New Analytical Methods for Finding Optimal Solution of Transportation Problems

by

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Dedicated to My

Respectable & Honorable Teacher

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Declaration

This is to certify that the thesis work entitled **"New Analytical Methods for Finding Optimal Solution of Transportation Problems"** has been carried out by Rijwana Kawser in the Department of Mathematics, Khulna University of Engineering & Technology, Khulna-9203, Bangladesh. The above thesis work or any part of the thesis work has not been submitted anywhere for the award of any degree or diploma.

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Approval

This is to certify that the thesis work submitted by Rijwana Kawser entitled **"New Analytical Methods for Finding Optimal Solution of Transportation Problems"** has been approved by the board of examiners for the partial fulfillments of the requirements for the degree of Master of Philosophy in the Department of Mathematics, Khulna University of Engineering & Technology, Khulna-9203, Bangladesh in February 2016.

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Abstract

Determining the efficient solution for large scale of transportation problems (TPs) is an important task in operations research. TPs can also be formulated as a linear programming problem (LPP). The TP is concerned for finding an optimal distribution plan for a single commodity from several sources to different destinations. A given supply of the commodity is available at the different number of sources and there is a specified demand for the commodity at each of the various numbers of destinations and the unit transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant during the period.

In this thesis, a modified Vogel's approximation method has been developed for obtaining a good primal solution of a large scale of TPs. Also, a direct analytical method has been developed for finding an optimal solution for a wide range of transportation problems. Numerical illustrations are considered and the optimality tests of the results are justified by these methods. The most attractive feature of these methods is that they require very simple arithmetical and logical calculations. So, it is very easy to understand and use for layman. These methods will be very worthwhile for those decision makers who are dealing with logistics and supply chain related issues. One can easily adopt the proposed methods among the existing methods for simplicity of the presented method.

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CHAPTER I

Introduction

Now a days, in turbulent economic environment, gaps between sale price and production cost have created big hazards in consumers market in Bangladesh. The sale prices of daily commodities hike up day by day breaking its limit. Dissatisfaction rises up among the common people. Authority is in big trouble to manage this discrepancy. High transportation cost is one of any manufacturing item goes up if the transportation cost of its raw material be high. Bangladesh is still following the manual system in transporting goods. This inconvenience can be handled by establishing a strong and effective transportation network. Effective transportation network can influence this field reducing the production cost and sale price of daily commodities. Also the profit of a manufacturing company can be increased by reducing the transportation cost of its raw materials. This thesis provides a mathematical framework in developing algorithms for solving transportation problems which influences the concerned field to minimize the total transportation cost (TTC).

The transportation problems (TPs) are concerned for finding an optimal distribution plan for sending a single commodity from different sources to several destinations. A given supply of the commodity is available at different number of sources and there is a specified demand for the commodity at each of the various numbers of destinations and the unit transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant during the period. The problem is to find the optimal distribution plan for transporting the products from different sources to several destinations that minimizes the TTC. TPs can also be formulated as linear programming problems that can be solved using either dual simplex or Big M method. Sometimes this can also be solved using interior approach method. There are many methods for solving TPs such as North West Corner Method (NWCM), Least Cost

Method (LCM), Vogel's Approximation Method (VAM). VAM gives approximate solution while Modified Distribution (MODI) and Stepping Stone (SS) methods are considered as standard techniques for obtaining the optimal solution of TPs. Since decade these two methods are being used for solving TPs. Goyal [1] has improved Vogel's approximation method (VAM) for the unbalanced TP. Ramakrishna [2] has discussed some improvement to Goyal's Modified VAM for unbalanced TPs. Moreover, Sultan [3], Sultan and Goyal [4] have studied initial basic feasible solutions and resolution of degeneracy in transportation problems. Few researchers have tried to give their alternate method for overcoming major obstacles over MODI and SS methods. Adlakha and Kowalski [5] have suggested an alternative solution algorithm for solving certain TP based on the theory of absolute point. Adlakha et al. [6] have presented a technique for solving transportation problems with mixed constraints. Ping and Chu [7] have discussed a new approach so called dual matrix approach to solve the TPs which gives also an optimal solution. Pandian and Natarajan [8] and Sudhankar *et al.* [9] have proposed two different methods for finding an optimal solution directly. Recently, Ullah and Uddin [10] have developed an algorithmic approach for calculating the minimum time of shipment in TPs. In this thesis, simple heuristic approach is proposed for finding optimal solutions of transportation problems with less number of iterations and very easy computations. The stepwise procedures of the proposed methods are carried out in this thesis and numerical examples are given for justifying the proposed methods and testing the optimality.

In **Chapter II,** the review of literature is presented. In **Chapter III,** a modified Vogel's approximation method has been developed for obtaining good primal solution of transportation problems. **In Chapter IV,** a direct analytical method has been presented for finding optimal solution of transportation problems. Finally, **in Chapter V,** some concluding remarks are given.

CHAPTER II

Literature Review

2.1 Preliminary Concept on Transportation Problems.

2.1.1 Transportation Problem

We explore a special type of linear programming (LP) model. Its structure and model is called transportation model and it can be solved using more efficient computational procedure than the simplex method. The transportation problem is also called network flow problem. The transportation problem deals with the distribution of goods from several points of supply (sources) to a number of points of demand (destinations). Usually, we have a given capacity of goods at each source and a given number of requirements for the goods at each destination.

.This model can be used for inventory control, employment scheduling, personnel and machine assignment, plant location, product mix problems, cash flow statements and many others so that the model is not really confined to transportation only. Transportation model plays a vital role to ensure the efficient movement and in time availability of raw materials and finished goods from sources to destinations.

2.1.2 Objective of the Research

Transportation cost is very important factor in business arena. In Operation Research (OR), one uses several traditional methods for minimizing the total transportation cost. At the beginning of this work, our aim was to achieve more knowledge about transportation network. We have gone through many books and journals aiming to know more about transportation problems (TP). We have earned knowledge for finding the initial basic feasible solution (IBFS) of the TPs. We have studied the existing algorithms as well as recently developed algorithms and analyzed them. We have developed two efficient algorithms for solving the TPs. The objective of our proposed methods is to determine the amount to be shipped from each source (factory /origin) to each destination (warehouse) such that the total transportation cost (TTC) is minimized.

2.1.3 General Transportation Table (GTT)

A specially designed table is constructed and used in order to solve the transportation problems called the transportation table.

Table: 2.1 General transportation table.

Origin (i)	Destination (j)				Supply (a_i)
	D_1	D_2	$\cdots$	D_n	
S_1	c_{11} x_{11}	c_{12} x_{12}	$\cdots$	c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{21}	$\cdots$	c_{2n} x_{2n}	a_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}	$\cdots$	c_{mn}	a_m
Demand (b_j)	b_1	b_2	$\cdots$	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

The above GTT consists of mn rectangles in m rows and n columns, where m denotes the number of rows and n denotes the number of columns. Each rectangle is called a cell. The cell in *ith* row and *jth* column is termed as cell (i, j) each unit cost component c_{ij} is placed at the middle of the corresponding cell. A component of a feasible solution $x_{ij}, i = 1,2,\cdots m, j = 1,2,\cdots n$ if any, is to be placed at the left-top of c_{ij}. Supply capacities a_i of the different origins are shown on the rightmost column corresponding to each row of the table and the demands of different destinations b_j are listed on the lowermost row corresponding to each column.

The total number of variables is mn, total number of constraints is $m+n$ and the total number of allocation cells in a feasible solution are $m+n-1$.

2.1.4 Definitions of Some Terminologies in Transportation Problem.

Definitions of some terminologies in transportation problem are given bellow:

Source

The supply nodes in a transportation problem, that is from where goods are being supplied to the demanding nodes are called sources. In the table 2.1 $S_1, S_2, \cdots, S_m$ are the sources. Sources are also termed as origins or factories.

Destination

The demanding nodes in a transportation problem, that is to where goods are being supplied from the sources are called destination. In the table 2.1 $D_1, D_2, \cdots, D_n$ are destinations. Destinations are also termed as warehoues.

Supply Limit

The commodity available in a source to meet the requirement of the demand nodes is called supply limit of that source.

Demand Requirement

The required amount of commodity to meet the need of a demanding node is called the demand requirement.

Unit Cost

In the table 2.1, the elements C_{ij} represents the cost required to transport one unit of a commodity from the source i to the destination j is called the unit cost.

Solution

The number of units transported from each source to each destination is called the solution of the Transportation Problem.

Basic Feasible Solution

Let us consider a TP with m sources and n destinations. If the solution consists $m+n-1$ basic variables is called the basic feasible solution (BFS).

Optimal Solution

A feasible solution (may not be basic feasible) is said to be optimal if it optimizes the total transportation cost.

Degenerate Basic Feasible Solution

A basic feasible solution which contains less than $m+n-1$ allocations is called a degenerate basic feasible solution.

Balanced Transportation Problem

A Transportation Problem is said to be balanced transportation problem if total number of supply is same as total number of demand.

$$\text{Supply}\left(\sum_{i=1}^{m} a_i\right) = Demand\left(\sum_{j=1}^{n} b_j\right)$$

Unbalanced Transportation Problem

A Transportation Problem is said to be unbalanced transportation problem if total number of supply is not same as total number of demand.

$$\text{Supply}\left(\sum_{i=1}^{m} a_i\right) \neq Demand\left(\sum_{j=1}^{n} b_j\right)$$

2.1.5 Existence of a Basic Feasible Solution

A necessary and sufficient condition for the existence of a basic feasible solution to a transportation problem with m sources and n destinations are

(i). Total supply and total demand must be equal *i.e.*, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ and

(ii). Total number of occupied cell or allocation must be equal to $m+n-1$.

2.1.6 Mathematical Statement of the Transportation Problems

The classical transportation problem can be stated mathematically as follows:

Total transportation cost

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,\cdots m \text{ (C apacity constraints)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,\cdots n \text{ (Demand constraints)}$$

$$x_{ij} \geq 0, \text{ for all } i \text{ and } j$$

where,

z = total transportation cost to be minimized.

c_{ij} = unit transportation cost of a commodity from each source i to destination j.

x_{ij} = number of units of allocations from source i to destination j.

a_i = level of supply at each source i.

b_j = level of demand at each destination j.

2.1.7 Existing Methods for Obtaining Basic Feasible Solution for Transportation Problems

The initial feasible solution of TPs is obtained by the following existing methods such as

(i). North West Corner Method (NWCM).

(ii). Least Cost Method (LCM).

(iii). Vogel's Approximation Method (VAM).

2.1.8 Algorithm of Vogel's Approximation Method (VAM)

The Vogel approximation method is an iterative procedure for computing a basic feasible solution of the transportation problems. In VAM, the following steps are applied.

1. Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.
2. Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column.
3. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.
4. If the penalties corresponding to two or more rows or columns are equal, select the top most rows and the extreme left column.

2.2 Background of the Research

Simplex algorithm is used to solve the Linear Programming Problem (LPP). But it is laborious task. For this reason, researchers try to develop the way for avoiding the complexity of simplex algorithm. Resultant of one such effort is Transportation Model. The Transportation Problem (TP) is a special type of LPP where a single commodity is to be shifted from a different number of sources to a several number of destinations in such a way that the total transportation cost (TTC) is minimizing while satisfying the total supply limits and the total demand requirements.

Frederick [11] has observed that in a machine oriented manufacturing house, one machine works while another waits for supply. As a consequence of weak supply network system, the production is hampered. He has tried to solve the weak supply system of raw material from machine to machine so that each machine may have adequate supply so that the final production is maximized. In the early scientific era, job scheduling was haphazard. A particular job was done by the first machine but would wait for acceptance for the next machine. Henry [12] has mapped each job from machine to machine, minimizing every delay. Now, with the Gantt procedure, it is possible to plan machine loadings months in advance and still quote delivery dates accurately. In 1959, Erlang [13] a Danish mathematician published his work on the problem of congestion of telephone traffic. The difficulty was that during busy period telephone operators were unable to handle calls moment they were made, resulting in delayed calls. A few years after its appearance, his work was accepted by the British Post Office as the basis for calculating circuit facilities. Erlang formulae on waiting time are of fundamental importance to the theory of telecommunication network.

In 1941 Hitchcock [14] presented a study entitled 'The Distribution of a Product from Several Sources to Numerous Localities'. The presentation is regarded as the first important contribution to the solution of the Transportation Problems. Later on, the first algorithm in solving the TPs is developed by Dantzig [15] and referred to as North West Corner (NWC) rule. This is the method of finding the initial basic feasible solution (IBFS) of the TPs which consider the NWC cost cell at every stage of allocation. Then the row minima method consists in allocating as much as possible in the lowest cost cell of the first row of the transportation table (TT) whereas the column minima method consists of allocating as much as possible in the lowest cost cell in the first column. The matrix minima method considers the lowest cost cell of the TT in making allocation in every stage. Vogel's approximation method (VAM) provides comparatively better IBFS. Researchers have also been adding new algorithms in order to solve the TPs. Balakrishnan [16] has developed modified Vogel's approximation method for unbalance transportation problem. Shore [17] has presented the transportation problems and the Vogel's approximation method. Pandian and Natarajan [18] have presented a new method for solving bottleneck-cost transportation problems. Korukoglu and Balli [19] have developed an improved Vogel's approximation method for the Transportation Problems. Sharma and Sharma [20] have developed a new dual based procedure for the transportation problem. Vannan and Rekha [21] have developed a new method for obtaining an optimal solution for transportation problems. Kircal and Satir [22] have developed a heuristic for obtaining an initial solution for the transportation problems. Mathirajan and Meenakshi [23] have experimented analysis of some variants of Vogel's approximation method. Many veteran writers like Harvey [24], Reinfield and Vogel [25], Winston [26], Dano [27], Phillips et al. [28], Taha [29], Chatterjee [30], Gupta and Mahan [31], Swarup et al. [32] have discussed TP in the field of operation research with linear and nonlinear programming in their written publications. Sharma and Prasad [33] have obtained a good primal solution to the uncapacitated TPs. Pandian and Natarajan [34] have solved a new approach for TPs with mixed constraints. Patel and Bhathawala [35] have presented the new global approach to a transportation problem for finding an optimal solution for a wide range of transportation problems directly. Luhandjula [37] has discussed linear programming with a possibility objective function. Shimshak et al. [38] have developed a modification of Vogel's approximation method through the use of heuristics. Issermann [39] worked linear bottleneck transportation problem. Varma and Kumar [40] have solved age-old transportation problems by nonlinear programming methods. Patel and Bhathwala [41] have developed the new global approach to a transportation problem.

CHAPTER III

A Modified Vogel's Approximation Method for Obtaining Good Primal Solution of Transportation Problems

3.1 Introduction

Transportation problem is a particular class of linear programming which is associated with day-to-day activities in our real life. Transportation models play an important role in logistics and supply chains. It helps in solving problems on distribution and transportation of resources from a place to another. The problem basically deals with the determination of a cost plan for transporting a single commodity from various sources to several destinations. The aim of such TPs is to minimize the total transportation cost (TTC) of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity. TPs can be solved by using general simplex based integer programming methods, however it involves time-consuming computations. We are going to propose a specialized algorithm with less iteration for TPs that are much more efficient than the simplex algorithm. The basic steps for solving TPs are:

Step 1. Determination the initial feasible solution.

Step 2. Determination optimal solution using the initial solution.

The most common method used to determine efficient initial solutions for solving the TPs (using a modified version of the simplex method) is Vogel's approximation method (VAM). This method involves calculating the penalty (difference between the lowest cost and the second-lowest cost) for each row and column of the cost-matrix, and then assigning the maximum number of units possible to the least-cost cell in the row or column with the largest penalty. Vogel's approximation method (VAM) gives approximate solution while MODI and Stepping Stone (SS) methods are considered as standard techniques for obtaining optimal solution of TPs. Goyal [1] has improved Vogel's approximation method (VAM) for the unbalanced transportation problems. Ramakrishna [2] has discussed some improvement to Goyal's modified VAM for unbalanced TPs. Moreover, Sultan [3], Sultan and Goyal [4] have studied initial basic feasible solutions and resolution of degeneracy in TPs. Few researchers have tried to give their alternate methods for overcoming major obstacles over MODI and SS methods. Adlakha and Kowalski [5] have suggested an alternative solution algorithm for solving certain TPs based on the theory of absolute point. Ullah and Uddin [10] have developed an algorithmic approach to calculate the minimum time of shipment in TPs. Balakrishnan [16] has discussed modified Vogel's approximation method for unbalanced TP. Sharma et al. [33] have studied on uncapacitated TP for obtaining a good primal solution. Shimshak et al. [38] have studied on modification of Vogel's approximation method through the use of heuristics. In this study, a simple heuristic modified Vogel's approximation method (MVAM) is proposed for obtaining good primal solution of wide range of TPs and the solution obtained by the proposed method is often very good in terms of minimizing TTC.

3.2 Algorithm of Proposed Modified Vogel's Approximation Method (MVAM)

Step1: Subtract the largest entry from each of the elements of every row of the transportation table and place them on the left-top of corresponding element.

Step 2: Subtract the largest transportation cost from each of the entries of every column of the transportation table and write them on the left-bottom of the corresponding element.

Step 3: From a reduced matrix elements whose elements are the summation of left-top and left-bottom elements of Steps1 Steps 2.

Step 4: Calculate the Distribution Indicators by subtracting of the largest and next-to-largest element of each row and each column of the reduced matrix and write them just after and below of the supply and demand amount respectively.

Step 5: Identify the highest distribution indicator, if there are two or more highest indicators, choose the highest indicator along which the largest element is present. If there are two or more largest elements present choose any one of them arbitrarily.

Step 6: Allocate $x_{ij} = \min(a_i, b_j)$, $i = 1, 2, \cdots m$, $j = 1, 2, \cdots n$ on the left bottom of the largest element in the $(i, j)th$ cell of the reduced matrix, a_i and b_j represent the supply capacity and required demand at each source and destination respectively.

Step 7: If $a_i < b_j$ leave the ith row and readjust b_j as $b'_j = b_j - a_i$. If $a_i > b_j$ leave the jth column and readjust a_i as $a'_i = a_i - b_j$. If $a_i = b_j$ then leave both the ith row and jth column.

Step 8: Repeat Steps 4 to step 7 until the rim requirement exhausted.

Step 9: Pull the allocations of the positive allocated cells of the reduced matrix to the original transportation table and calculate the TTC, $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ where x_{ij} is the total allocation of the $(i, j)th$ cell and c_{ij} is the corresponding unit transportation cost.

3.3. Numerical Examples

Let us consider the following TP to find out the minimum TTC with three sources and four destinations.

Table 3.1 Transportation table

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	4	19	22	11	$a_1 = 100$
F_2	1	9	14	14	$a_2 = 30$
F_3	6	6	16	14	$a_3 = 70$
Demand	$b_1 = 40$	$b_2 = 20$	$b_3 = 60$	$b_4 = 80$	200 (Total)

At first we find the row differences and column differences which are shown in Table 3.2.

Table 3.2 Distribution indicator table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	${}^{18}_{2}4$	${}^{3}_{0}19$	${}^{0}_{0}22$	${}^{11}_{3}11$	100
F_2	${}^{13}_{5}1$	${}^{5}_{10}9$	${}^{0}_{8}14$	${}^{0}_{0}14$	30
F_3	${}^{10}_{0}6$	${}^{10}_{13}6$	${}^{0}_{6}16$	${}^{2}_{0}14$	70
Demand	40	20	60	80	200 (Total)

Now we can form a reduced matrix as follows in Table 3.3.

Table 3.3 Reduced matrix of given TP.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	20	3	0	14	100
F_2	18	15	8	0	30
F_3	10	23	6	2	70
Demand	40	20	60	80	200 (Total)

Now we determine the distribution indicators for each row and each column by subtracting the largest and next-to-largest element. Among the distribution indicators, 13 is the highest one and $c_{32} = 23$ is the largest element. We allocate $x_{32} = \min(a_3, b_2) = \min(70, 20) = 20$ on the left bottom of c_{32}.

Table 3.4 First row and column distribution indicator table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators
F_1	20	3	0	14	100	6
F_2	18	15	8	0	30	3
F_3	10	$_{20}23$	6	2	50	13
Demand	40	0	60	80	180 (Total)	
Column distribution indicators	2	8	2	12		

b_2 is exhausted and readjust a_3 as $a_3' = a_3 - b_2 = 70 - 20 = 50$. Among the distribution indicators in the second step, 12 is the highest one and $c_{14} = 14$ is the largest element. We allocate $x_{14} = \min(a_1, b_4) = \min(100, 80) = 80$ on the left bottom of c_{14}.

Table 3.5 Second row and column distribution indicator table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators	
F_1	20	3	0	$_{80}14$	20	6	6
F_2	18	15	8	0	30	3	10
F_3	10	$_{20}23$	6	2	50	13	4
Demand	40	0	60	0	100 (Total)		
Column distribution indicators	2	8	2	12			
	2	×	2	12			

Here, b_4 is exhausted and readjust a_1 as $a_1' = a_1 - b_4 = 100 - 80 = 20$. Among the distribution indicators in the third step, 20 is the highest one and $c_{11} = 20$ is the largest element. We allocate $x_{11} = \min(a_1', b_1) = \min(20, 40) = 20$ on the left bottom of c_{11}.

Table 3.6 Third row and column distribution indicator table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators		
F_1	$_{20}20$	3	0	$_{80}14$	0	6	6	20
F_2	18	15	8	0	30	3	10	10
F_3	10	$_{20}23$	6	2	50	13	4	4
Demand	20	0	60	0	80 (Total)			
Column distribution indicators	2	8	2	12				
	2	×	2	12				
	2	×	2	×				

Here a_1 is exhausted and readjust b_1 as $b_1' = b_1 - a_1' = 40 - 20 = 20$. Among the distribution indicators in the next step, 10 is the highest one and $c_{21} = 18$ is the largest element. We allocate $x_{21} = \min(b_1', a_2) = \min(20, 30) = 20$ on the left bottom of c_{21}.

Table 3.7 Fourth row and column distribution indicator table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators			
F_1	$_{20}20$	3	0	$_{80}14$	0	6	6	20	×
F_2	$_{20}18$	15	8	0	10	3	10	10	10
F_3	10	$_{20}23$	6	2	50	13	4	4	4
Demand	0	0	60	0	60 (Total)				
Column distribution indicators	2	8	2	12					
	2	×	2	12					
	2	×	2	×					
	8	×	2	×					

Here, b_1 is exhausted and readjust a_2 as $a_2' = a_2 - b'_1 = 30 - 20 = 10$. Having no other alternatives, next two allocations are automatically 10 and 50 to the cell with cost c_{23} and c_{33} respectively.

Table 3.8 Supply and demand exhausted table.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply	Row distribution indicators			
F_1	$_{20}20$	3	0	$_{80}14$	0	6	6	20	×
F_2	$_{20}18$	15	$_{10}8$	0	0	3	10	10	10
F_3	10	$_{20}23$	$_{50}6$	2	0	13	4	4	4
Demand	0	0	0	0	0 (Total)				
Column distribution indicators	2	8	2	12					
	2	×	2	12					
	2	×	2	×					
	8	×	2	×					

Now all the rim requirements are satisfied and the initial basic feasible solution is $x_{11} = 20, x_{14} = 80, x_{21} = 20, x_{23} = 10, x_{32} = 20, x_{33} = 50$ which we allocate to the original transportation table.

Table 3.9 Optimum allocated table of the proposed method.

Destination → / Source ↓	D_1	D_2	D_3	D_4	Supply
F_1	$_{20}4$	19	22	$_{80}11$	$a_1 = 100$
F_2	$_{20}1$	9	$_{10}14$	14	$a_2 = 30$
F_3	6	$_{20}6$	$_{50}16$	14	$a_3 = 70$
Demand	$b_1 = 40$	$b_2 = 20$	$b_3 = 60$	$b_4 = 80$	200 (Total)

The TTC is, $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

$$= c_{11}\, x_{11} + c_{14}\, x_{14} + c_{21}\, x_{21} + c_{23}\, x_{23} + c_{32}\, x_{32} + c_{33}\, x_{33}$$
$$= 4 \times 20 + 11 \times 80 + 1 \times 20 + 14 \times 10 + 6 \times 20 + 16 \times 50 = 2040.$$

3.4 Comparison of Total Transportation Cost Obtained in Different Methods

Comparison of TTC obtained in different methods is given in Table 3.10 obtained by the proposed MVAM and the existing methods with optimal solution obtained by MODI method.

Table 3.10 Comparison of IBFS among four methods.

Name of the Methods	Primal Solution	No. of Iterations to Get an Optimal Solution
North-West Corner Method	2820	5
Least Cost Method	2090	2
Vogel's Approximation Method	2170	3
MVAM (Proposed)	**2040**	1

The optimal solution obtained by adopting Modified distribution method (MODI) is 2040. It is seen that the value of the objective function obtained by the proposed MVAM is same as the optimal value obtained by MODI method.

To apply and justify the efficiency of the proposed MVAM, we also have considered the following several supply chain (Problems 1-5) TPs from different sources to several destinations.

Problem 1:

Destination → / Source ↓	E	F	G	H	Supply
A	4	5	8	4	52
B	6	2	8	1	57
C	8	7	9	10	54
Demand	60	45	8	50	163(Total)

Problem 2:

Destination → / Source ↓	E	F	G	H	Supply
A	6	3	8	7	110
B	8	5	2	4	60
C	4	9	8	4	54
D	7	8	5	6	30
Demand	20	70	78	86	254(Total)

Problem 3:

Destination → / Source ↓	F	G	H	I	Supply
A	5	2	4	1	30
B	5	2	1	4	20
C	6	4	8	2	12
D	4	6	5	4	30
E	2	8	4	5	46
Demand	31	50	30	27	138(Total)

Problem 4:

Destination → / Source ↓	E	F	G	H	I	Supply
A	100	150	200	140	35	400
B	50	70	60	65	80	200
C	40	90	100	150	130	150
Demand	100	200	150	160	140	750(Total)

Problem 5:

Destination → / Source ↓	E	F	G	Supply
A	3	15	17	580
B	45	30	30	240
C	13	25	42	330
Demand	310	540	300	1150(Total)

3.5 Initial Basic Feasible Solutions of several TPs

Initial basic feasible solutions of the above TPs (1-5) obtained by all existing methods as well as proposed modified VAM are compared with the optimal solution obtained by MODI method in Table 3.11 with number of iterations.

Table 3.11 Comparisons of IBFS of the several TPs with optimal solution obtained by all procedures.

No. of Problems		Methods				Optimal Solution (MODI)
		NWCM	LCM	VAM	MVAM (Proposed)	
1.	Primal Solution	914	674	750	**674**	674
	No. of Iterations to Get an Optimal Solution	4	1	2	**1**	
2.	Primal solution	1010	988	988	**968**	968
	No. of Iterations to Get an Optimal Solution	3	2	2	**1**	
3.	Primal solution	621	423	391	**381**	381
	No. of Iterations to Get an Optimal Solution	7	4	2	**1**	
4.	Primal solution	92450	63550	66300	**63300**	63300
	No. of Iterations to Get an Optimal Solution	6	2	3	**1**	
5.	Primal solution	25530	21450	21030	**20550**	20550
	No. of Iterations to Get an Optimal Solution	3	2	3	**1**	

3.6 Results and Discussion

From the **Table 3.11**, it is clear that the value of the objective function (TTC) obtained by the proposed MVAM is same as the optimal value obtained by the MODI method. MODI and Stepping Stone (SS) method are considered as a standard technique for obtaining to optimal solution. We also have solved the different real life problems by the traditional methods and observed that the presented method provides better solutions than the other existing methods with less number of iterations.

CHAPTER IV

A Direct Analytical Method for Finding Optimal Solution of Transportation Problems

4.1 Introduction

The description of a transportation problem is as follows. Given are M plants and N markets with cost of shipping unit goods from any plant m to any market n being known. Each plant has known supply and each market has known demand such that sum of all supplies is equal to sum of all market demands. Transportation problem is a special variety of classical linear-programming problem. The objective of the transportation problem is to provide the following information to the decision makers: what quantity should be transported from a manufacturing unit to all possible destinations? And what would be the cost for this allocation? Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from place to another. Transportation problems deal with transportation of a single product manufactured at different plants (supply origins) to a number of different warehouses (demand destination). The objective of the transportation model is to prepare a minimal cost shipment plan from plants to markets such that demands at all market points are met without exceeding the supply available at any plant.

Goyal [1] has improved Vogel's approximation method (VAM) for the unbalanced transportation problem. Ramakrishna [2] has discussed some improvement to Goyal's modified VAM for unbalanced transportation problem. Moreover, Sultan [3], Sultan and Goyal [4] have studied initial basic feasible solutions and resolution of degeneracy in transportation problem. Few researchers have tried to give their alternate method for overcoming major obstacles over MODI and Steeping Stone (SS) methods. Adlakha and Kowalski [5] have suggested an alternative solution algorithm for solving certain TP based on the theory of absolute point. Ji and Chu [7] have discussed a new approach so called dual matrix approach to solve the transportation problem which gives also an optimal solution. Pandian and Natarajan [8] and Sudhakar et al. [9] have proposed two different methods in 2010 and 2012 respectively for finding an optimal solution directly. In this chapter, a simple heuristic approach is proposed for finding an optimal solution of a transportation problem directly with less number of iterations and very easy computations. The stepwise procedure of the proposed method is carried out as follows and a numerical example is given for testing optimality.

4.2 Algorithm of Proposed Direct Analytical Method

Step 1: Select the first column (destination) and verify the row (source) which has minimum unit cost. Write that destination under column 1 and corresponding source under column 2. Continue this process for each destination. However, if any destination has more than one same minimum value in different sources then write all these sources under column 2.

Step 2: Select those destinations under column-1 which have unique source. For example, under column-1, destinations are D1, D2, D3 have minimum unit cost which represents the sources K1, K1, K3 written under column 2 respectively. Here K3 is unique and hence allocate cell (K3, D3) a minimum of demand and supply. For an example, if corresponding to that cell supply is 8, and demand is 6, then allocate a value 6 for that cell. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand is exhausted.

Step 3: If source under column-2 is not unique then select those destinations where sources are identical. Next find the difference between minimum and next minimum unit cost for all those destinations where sources are identical.

Step 4: Check the destination which has maximum difference. Select that destination and allocate a minimum of supply and demand to the corresponding cell with minimum unit cost. Delete that row/column where supply/demand is exhausted.

If the maximum difference for two or more than two destinations appear to be same then find the difference between minimum and next to next minimum unit cost for those destinations and select the destination having maximum difference. Allocate a minimum of supply and demand to that cell. Next delete that row/column where supply/demand is exhausted.

Step 5: Repeat step 1 to step 4 until all the demand and supply are exhausted.

Step 6: Total cost is calculated as the sum of the product of unit cost and corresponding allocated number of units of supply / demand. That is,

$$\text{Total transportation cost } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

where c_{ij} = Transportation cost per unit of commodity and x_{ij} = Number of allocated units.

4.3 Numerical Example

Consider the following real life transportation problem with four sources and four destinations.

Table 4.1 Given transportation problem

Destination →	E	F	G	H	Supply
Source ↓					
A	2	4	5	8	52
B	5	7	6	7	59
C	16	20	10	12	28
D	19	18	17	28	94
Demand	40	55	68	70	233(Total)

Step 1: The minimum cost value for the corresponding destinations E, F, G, and H are 2, 4, 5 and 7 which represent the sources A, A, A and B respectively which is shown in Table 4.2.

Table 4.2 First minimum costs represent by several sources to corresponding different destinations.

Column 1	Column 2
E	A
F	A
G	A
H	B

Step 2: Here the source B is unique for destination H and allocate the cell (B, H), min(59,70)=59. This is shown in table 4.3.

Table 4.3 First allocation table.

Destination / Source	E	F	G	H	Supply
A	2	4	5	8	52
B	5	7	6	59 7	59
C	16	20	10	12	28
D	19	18	17	28	94
Demand	40	55	68	70	

Step 3: Delete row B as this supply is exhausted and adjust demand as (70-59) =11. Next the minimum cost value for the corresponding destinations E, F, G and H are 2, 4, 5 and 8 which represent the sources A, A, A and A respectively which is shown in table 4.4.

Table 4.4 Second minimum costs represent by several sources to corresponding different destinations.

Column 1	Column 2
E	A
F	A
G	A
H	A

Step 4: Here the sources are not unique because destinations E, F, G and H have identical source A. So, we find the difference between minimum and next minimum unit cost for the destinations E, F, G and H. The differences are 14, 14, 5 and 4 respectively for the destinations E, F, G and H. Since the maximum difference 14 occurs for two destinations. Now find the difference between minimum and next minimum unit cost for the destinations E and F. The differences are 17 and 16 for the destinations E and F respectively. Here the maximum difference is 17 which represents destination E. Now allocate the cell (A, E), min (40, 52) =40 which is shown in Table 4.5.

Table 4.5 Second allocation table.

Destination / Source	E	F	G	H	Supply
A	40 2	4	5	8	52
C	16	20	10	12	28
D	19	18	17	28	94
Demand	40	55	68	11	

Step 5: Delete the column E as this demand is exhausted and adjust supply as (52-40) =12. Next the minimum unit cost for the corresponding destinations F, G and H are 4, 5 and 8 which represent the sources A, A and A respectively which is shown in Table 4.5.

Table 4.6 Third minimum costs represent by several sources to corresponding different destinations.

Column 1	Column 2
F	A
G	A
H	A

Step 6: Repeat step 3 and step 4, we get the following table 4.7.

Table 4.7 Third allocation table.

Destination / Source	F	G	H	Supply
A	12 4	5	8	12
C	20	10	12	28
D	18	17	28	94
Demand	55	68	11	

Step 7: Delete row A as this supply is exhausted and adjust demand as (55-12) = 43. Next the minimum cost value for the corresponding destinations F, G and H are 18, 10, and 12 which represent the sources D, C and C respectively which is shown in Table 4.8.

Table 4.8 Fourth minimum costs represent by several sources to corresponding different destinations.

Column 1	Column 2
F	D
G	C
H	C

Step 8: Repeat step 2 we get the following Table 4.9.

Table 4.9 Fourth allocation table.

Destination / Source	F	G	H	Supply
C	20	10	12	28
D	[43] 18	17	28	94
Demand	43	68	11	

Step 9: Delete the column F as this demand is exhausted and adjust supply as (94-43) =51. Next the minimum unit cost for the corresponding destinations G and H are 10 and 12 which represent the sources C and C respectively which is shown in Table 4.10.

Table 4.10 Fifth minimum costs represent by several sources to corresponding different destinations.

Column 1	Column 2
G	C
H	C

Step 10: Repeat step 3 and step 4 we get the following Table 4.11.

Table 4.11 Fifth allocation table.

Destination / Source	G	H	Supply
C	10	[11] 12	28
D	17	28	51
Demand	68	11	

Step11: Delete column H as this demand is exhausted and adjust supply as (28-11) =17. However, only one destination remains. So allocate the remaining supply 17 and 51 to the corresponding cells (C, G) and (D, G) which is shown in Table 4.12.

Table 4.12 Sixth allocation table.

Destination / Source	G	Supply
C	[17] 10	17
D	[51] 17	51
Demand	68	

Step 12: Since all the demand and supply are exhausted. So, we calculate total cost as the sum of the product of the unit cost and its corresponding allocated number of units of supply or demand which is shown in Table 4.13

Table 4.13 Optimum solution applying the proposed method.

Destination → / Source ↓	E	F	G	H	Supply
A	[40] 2	[12] 4	5	8	52
B	5	7	6	[59] 7	59
C	16	20	[17] 10	[11] 12	28
D	19	[43] 18	[51] 17	28	94
Demand	40	55	68	70	233(Total)

The total cost associated with these allocations is

$$z = (40\times2)+(12\times4)+(59\times7)+(17\times10)+(11\times12)+(43\times18)+(51\times17) = 2484.$$

4.4 Optimality Check

To find initial basic feasible solution for the above example by using NWCM, LCM and VAM then the allocations are obtained as follows:

Table 4.14 Initial basic feasible solution by using NWCM

Destination / Source	E	F	G	H	Supply
A	[40] 2	[12] 4	5	8	52
B	5	[43] 7	[16] 6	7	59
C	16	20	[28] 10	12	28
D	19	18	[24] 17	[70] 28	94
Demand	40	55	68	70	233(Total)

The initial transportation cost associated with these allocations is

$$= (40\times2)+(12\times4)+(43\times7)+(16\times6)+(28\times10)+(24\times17)+(70\times28) = 3173.$$

Table 4.15 Initial basic feasible solution by using LCM.

Destination / Source	E	F	G	H	Supply
A	[40] 2	[12] 4	5	8	52
B	5	7	[59] 6	7	59
C	16	20	[9] 10	[19] 12	28
D	19	[43] 18	17	[51] 28	94
Demand	40	55	68	70	233(Total)

The initial transportation cost associated with these allocations is

$$= (40\times2)+(12\times4)+(59\times6)+(9\times10)+(19\times12)+(43\times18)+(51\times28) = 3002.$$

Table 4.16 Initial basic feasible solution by using VAM.

Destination / Source	E	F	G	H	Supply
A	2 (40)	4 (12)	5	8	52
B	5	7 (43)	6	7 (16)	59
C	16	20	10	12 (28)	28
D	19	18	17 (68)	28 (26)	94
Demand	40	55	68	70	233(Total)

The initial transportation cost associated with these allocations is

$$= (40\times 2)+(12\times 4)+(43\times 7)+(16\times 7)+(28\times 12)+(68\times 17)+(26\times 28)=2761.$$

To get an optimal solution by adopting Modified Distribution Method, the optimal solution is obtained as 2484. It can be seen that the value of the objective function obtained by proposed method is same as the optimal value obtained by MODI method. Thus, the total transportation cost obtained by direct analytical method is also optimal.

4.5 Comparison of Total Transportation Cost Obtained in Different Methods

Comparison of the total transportation cost is given in the following table 4.17 obtained by the proposed direct analytical method and the existing methods with optimal solution obtained by MODI method.

Table 4.17 Comparison of IBFS among the four methods with optimal solution.

Methods	IBFS	Optimal solution obtained by MODI method
Proposed method	**2484**	**2484**
NWCM	3173	
LCM	3002	
VAM	2761	

4.6 Results and Discussion

The optimal solution by adopting Modified Distribution Method (MODI) is 2484. It is noticed that the IBFS solution is meet to the optimal solution obtained by MODI method. Thus, the total transportation cost obtained by a direct analytical method is also optimal.

CHAPTER V

Conclusions

In every stage of our daily life we have to send and receive goods and services as essential components of life which is an ongoing process of modern civilization. But it is required to do these jobs in affordable transportation cost in expected time. Transportation networks are built up in order to save transportation cost and time so that the market prices of daily commodities remain affordable. Also the managerial works of manufacturing company are influenced by scheduling the right allocation of raw materials and machines in producing the right amount of products which are essential for optimal profit. Though several established transportation algorithms are available in concerned arena, we try to add new algorithms in this field. We have developed two algorithms and apply them as the techniques for solving TPs.

We have got the initial basic feasible solutions (IBFS) that are very closer to the optimal solutions and they take less iteration to get the optimal ones. Also our proposed algorithms can help the managers of manufacturing companies in order to make the exact allocation of machines in producing the right amount of different products that optimizes the profit.

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