339.

Soundalgeker, V. M. and Ramanamurthy, T. V. (1980). J. Engg. Maths. **14**, 155.

Spalding, D. R. (1977). CENMIX, Pergamon Press, Oxford, UK.