A Weighted Least Cost Matrix Approach in Transportation Problem

by

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Declaration

This is to certify that the thesis work entitled "**A Weighted Least Cost Matrix Approach in Transportation Problem**" has been carried out by **Fatima Jannat** in the Department of Mathematics, Khulna University of Engineering & Technology, Khulna, Bangladesh. The above thesis work or any part of this work has not been submitted anywhere for the award of any degree or diploma.

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This is to certify that the thesis work submitted by **Fatima Jannat** entitled "**A Weighted Least Cost Matrix Approach in Transportation Problem**" has been approved by the board of examiners for the partial fulfillment of the requirements for the degree of **Master of Science** in the Department of Mathematics, Khulna University of Engineering & Technology, Khulna, Bangladesh in July 2018.

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Dedication

To

My Parents

Syed Humayun Kabir & Tahamina Begum

Acknowledgment

I am highly grateful to the Almighty Allah for granting me time and effort to undertake this thesis and complete it.

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Abstract

Transportation models are of multidisciplinary fields of interest. Mainly in the business arena, it is common to encounter the problem of transportation of some products/goods from source/origin to sink/destination so that cost of transportation is minimal and also satisfy the constraints related to demand and the supply. A huge number of physical problems are modeled as Transportation problem (TP) which includes inventory problem, assignment problem, traffic problem and so on. TPs are required for analyzing and formulating such models. In order to minimize the transportation cost satisfying all constraint, transportation model first provides the Initial Basic Feasible Solution (IBFS) and then IBFS be optimized by some related optimization algorithm if IBFS is not optimized. So the primary objective of transportation model is to find out a good IBFS of TPs.

In classical transportation approaches, the flow of allocation is controlled by the cost entries such as West Corner Method (WCM), Least Cost Method (LCM) etc. and/or manipulation of cost entries, so called Distribution Indicator (DI) or Total Opportunity Cost (TOC) like Vogel's Approximation Method (VAM) and its variations. In LCM, the flow of allocation is directly controlled by the cost entries i.e. lowest cost prefers first. On the other hand, for examples, on VAM, its variants and some other methods, the flow of allocations is controlled by the DI or TOC tables. But these DI or TOC tables are formulated by the manipulation of cost entries only. None of them considers demand and/or supply entry to formulate the DI/ TOC table.

In this thesis, we have first developed a new procedure of control of allocation named Weighted Opportunity Cost (WOC) matrix by incorporating supply/demand entries. At first, weight factors are formulated by using demand and supply entries which is off-course statistically valid. Then virtual weighted cost entries are formulated by manipulation of cost entries along with weight factors. Finally WOC matrix is formulated in which supply/demand entries acts as weight factor upon corresponding cost entries. Several examples are provided to demonstrate the concept of WOC matrix.

After successfully development of WOC, our intension is go upon the development of an algorithm to find out IBFS of TPs. It is known that, in Least Cost Matrix method, the flows of allocations are controlled by the cost entries only. The flows of allocations are predefined according to the cost entries i.e the ascending order of cell cost and the whenever identical costs are encountered whatever be the structure of demand/supply entries. So, the algorithm does not need to update allocation direction in subsequent steps. On the other hand in VAM, the flow of allocation is controlled by the DI table rather than directly cost matrix. By incorporating these two ideas, we have proposed a Weighted Opportunity Cost based on LCM (WOC-LCM) approach. In this proposed approach, the flow of allocation is controlled by the WOC matrix rather than cost matrix as in LCM approach. But WOC matrix is invariant through all over the allocation procedures like cost matrix in LCM method whereas DI table is updated after each step of allocations. Some experiments have been carried out to justify the validity and the effectiveness of the proposed WOC-LCM approach. Experimental results have shown that the WOC-LCM approach outperforms LCM. Moreover, sometime this approach is able to find out optimal solution too.

Publications

The following articles have been extracted from this thesis work:

1. A. R. M. Jalal Uddin Jamali, Pushpa Akhtar and **Fatima Jannat**, "Weighted cost based distribution opportunity table in transportation problem", Poster Presentation of University Day, 2017, Khulna University of Engineering & Technology, Khulna.
2. A. R. M. Jalal Uddin Jamali, Pushpa Akhtar and **Fatima Jannat**, "Weighted cost based distribution opportunity table in transportation problem", Proceedings of the 4^{th} International Conference on Mechanical Engineering and Renewable Energy, (ICMERE 2017) Chittagong, Bangladesh, 18 – 20, December 2017.
3. Jamali, A. R. M. Jalal Uddin, **Fatima, J**. and Pushpa, A., 2017, "Weighted cost opportunity based algorithm for initial basic feasible solution: A new approach in transportation problem", Journal of Engineering Science, Vol. 8, no. 1, pp. 63-70.

Contents

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

Introduction

1.1 Background

Transportation Problem (TP) is a specific category of linear programming which is linked with day to day performances in our life and mainly acts with logistics. TP is an essential job in Operation Research (OR). Transportation of a product from multi-source to multi-destination with minimal total transportation cost plays an important role in logistics and supply chain management. Researchers have given considerable attention in minimizing this cost with fixed supply and demand quantities. Now-a-days, networks in communication lines, railroad networks, pipeline systems, road networks, shipping lines, aviation lines etc are examples of network. In all these networks, we are interested to send some specific commodity from certain supply points to some demand points. Many of these network flow problems can be formulated as TP.

As TP is a Liner Programming Problem (LPP) so it can be solved by regular simplex method but this method is a laborious works. Its special structure allows to develop special techniques called transportation techniques which are computationally more efficient. TP received this name because many of its applications involve in determining how to optimally transport goods [Asase (2011)].

TP deals with the transportation of some products from source to destination so that cost of transportation should be minimal and also satisfy the constraints related to demand and the supply. TP can be utilized in inventory, assignment, traffic and so on. TPs are required for analyzing and formulating such models [Aizemberg et al. (2014), Juman (2015)]. Therefore, primary objective of TP is to find out the Initial Basic Feasible Solution (IBFS). To obtain optimal solution it is required to start from the IBFS. Therefore IBFS affects the optimal solution of TP. In fact, we can say that finding IBFS would be significant to obtain optimal solution. Therefore, the researchers pay much attention in TPs to find out IBFS as well as its optimal solution.

1.2 Literature Review

In the development of the subject LPP the name G. B. Dantzig (1951) remains in the top although the similar problem was first formulated in 1939 by the Russain Economist and Mathematician L. V. Kantorovich in product allocation problem. Later in 1947 the problem was formulated by G. B. Dantzig and devised a method of solving such problem named Simplex Method. Simplex algorithm was used to solve the LPP. The simplex algorithm is not that much easy and for that reason, researchers try, wherever possible, to simplify the way of calculation. Resultant of one such effort is Transportation Model.

The initial transportation problem was basically developed by Hitchcock (1941) and later discussed in details by Koopman (1949). Efficient methods of solution are derived from the Simplex algorithm and were developed by him in 1947. It is noticed that TP can be converted to a standard LPP and can be solved by Simplex method too but it is time consuming.

The Simplex process for finding an IBFS was proposed by Dantzig and was termed as the North-West Corner (NWC) rule by Charnes and Cooper (1954). Later Dantzing (1963) proposed the method of solving LPP by Simplex method. Karmarkar (1984a, 1984b) developed a new method to solve LPP. Charnes and Cooper (1954) developed the Stepping Stone Method which provides an alternative way of determining the Simplex method. Dantzig (1963) used the Simplex method in the TP as the Primal Simplex transportation method. It may be noted that an IBFS for the TPs can be obtained by using the NWC rule.

After Dantzig (1963), many researchers were devoted to develop some efficient transportation algorithms and also till now researchers are devoted to find out much more efficient one. It is worthwhile to mention here that for finding IBFS much of the research works are concerned with cost entries and/or manipulation of cost entries. It is also noted that in TP, all most all the optimized algorithms initially need an IBFS and then by using this IBFS some associate algorithm are developed for obtaining optimal solutions [Korte and Vygen (2012), Sifaleras (2013) and Bazaraa (2009)].

There are various simple heuristic methods available to get an IBFS, such as, North-West Corner (NWC) method, Row minimum method, Column minima method and Least Cost

Matrix (LCM) method etc. [Taha (2003)]. In NWC, the process begins from the northwest corner cell in the transportation table. In LCM, a cell having lower cost is selected sooner than a cell with higher cost. In fact, the process starts with the cell having the least cost of the transportation table. In most of the cases the LCM find IBFS better than NWCM because LCM algorithm uses costs during the allocation but NWCM uses positions of the costs whatever be it value. In LCM, all processes used in NWCM are repeated the only difference is that cell having minimum cost is selected first instead the northwest cell. Among all the simple heuristic methods, the LCM (Matrix Minima) is relatively efficient and this method considers the lowest cost cell of the Transportation Table (TT) for making allocation in every stage.

There is another well-known algorithm for IBFS is Vogel's Approximation Method (VAM) [Reinfeld and Vogel (1958)]. In VAM penalties are determined from the difference of smallest and next-to-the smallest cost entries and denoted as Distribution Indicator (DI). VAM provides comparatively better IBFS. After VAM method, researchers proposed several versions of the VAM method by modifying some tricks [Shimshak et al., (1981), Goyal (1984), Ramakrishnan (1988), Balakrishnan (1990), Islam (2012), Korukoglu and Balli (2011), Hakim (2012), Deshmukh (2012), Kawser (2016) etc.].

Balakrishnan (1990) proposed a modified VAM method for unbalanced transportation problems. On the other hand Islam et al. (2013) proposed a modified VAM for finding IBFS in the case of maximization of profit. Das et al. (2014) proposed another modified embed VAM and named Logical Development of Vogel's approximation method (LD-VAM) to find IBFS for TP.

Arsham and Khan (1989) introduced a new algorithm for solving TPs. They proposed method the Gauss Jordan pivoting method to solve the TPs. This algorithm is faster than Simplex method. Recently Sharma and Bhadane (2016) have presented an alternative method to NWC method by using Statistical tool called Coefficient of Range (CoR). On the other hand, very recently, Azad et al. (2017) at first developed TOC then they formed DI tableau for allocation by considering the average of TOC of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of TOC of cells along each column identified as Column Average Total Opportunity Cost (CATOC).

Allocations of costs are started in the cell along the row or column which has the highest RATOCs or CATOCs. Sharma and Bhadane (2016) presented an alternative method to NWC method by using statistical tool called Coefficient of Range (CoR).

Ramadan and Ramadan (2014) proposed Hybrid two-stage algorithm for solving TPs. The proposed algorithm consists of two stages: the first stage uses genetic algorithm (GA) to find an improved non-artificial feasible solution for the problem and the second stage utilizes this solution as a starting point in the Revised Simplex Method (RSM) algorithm to find the optimal solution for the problem. Hosseini (2017) proposed three new algorithms named Total Difference Method (TDM) for solving TP in environment. The proposed three algorithms, TDM1, TDM2 and TDSM, have been constructed from scrutiny of relationship of costs in rows and columns.

Juman and Hoque (2014), first demonstrate a deficiency of a recently developed method in obtaining a minimal cost solution to the TPs. Then a new heuristic approach is proposed to obtain efficient IFS (optimum or very near optimum) for TP. Later Juman and Hoque (2015) carried out the theoretical analyses on developing the lower and the upper bound of the heuristic solution techniques regarding TP. They show that the proposed heuristic approach algorithm is polynomial time ($O(N^3)$) regarding computational cost though the TP problem is NP- Hard.

Very recently Paul (2018) has proposed a novel approximation method to find out the IBFS of the TPs. For flow of allocation he calculate the distribution indicators by subtracting the smallest and next-to smallest element of each row and each column of the reduced matrix which formulated by subtracting elements of each row with the least element of each row and subtracting elements of each column with the least element of each column. Then allocating to the cell corresponds to the highest distribution indicator.

Many researchers have developed a numbers of transportation algorithms which are mainly devoted to finding a good Initial Basic Feasible Solution and also research works are ongoing for better results.

1.3 Objectives of the Research Work

It is observed that all the approaches discussed above are concerned with the cost entries and /or the manipulation of cost entries to form DI or TOC table whatever the structure of supply and demand be. None of them considered to treat the cost elements by manipulating supply/ demand to find DI or TOC in allocation procedures. But it might be assumed that, supply and demand play a vital role in the formulation of cost allocation table to obtain a better solution.

Consequently, the first objective of this research is to formulate a weighted allocation flow matrix by considering supply and demand entries as a weight factor upon the transportation cost entries. Our main goal is to develop a new algorithm based on Least Cost Matrix (LCM) algorithm in which flow of allocation will be controlled by the weighted allocation flow matrix. Experiments have been carried out to justify the validity and the effectiveness of the proposed weighted allocation flow matrix based Least Cost Matrix method.

1.4 Arrangement of the Thesis

In **Chapter I** the background and the state-of-the-arts of the TP regarding find out IBFS are briefly discussed. Moreover objective as well as arrangement of the thesis is presented in this Chapter. The necessary preliminaries related to the thesis are presented in the **Chapter II**. The formulation of weight based opportunity cost matrix as well as its validity is discussed in **Chapter III**. Some numerical illustrations are also given in this chapter to clarify each possible case in weight based opportunity cost matrix. The proposed algorithm to find out IBFS is presented in **Chapter IV**. For the test of validity, the detail procedure of the algorithm is illustrated through a numerical example. Moreover to justify the effectiveness as well as efficiency of the proposed algorithm some experiments have been carried out upon some numerical instances and compared them with LCM method. Finally conclusion has been drawn in the **Chapter V**.

CHAPTER II

Basic Preliminaries

2.1 Introduction

Mathematical Programming (MP) is a Programming in the sense of planning which deals with situations where a number of resources, such as men, materials, machines etc. are to be combined to yield one or more products. Linear Programming is a mathematical technique applied for identifying optimal maximum or minimum values of a problem subject to certain linear constraints. Transportation Problem is a special type of linear programming problem. It deals with the situation in which a particular commodity shipped from Origins to Destinations in such a way that the transportation cost is minimum, while satisfying both the supply limits and the demand requirements. Thus we have the places of production as origins and the places of supply as destination. Sometimes the origins and destinations are also termed as sources and sinks, respectively.

In the following sub sections, we will present some basic concepts related to this research work. Mainly, we will focus on the Linear Programming Problem (LPP) and Transportation problem (TP). At first we will present mathematical model of LPP and some associates definitions and characteristics of LPP. And then we will focuses on some basic information on TP.

2.2 Linear Programming Problem

LPP is the classes of optimization problem, when a linear function of several variables is to optimize subject to a number of conditions called constraints, which are also the linear functions of those involved variables, with the restrictions that involved variables are non-negative. The word optimization means maximization or minimization. So Linear Programming is the branch of mathematics which is widely used in the field of social sciences and business arena.

2.2.1 The General form of Linear Programming Problem

Mathematically a general linear programming problem can be stated as follows:

Find the values of variables $x_1, x_{2,..}, x_n$ which maximize (or minimize) an objective function which is a linear function of variables, such as [Gupta and Hira (2007)].

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \tag{2.1}$$

Subject to the constraints,

$$\left.\begin{matrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n(\leq, =, \geq)b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n(\leq, =, \geq)b_2 \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}\ (\leq, =, \geq)b_m \end{matrix}\right\}, \tag{2.2}$$

And meet the non-negativity restrictions

$$x_1, x_2, \cdots, x_n \geq 0. \tag{2.3}$$

For each constraint one and only one of signs $(\leq, =, \geq)$ holds but the sign may vary from one constraint to another. Here, for i=1,2,…,m; j=1,2,…,n, the variables x_j c_j, a_{ij} and b_i are called decision variables, coefficients of objective function, structural constants and stipulations respectively. Also in general LPP, $m<n$. Because of the variety of notations in common use, one finds the general LPP stated in many forms. Some of them are

(i) Compact form (by using the summation (sigma) sign:

Maximize (or minimize)

$$Z = \sum_{j=1}^{n} c_j\, x_j, \tag{2.4}$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j\ (\leq, =, \geq)b_i, i = 1,2, \cdots m\ , \tag{2.5}$$

and $\quad x_j \geq 0, j = 1,2, \cdots, n.$ (2.6)

Matrix-vector form:

Maximize (or minimize)

$$Z = \mathbf{cx}, \tag{2.7}$$

Subject to

$$\mathbf{Ax}\ (\leq, =, \geq)\mathbf{b}, \tag{2.8}$$

and $\quad \mathbf{x} \geq 0,$ (2.9)

Where **A** is a $(m \times n)$ matrix, **x** is a $(n \times 1)$ column vector, **b** is a $(m \times 1)$ column vector and **c** is a $(1 \times n)$ row vector namely shown below

$$\mathbf{A}_{(m\times n)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{13} & & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \tag{2.10}$$

$$\mathbf{x}_{(n\times 1)} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \tag{2.11}$$

$$\mathbf{b}_{(\boldsymbol{m}\times \mathbf{1})} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \tag{2.12}$$

and $\mathbf{c}_{1\times n} = [\, c_1, c_2, \cdots, c_n].$ (2.13)

Where **A** is called the *coefficient matrix*, **x** is the *decision* vector, **b** is the requirement vector, **c** is the cost (price or profit) vector of the LPP and **0** is an *n*-dimensional null column vector. A typical element a_{ij} of the matrix **A** may be considered as an indicator of the amount of *i*th type of resource necessary to manufacture one unit of product *j*. Hence in a way, these elements represent the activity of the operational system. Therefore, matrix **A** is also called the *activity matrix* and its elements are activity coefficients. For an identical reason, vector **b** is also referred to as the *resource* vector or *availability* vector.

2.2.2 The Canonical form of Linear Programming Problem

The general linear programming problem discussed in section 2.2.1 can always be put in the following form, called the canonical form [Gupta and Hira (2007)]:

Maximize

$$Z = \sum_{j=1}^{n} c_j \, x_j \ ,$$

Subject to

$$\sum_{j=1}^{n} a_{ij} \, x_j \leq b_i, \;\; i = 1,2, \cdots m,$$

And $x_j \geq 0, \qquad j = 1,2, \cdots n.$

The characteristics of this form are:

(i) Objective function is of maximization type,

(ii) All constraints are of the ($\leq$) type (except non-negativity restrictions which are of ($\geq$) type.

(iii) All decision variables are non-negative as well.

2.2.3 The Standard form of Linear Programming problem

The general LP problem discussed in section 2.2.1 can always be put in the following form, called the standard form [Gupta and Hira (2007)]:

Maximize (or minimize)

$$Z = \sum_{j=1}^{n} c_j \, x_j \ ,$$

Subject to

$$\sum_{j=1}^{n} a_{ij} \, x_j = b_i, \quad \text{s.t.}\, b_i \geq 0 \text{ and } i = 1,2,\cdots m,$$

$$x_j \geq 0, \qquad\qquad j = 1,2,\cdots n.$$

which can be represented in matrix-vector form as follows:

Maximize (or minimize)

$$Z = \mathbf{cx} \, ,$$

Subject to

$$\mathbf{Ax} = \mathbf{b}$$

where $x_i \geq 0$, $b_j \geq 0$; $\forall\, i = 1,2,\cdots,m$, $\forall\, j = 1,2,\cdots,n$.

The characteristics of the standard form are

(i) The right- hand side of each constraint is non-negative.

(ii) All constraints are expressed as equations.

(iii) Objective function may be of maximization or minimization type

(iv) All variables are non-negative.

2.3 Limitations of Linear Programming Problem

(**a**) For large problem having many limitations and constraints the computational difficulties are enormous.

(**b**) The model does not take into account the effect of time.

(**c**) According to the linear programming problem, the solution variable can have any value, integers or fractions; whereas sometimes it happens that some of the variables can have only integer values.

(**d**) In many situations, it is not possible to express both the objective function and constraints in linear form.

2.4 Some Definitions related to Transportation Problem

(a) Objective function

In the above mathematical model of LPP (section 2.2.1) the linear function $Z = \mathbf{c}^T\mathbf{x}$ (i.e. $= c_1x_1 + c_2x_2 + \cdots + c_nx_n$) which is to be maximized (or minimized) is called objective function.

(b) Constraints

In the above mathematical model of LPP (section 2.2.1), the inequalities $\mathbf{Ax}(\leq, =, \geq)\mathbf{b}$ are called the constraints of the LPP.

(c) Non-negative restrictions

The set of inequalities $x_j \geq 0$; j=1, 2,..., n are called the non-negative restriction of the general LPP.

(d) Feasible solution (FS)

A set of non-negative allocations $x_{ij} \geq 0$ which satisfies constraints of the LPP i.e. $\mathbf{Ax}$ $(\leq, =, \geq)\mathbf{b}$ is known as feasible solution.

(e) Basic feasible solution (BFS)

A solution of LPP will be a basic feasible solution (BFS) if all components of the solution set corresponding to the basic variables are non-negative.

(f) Degenerate and Non-degenerate B.F.S

The feasible solution of a LPP of which some components corresponding to the basic variables are zero, is known as the Degenerate solution.

The solution set of a LPP of which all components corresponding to the basic variables are non-zero positives quantities is known as Non-degenerate Basic feasible solution.

(g) Optimal solution

A feasible solution to a LPP which makes the objective function an optimal is known as the optimal solution. It gives the maximum or minimum value of the objective function provided the maximum or minimum value exist.

2.5 Transportation Problem

The Transportation Problem is a special type of linear programming problem, where the objective is to minimize the cost of distributing a product from a number of sources to a number of destinations. In a transportation problem, we have certain origins, which may represent factories where products are produced and certain destinations where products to be supplied. This must be done in such a way as to minimize the cost. Sometimes the origins and destinations are also termed as sources and sinks.

2.5.1 Mathematical Model of Transportation Problem

As it is mentioned earlier that TP is a special type of LPP in which the solution variables should satisfy simultaneously two constraints namely supply (availability) and demand (requirement). The Mathematical model of TP expressed in Linear Programming model as follows:

Minimize

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \qquad (2.14)$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} \leq a_i; \; i = 1, 2, 3, ..., m \text{ (supply constraints)} \qquad (2.15)$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j \quad \text{For } j = 1, 2, 3, .., n \text{ (demand constraints)} \qquad (2.16)$$

$$x_{ij} \geq 0; a_i \geq 0; b_j \geq 0; \; \forall i = 1,2,\cdots,m \,; \; j = 1,2,\cdots,n \qquad (2.17)$$

where

Z : Total transportation cost, to be minimized, which is the objective function.

c_{ij} : Unit transportation cost of the commodity from each source *i* to destination *j*.

x_{ij} : Number of units of commodity sent from source *i* to destination *j*.

a_i : Number of commodity to be supplied from source *i*.

b_j: Number of commodity to be supplied to destination *j*.

The Equations (2.15) indicate supply constraints and (2.16) indicate Demand constraints. In brief Equations (2.15) and (2.16) are called *Capacity constraints* whereas constraint (2.17) is called *non-negative Restrictions* conditions. Note that the problem called balanced if

Total Supply= Total Demand.

Mathematically,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (2.18)$$

Otherwise the problem will be unbalanced. i.e. if

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \quad (2.19)$$

Now Equation (2.14) indicates that the above TP has mn variables, Equation (2.15) have m constraints and Equations (2.16) have n constrains. That is the above TP has $m+n$ constraints excluding the non-negativity of the constraints presented by (2.17). Now for a balanced transportation problem we have $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. A consequence of this is that the problem is defined by $m+n$-1 supply and demand variables. Since, if a_i, i=1,2,..,m and b_j, j=1,2,...,n are specified, then one of a_i can be readily found from (2.18) .This means that one of the constraint equations is not required. Thus, a balanced transportation model needs $m+n$-1 independent constraint equations. Science the number of basic variables in a basic solution is the same as the number of constraints, solutions of this problem should have $m+n$-1 basic variables x_{ij} (which is non-zero for non-degenerated solution) and all the remaining variables will be non-basic and thus have the value zero. For a feasible solution to exist, it is necessary that total capacity equals to the requirements and there will be ($m+n$-1) basic independent variables out of ($m \times n$) variables.

2.5.2 The characteristics of Transportation Problem

(**a**) Only a single type of commodity is being shipped from an origin to a destination.

(**b**) a_i (Supply) and b_j (demand) are all positive integers.

(**c**) The unit transportation cost of the item from all the sources to destinations is certainly and preciously known.

(d) The objective is to minimize the total cost.

In addition, for balanced case: $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

2.5.3 Transportation Tableau

Table 2.1: A transportation Tableau of a TP with m origins and n destinations

		Destinations					
		D_1	D_2	⋯	D_{n-1}	D_n	Supply
Origins	O_1	c_{11}	c_{12}	⋯	c_{1n-1}	c_{1n}	a_1
	O_2	c_{21}	c_{22}	⋯	c_{2n-1}	c_{2n}	a_2
	O_3	⋮	⋮	⋯	⋮	⋮	a_3
	:	⋮	⋮	⋯	⋮	⋮	⋮
	O_m	c_{1m}	c_{2m}	⋯	c_{mn-1}	c_{mn}	a_m
Demand		b_1	b_2	⋯	b_{n-1}	b_n	

By exploiting the special characteristics of Transportation problem, a specially designed tableau called Transportation Tableau (TT). It can be formulated typical view of Transportation Table is shown in the Table 2.1. In the Transportation Tableau O_i indicates ith source with amount of availability is a_i which is shown in the far right column. On the other hand D_j denotes jth destination with demand b_j, which is shown in the bottom row of the tableau.

In this table there are $m \times n$ squares in m rows and n columns which indicate cost entries. Each square is called a cell. The cell in ith row and jth column is called (i, j) th cell and denoted as c_{ij}. Which represents the unit shipping cost from ith origin to jth destination.

2.5.4 Mathematical model of a physical problem

In order to demonstrate the mathematical model of transportation problem representing as in Linear Programming Problem as well as Transportation Tableau, we consider a simple real life physical problem given in Example 2.1 as follows:

Example 2.1: A company has 3 production centers i.e. factories O_1, O_2 and O_3 at different locations with production capacities of 5, 11 and 14 tons (per day) respectively, of a certain product with which it must supply to 3 warehouses: D_1, D_2 and D_3 where the demand of the warehouses are 10, 5 and 15 tons (per day) respectively. The unit costs of transportation, from factory O_i to destination (warehouse) D_j is shown in the (i,j)th, cell c_{ij}, of the cost matrix $[c_{ij}]$which is shown in the Table 2.2.

Table 2.2: The unit cost of transportation from the factories to the warehouses

Warehouse

		D_1	D_2	D_3
Factories	O_1	**1**	**3**	**5**
	O_2	**3**	**5**	**7**
	O_3	**5**	**7**	**9**

2.5.5 Formulation of mathematical model namely LP and TT of the transportation problem

Step 1: Key decision to be made is to find how much quantity of production from which factories will be shipped to which warehouse so as to satisfy the constraints and minimize the cost. Since there are 3 factories (Origins) and 3 warehouses (Destinations), so there are 3×3 i.e. 9 possible variables: x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32} and x_{33}. These variables represent the quantities of product to be shipped from different factory and can be represented in the form of a matrix shown in the Table 2.3 below:

Table 2.3: The amount of transportation commodity in matrix form

Warehouse

Factories	D_1	D_2	D_3
O_1	x_{11}	x_{12}	x_{13}
O_2	x_{21}	x_{22}	x_{23}
O_3	x_{31}	x_{32}	x_{33}

In general, we can say that the key decision to be made is to find the quantity of units to be transported from each origin to each destination. Thus, if there are m origins and n destination, then x_{ij} are the decision variables (quantities to be found), where i=1, 2,..,m and j=1, 2,..,n.

Step 2 (set non negative constraint): Feasible alternatives are sets of values of x_{ij}, where $x_{ij} \geq 0$.

Step 3 (set objective function): Objective is to minimize the cost of transportation i.e.

Minimize

$$Z=\{1x_{11} + 3x_{12} + 5x_{13}$$
$$+3x_{21} + 5x_{22} + 7x_{23}$$
$$+5x_{31} + 7x_{32} + 9x_{33}\}$$

In general, we can say that if c_{ij} is the unit cost of shipping from ith source to jth destination, the objective is

Minimize

$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

Step 4 (set capacity constraints): Constraints are

(i) According to the availability or supply:

$x_{11} + x_{12} + x_{13} = 5$ (for factory 1)

$x_{21} + x_{22} + x_{23} = 11$ (for factory 2)

$x_{31} + x_{32} + x_{33} = 14$ (for factory 3)

Thus in all, there are 3 constraints (equal to the number of factories).

In general, there will be *m* constraints, if number of origins is *m*, with *n* number of destinations, which can be expressed as

$\sum_{j=1}^{n} x_{ij} = a_i,$ $i=1, 2, 3,.., m.$

(ii) According to the requirement or demand:

$x_{11} + x_{21} + x_{31} = 10$ (for warehouse 1)

$x_{12} + x_{22} + x_{32} = 5$ (for warehouse 2)

$x_{13} + x_{23} + x_{33} = 15$ (for warehouse 3)

In general, there will be *n* constraints, if number of destinations is *n*, with m number of origins, which can be expressed as

$\sum_{i=1}^{m} x_{ij} = b_{j,}$ $j=1, 2, 3,..,n.$

Thus we have found that the given situation involves $(3 \times 3 =)9$ variables and $(3 + 3 =)6$ constraints. In general, such a solution will involve $(m \times n)$ variables and $(m + n)$ constraints.

Step 5 (Balanced TP): It is observed that total availability of factories (supply) is

$\sum_{i=1}^{3} a_i = 30, (= 5 + 11 + 14).$

And total capacity of warehouse (demand) is

$\sum_{j=1}^{3} b_j = 30, (= 10 + 5 + 15).$

i.e. $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j$, the problem is balanced TP.

In general, if number of origins is *m* and number of destinations is *n*, then for balanced problem

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$

(a) Therefore the Mathematical model of the given physical problem can be represented as LPP in Matrix form as follows:

$$Minimize \quad Z = (1 \quad 3 \quad 5 \quad 3 \quad 5 \quad 7 \quad 5 \quad 7 \quad 9)\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{31} \\ x_{32} \\ x_{33} \end{bmatrix}$$

s.t.
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{31} \\ x_{32} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ 14 \\ 10 \\ 5 \\ 15 \end{bmatrix}$$

and $x_{ij} \geq 0$.

(b) Since, in general, the transportation model is balanced, so one of these constraints must be redundant. Thus, the model has *m*+*n*-1 independent constraint equations, which means that the starting basic feasible solution consists of *m*+*n*-1 basic variables. It is observed that in this model, the objective function and the constraints are linear functions. The following points may be noted in a transportation model.

(i) All supply as well as demand constraints are of equality type.

(ii) They are expressed in terms of only one kind of unit.

(iii) Each variable occurs only once in the supply constraints and only once in the demand constraints.

(iv) Each variable in the constraints has unit coefficient only.

Therefore, the transportation model is a special case of general LP model where in the above four conditions hold good and can be solved by a special technique called the transportation technique namely *Transportation tableau* (TT) which is easier and shorter than the other technique. Therefore the transportation tableau of the given problem is displayed in the Table 2.4.

Table 2.4: A transportation Table of the given below in problem

	Destinations				
Origins		D_1	D_2	D_3	Supply
	O_1	**1**	**3**	**5**	5
	O_2	**3**	**5**	**7**	11
	O_3	**5**	**7**	**9**	14
	Demand	10	5	15	Total = 30

After formulation of transportation tableau (TT) we have to check whether $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is true or not. If yes, the problem is said to be a balanced or self contained or standard problem.

2.6 Graphical view of Transportation Problem:

The transportation problem can be represented as network flow diagram which is shown in the Figure 2.1. The production can be represented as sources whereas warehouses are represented as sinks. In the figure, there are m sources and n sinks each of which is denoted as node. The arcs (joining each source node to each sink node) represent the routes linking the sources and sinks. Each arc (i,j), for i =1, 2, 3, …, m ; j =1, 2, 3, …, n , joining source i to sink j carries two types of information: the unit transportation cost, c_{ij}, and the amount of goods shipped, x_{ij}. The amount of supply at source each i is a_i i =1, 2, 3, …, m and the amount of demand at sink j is b_j, j =1, 2, 3, …, n.

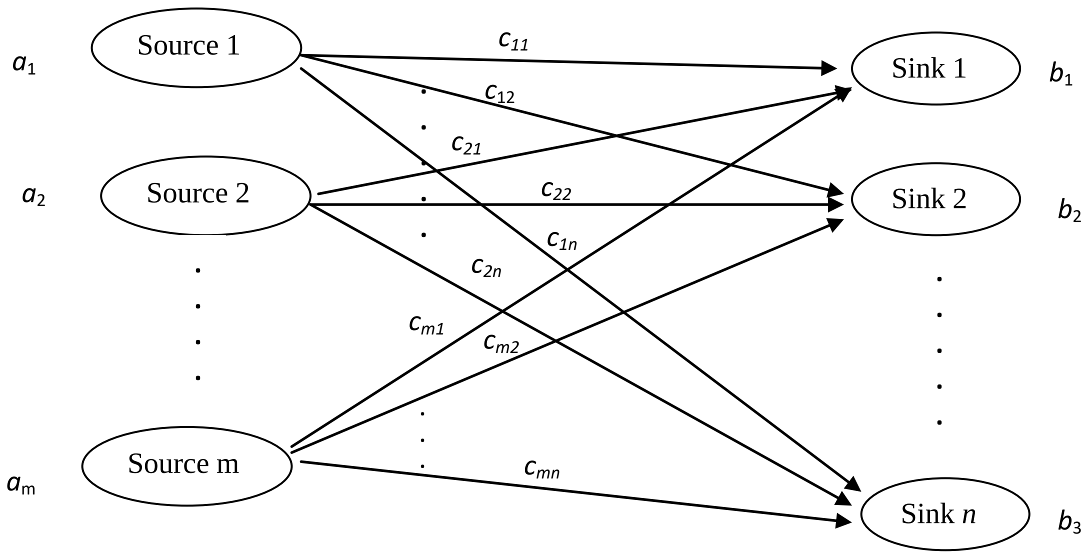


Fig. 2.1: A diagram of Transportation Network

2.7 Solution Algorithm for the Transportation problem:

The solution algorithm to a transportation problem can be summarized into following steps:

(a) Formulate the problem and set up in the matrix form:

The formulation of transportation problem is similar to LP problem formulation. Here the objective function is the total transportation cost and constrains are the supply and demand available at each source and sink, respectively.

(b) Obtain an initial feasible solution:

There are several methods exist in literature to find out initial Basic Feasible Solution (IBFS) like Least Cost Method (LCM), Vogel's Approximation Method (VAM) etc.

(c) Find out optimal solution:

There are also several methods exist in literature to find out optimal solution from the IBFS such as Modified Distribution (MODI) method and Stepping Stone Method.

The algorithm of LCM method is presented briefly as our proposed algorithm is developed based on the basis of the method.

2.7.1 Least Cost Method (LCM)

Least Cost Matrix or Matrix minimum method is a method for computing IBFS of a transportation problem where the basic variables are chosen according to the minimum unit cost of transportation. The main steps of LCM algorithm are given bellow:

Step1. Identify the smallest cost in the cost matrix of the transportation tableau, and then allocate minimum shipping cost corresponding to the cell.

Step2. Cross out the satisfied row or column (which corresponds to min {demand, supply}.

Step3. Repeat steps 1 and 2 for the resulting reduced transportation table until all the requirements are satisfied.

Note that whenever the minimum cost is not unique, make an arbitrary choice among the minima.

2.8 Optimality Test and procedure for Optimal Solution of TP:

A feasible solution is said to be optimal if it minimizes the total transportation cost. There are basically two well known methods available to test the optimality as well as to find optimal solution from the obtained IBFS of TP. The well-known methods are given below:

(a) Modified Distribution (MODI) method.

(b) Stepping Stone (SSM) method.

In the following, we have discussed the MODI method only. The main steps of the method are as follows:

Steps

1. Determine an initial basic feasible solution using any method such as Least Cost Method.
2. Determine the values of dual variables, u_i and v_j, using $u_i+v_j=c_{ij}$
3. Compute the opportunity cost using $\Delta_{ij} =(u_i+v_j)-c_{ij}$
4. Check the sign of each opportunity cost.
    (a) If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimal solution.
    (b) If one or more unoccupied cell has negative opportunity cost, the given solution is not an optimal solution and further savings in transportation cost are possible.
5. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.
6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Note that, the right angle turn in path is permitted only at occupied cells and at the original unoccupied cell.
7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.
8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.
9. Repeat the whole procedure until an optimal solution is obtained.

CHAPTER III

Weighted Opportunity Cost Matrix

3.1 Introduction

In the previous chapter we have discussed some important issues regarding LP as well as TPs. Moreover, we have presented briefly an algorithm of TP namely Least Cost Matrix (LCM) as its concepts will be incorporated in our proposed control the allocation procedures as well as algorithm of the approach. It is known that the allocation procedure of LCM approach are controlled by the Cost matrix according to the minimum cost of the cost matrix whatever be the supply /or demand entries. Not only the LCM but almost all existing algorithms, in literature, consider only cost and/or manipulation of cost entries to control the allocation procedures.

3.2 Transportation Tableau

At first we recall the physical problem **Example 2.1:** (chapter 2) and rename it as **Example 3.1** whose mathematical model in TP is again displayed in Table 3.1.

Table 3.1: A Transportation Tableau of the problem 2.1

	Destinations				
Origins		D_1	D_2	D_3	Supply
	O_1	**1**	**3**	**5**	5
	O_2	**3**	**5**	**7**	11
	O_3	**5**	**7**	**9**	14
	Demand	10	5	15	Total= 30

It is observed that the given problem is a balanced problem ($\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$)

3.3 Formulation of Distribution Indicator Matrix

In classical transportation approaches, like North-West Corner Rule, Least Cost Matrix (LCM) method etc, the flow of allocation is controlled by the cost entries only. Again some other classical transportation approaches, like Vogel's Method Approximation (VAM) method or its all variants, the Coefficient of Range (CoR) method etc, the flow of allocation is controlled by the cost entries and/or manipulation of cost entries-so called Distribution Indicator (DI) or TOC. But this DI/TOC table/Matrix is formed by the manipulation of cost entries only. None of them considers demand and/or supply entry to formulate the DI/TOC table/Matrix.

We have a new idea for the control of the flow of allocations we have formulated a *new flow of allocation matrix* in which demand/supply entries are incorporated as a weight factor to the corresponding cost entries and is named as Weighted Opportunity Cost (WOC) matrix. It may be noted that, these weight factor are formulated by supply and demand entries along with the manipulation of cost entries. In this WOC matrix, the supply and demand entries act as weight factors on manipulated cost entries.

3.3.1 Finding cell weight

At first we will find out the maximum possible allocation of any cell C_{ij} (the cell correspond to origin i and destination j) whose corresponding transportation cost is c_{ij} (unit cost from origin i to destination j). Since the availability from the origin i is a_i (units) and the demand at destination j is b_j(units), so the maximum possible allocation at cell C_{ij} is obviously *min* (a_i,b_j). Now since the maximum possible ability of allocation of each cell C_{ij} is *min* (a_i,b_j), so the total possible maximum allocation of all cells be

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\min\{a_i,b_j\}.$$

Therefore for each cell C_{ij}, we have defined its weight factor w_{ij} as

$$w_{ij} = \min\ \{a_i,b_j\} \,/\, \sum_{i=1}^{m}\sum_{j=1}^{n}\min\{a_i\,,b_j\} \qquad (3.1)$$

inconsequence we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} = 1.$$

But since the factor "$1/\sum_{i=1}^{m}\sum_{j=1}^{n} \min\{a_i, b_j\}$" is common to w_{ij} $\forall i, j$, so without loss of generality we can ignore this factor in allocation flow matrix. Therefore, the weight factor corresponding to the cell C_{ij} can be combined as

$$w_{ij} = \min\{a_i, b_j\}. \tag{3.2}$$

It is noted that this reduces a significant amount of computational cost. *The significance of this weight is that larger weight poses larger possibility to flow of allocation.*

3.3.2 Finding appropriate weighted opportunity cost entries

After successful formulation of cell weight, our task is to formulate the Weighted Opportunity Cost (WOC) based on Distribution Indicator Matrix. But now we have to face a problem regarding accumulation of this weight to cost entries. Since cell with lower cost has preference for allocation first, on the other hand the cell with large weight has preference for allocation. So it is not directly possible to formulate WOC matrix just by simply multiply weight to cell cost to find out meaningful elements of WOC matrix. To overcome this difficulty and for the formulation of meaningful WOC matrix, we should transform one of the two so that the multiplication of the two will be meaningful. This can be done by inversing the cost elements. Therefore, the weighted opportunity cost corresponding to the cell cost C_{ij} be

$$w_{c_{ij}} = \frac{1}{c_{ij}} \times \min\{a_i, b_j\} \tag{3.3}$$

Here $w_{c_{ij}}$ and c_{ij} denote the virtual weighted cost and actual cost corresponding to the cell C_{ij} respectively.

But another problem takes place again. What happen if cost cell is zero? Since when $c_{ij} = 0$ then $\frac{1}{c_{ij}}$ becomes undefined. So, to overcome this difficulty we need some more special attentions. We can overcome this shortcoming by considering zero costs and costs which is greater than zero but less than one specially. So, if there exist any cell whose cost entry is zero, then we can formulate the virtual weighted cost to the cell C_{pq} as follows:

(a) If $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\} = \varphi$, (i.e. null set) then set
$w_{c_{pq}} = N \times \min\{a_p, b_q\}$; Where $N = \max\{a_i, b_j; \forall i, j\}$.

(b) Else if $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\} \neq \varphi$ (i.e. not null set), then set
$w_{c_{pq}} = M \times \min\{a_p, b_q\}$; where $M = \max\{a_i, b_j; \forall i, j\} / [\min\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\}]$.

3.3.3 Formulation of Weighted Opportunity Cost (WOC) matrix

After development of virtual weighted cost entries, we can easily formulate the Weighted Cost (WOC) Matrix $[w_{c_{pq}}]$which is as follows:

(a) If $c_{pq} > 0, \forall i, j$ then set

$$w_{c_{pq}} = \frac{1}{c_{pq}} \times \min\{a_p, b_q\}$$

(b) Else if $c_{pq} = 0$ and $\{ c_{ij}: 0 < c_{ij} < 1, \forall i, j\} = \varphi$(i.e. null set), then set

$w_{c_{pq}} = N \times \min\{a_p, b_q\}$; Where $N = \max\{a_i, b_j; \forall i, j\}$

(c) Else if $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\} \neq \varphi$ (i.e. not null set), then set

$w_{c_{pq}} = M \times \min\{a_p, b_q\}$; Where $M = \max\{a_i, b_j; \forall i, j\} / [\min\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\}]$.

3.4 Numerical Illustration

For clear understanding, a numerical example namely **Example 2.1** given in previous chapter is considered. The TT of the above problem is displayed in the above Table 3.1. But for the better visualization, we again copy it here - which is shown at the table 3.4. (a). It is observed in the table 3.4. (a) that there is no any cost entry which is zero. That is all the cost entries are greater than zero ($c_{ij} > 0$), so the algorithm executes only the first case (case (a)) of the WOC algorithm (section 3.3.2). Formally

(a) If $c_{ij} > 0, \forall i, j$ then set

$$w_{c_{ij}} = \frac{1}{c_{ij}} \times \min\{a_i, b_j\}$$

So, for the cell $c_{11} > 0$, the corresponding virtual weighted cost is

$$w_{c_{11}} = \frac{1}{c_{11}} \times \min\{a_1, b_1\} = \left(\frac{1}{1}\right) \times \min(5,10) = \frac{5}{1}$$

Similarly we can find out all the weighted opportunity cost according to the algorithm of WOC. The final WOC Tableau (WOC with supply and demand entries) corresponding to the Transportation Tableau 3.4(a) is shown in the Table 3.4(b).

Table 3.4: Transportation Tableau and corresponding WOC Tableau

(a) Transportation Tableau

	D_1	D_2	D_3	S
O_1	1	3	5	5
O_2	3	5	7	11
O_3	5	7	9	14
D	10	5	15	

(b) WOC Tableau

	D_1	D_2	D_3	S
O_1	5/1	5/3	5/5	5
O_2	10/3	5/5	11/7	11
O_3	10/5	5/7	14/15	14
D	10	5	15	

As in the example 2.1 all the cost entries are greater than zero, so we have considered another **Example 3.2**.

Example 3.2: The transportation tableau of a transportation cost of goods from the factories to the warehouses as well as demand and supply are shown in the Table 3.5.

Table 3.5: Transportation Tableau of the **Example 3.2**

	D_1	D_2	D_3	Supply
O_1	0	3	0	14
O_2	3	5	7	11
O_3	5	7	9	5
Demand	10	5	15	

It is observed in the Table, there are two entries (cost) whose values are zero namely C_{11}= C_{13}=0. Moreover, we observed in the Table 3.5 that there is no any cell cost lies between zero and one. So for finding out virtual weighted cost for cells C_{11} and C_{13} we need to evaluate case (b) of the WOC algorithm and for the remain cells we need to evaluate case (a) as well. Since except cells C_{11} and C_{13}, all other cell costs are greater than. Formally

(a) If $c_{pq} > 0, \forall i, j$ then set

$$w_{c_{pq}} = \frac{1}{c_{pq}} \times \min\{a_p, b_q\}$$

(b) Else if $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\} = \varphi$, then set

$$w_{c_{pq}} = N \times \min\{a_p, b_q\} \text{ where } N = \max\{a_i, b_j; \forall i, j\}$$

Now to find out virtual weighted cost of the cells whose cost entries are zero, we need to evaluate the value N, where

$N = max\{a_i, b_j; \forall i, j\}$. It is observed that $N = max\{14, 11, 5, 10, 5, 15\} = 15$.

Then for cell C_{11},

$$min\ \{\text{supply, demand}\} = min\{a_1, b_1\} = min\{14, 10\} = 10.$$

And for the cell C_{13}

$$min\ \{\text{supply, demand}\} = min\{a_1, b_3\} = min\{14, 15\} = 14.$$

Therefore, the virtual weighted cost for the cell C_{11} and C_{13} are as follows:

$$w_{c_{11}} = N \times \min\{a_1, b_1\} = 15 \times 10 = 150.$$

$$w_{c_{13}} = N \times \min\{a_1, b_3\} = 15 \times 14 = 210.$$

On the other hand for any nonzero cell say C_{12}

$$w_{c_{12}} = \frac{1}{c_{12}} \times min\{a_1, b_2\} = \frac{1}{3} min\{14,5\} = \frac{1}{3} \times 5$$

Similarly, we can find out all virtual weighted cost entries of the WOC and the complete WOC is displayed in the Table 3.6(b).

Table 3.6: WOC tableau and TT for the **Example 3.2**

(a) Transportation Tableau

	D_1	D_2	D_3	S
O_1	0	3	0	14
O_2	3	5	7	11
O_3	5	7	9	5
D	10	5	15	

(b) WOC Tableau

	D_1	D_2	D_3	S
O_1	150	5/3	**210**	14
O_2	10/3	5/5	11/7	11
O_3	5/5	5/7	5/9	5
D	10	5	15	

In the Table 3.6 it is observed that though both the cell C_{11} and C_{13} have zero transportation costs but their corresponding virtual weighted costs are not identical. Therefore according to LCM method both cell have equal preference but according to the WOC matrix cell C_{13} prefers to first as its weight factor is larger than that of C_{11} i.e. $w_{c_{13}}$= (210) > $w_{c_{11}}$ (= 150).

In Examples 3.1, and 3.2, the cost entries are either zero or greater than one; no any cost entry lies between zero and one. So, we have considered another **Example 3.3**.

Example 3.3: The transportation tableau of a transportation cost of goods from the factories to the warehouses as well as demand and supply are shown Table 3.7.

Table 3.7: Transportation Tableau of the **Example 3.3**

	D_1	D_2	D_3	Supply
O_1	0	3	0.5	5
O_2	3	5	7	11
O_3	5	0.7	9	14
Demand	5	15	10	

It is observed in the Table that there are one entries whose values are zero i.e. C_{11}= 0 and two entries whose values are proper fractions namely C_{13}= 0.5 and C_{32} = 0.7 i.e. cell cost lies between zero and one and remaining entries are greater than one. As the cell costs are zero, greater than zero but less than one and also greater than one, so we have to evaluate cases case (a) for nonzero cell cost and case (c) for zero cell cost. So for finding out virtual weighted cost for cell C_{11}, we need to evaluate case (c) of the WOC algorithm. Formally

(b) Else if $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\} \neq \varphi$ (i.e. not null set), then set $w_{c_{pq}} = M \times \min\{a_p, b_q\}$ where $M = max\{a_i, b_j; \forall i, j\} /[\min\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\}]$.

So for the virtual weighted cost of the cell whose cost entries is zero, namely cells C_{11} we need to evaluate the value M :

$$M = max\{a_i, b_j; \forall i, j\} /[\min\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\}].$$

$$= max\{5, 11, 14, 5,15,10\} / [min\{0.5, 0.7\}]$$

$$= \frac{15}{0.5} = 30$$

Further that, the cell C_{11},

min {supply, demand}$= min\{a_1, b_1\} = min\{5,5\} = 5$

Therefore the virtual weighted cost for the cell C_{11} is

$$w_{c_{11}} = M \times \min\{a_1, b_1\} = 30 \times 5 = 150$$

Since there is no further cell with zero cost so for the remaining cells we have to evaluate case (a) only. For example of cells C_{13} and C_{32}

$$w_{c_{13}} = \frac{1}{c_{13}} \times min\{a_1, b_3\} = \frac{1}{0.5} \times min\{5,10\} = \frac{1}{0.5} \times 5 = 10$$

$$w_{c_{32}} = \frac{1}{c_{32}} \times min\{a_3, b_2\} = \frac{1}{0.7} \times min\{14,15\} = \frac{1}{0.7} \times 14 = 20$$

Similarly we can find out all virtual weighted cost entries of WOC matrix. The complete WOC matrix is displayed in the Table 3.8(b).

It is observed in the Table 3.8 that thought the cell cost of C_{13} is smaller than that of C_{32} but virtual weighted cost of cell C_{13} is smaller than that of C_{32}. Since the possible amount of allocation of cell C_{32} is much greater than that of the cell C_{13}. According to WOC, the cell C_{11} prefers first then cell C_{32} and then cell C_{13} and so on.

Table 3.8: The WOC and TT tableau of Example 3.3

(a) Transportation Tableau

	D_1	D_2	D_3	S
O_1	**0**	3	0.5	5
O_2	3	5	7	11
O_3	5	0.7	9	14
D	5	15	10	

(b) WOC Tableau

	D_1	D_2	D_3	S
O_1	**150**	5/3	**10**	5
O_2	10/3	5/5	11/7	11
O_3	10/5	**20**	11/9	14
D	5	15	10	

CHAPTER IV

Weighted Opportunity Based Algorithm

4.1 Introduction

In the prior chapter we have successfully developed weighted opportunity cost (WOC) matrix by incorporating supply and demand as weight factor. Now we need to develop an algorithm in which WOC will be incorporated to solve the TP. In order to develop an algorithm by incorporating WOC matrix, we need a base algorithm which is able to solve the transportation problem (TP). For the base algorithm, we have considered LCM method which is simple and straight forward.

4.2 WOC based Algorithm

It is known that Least Cost Matrix (LCM) is very simple and effective to find out IBFS of TP. In LCM, the allocation flows are directly controlled by cost matrix – least cost prefers first over larger costs for allocation. In LCM, cost matrix itself is the Distribution Indicator (DI) and there is no any further Distribution Indicator, which forces the direction of flow of allocation. On the other hand, VAM and its variants methods contain further Distribution Indicator which controls the flow of allocations. Finally, we have developed an algorithm based on LCM embedded with WOC in which flow of allocations are controlled by the WOC matrix rather than only cost matrix to find out IBFS of TP and named as WOC-LCM approach.

In the proposed WOC-LCM, the flow of allocation is controlled by the WOC matrix rather than cost matrix such that the cell with larger weight factor is preferred first for allocation rather than smallest cost as in LCM approach. But after each step of each allocation procedures, WOC matrix is changed as cost matrix is unchanged in LCM approach. The algorithm of WOC-LCM is given below:

Alg. WOC-LCM

Step 1 (Input): read cost matrix [c_{ij}], supply [a_i] and demand [b_j].

Step 2 (Find Allocation Units): find possible maximum allocation units of each cell [c_{ij}]: $min\ \{a_i, b_j\}$.

Step 3 (Find Weighted Opportunity Cost of each Cell $w_{c_{ij}}$):

(a) If $c_{ij} > 0$, set $w_{c_{ij}} = \frac{1}{c_{ij}} \times \min\{a_i, b_j\}$.

(b) Else if $c_{ij} = 0$ and $\{\ c_{pq}: 0 < c_{pq} < 1, \forall p, q\} = \varphi$, (i.e. null set), then set $w_{c_{ij}} = N \times \min\{a_i, b_j\}$; where $N = max\{a_p, b_q; \forall p, q\}$

(c) Else if $c_{ij} = 0$ and $\{c_{pq}: 0 < c_{pq} < 1, \forall p, q\} \neq \varphi$ (i.e. not null set), then set $w_{c_{ij}} = M \times \min\{a_i, b_j\}$; where $M = max\{a_p, b_q; \forall p, q\}/[\min\{c_{pq}: 0 < c_{pq} < 1, \forall p, q\}]$.

Step 4 (Formulation of WOC matrix i.e. **[** $w_{c_{ij}}$]: Do Step 2 and Step 3 for each i and j until the WOC matrix [$w_{c_{ij}}$] is formed.

Step 5 (Allocation Procedure):

Allocate amount of $min\ \{a_i, b_j\}$ at cell C_{ij} s.t. $w_{c_{ij}} = max\ \{w_{c_{pq}}; \forall \text{p}, \text{q}\}$

Step 6 (Updating transportation tableau):

(a) If $a_i = min\ \{a_i, b_j\}$ then set $a_i = 0$ and cross out c_{iq} and $w_{c_{iq}} \forall \text{q}$ and then update demand $b_j^{'} = |b_j - a_i|$;

(b) Else set $b_j = 0$ and cross out c_{pj} and $w_{c_{pj}} \forall p$ and then update supply $a_i^{'} = |b_j - a_i|$;

Step 7 (Termination Condition): Repeated the **Step 5** and **Step 6** unless **termination condition meets** i.e. $a_i = 0, \forall i$ and $b_j = 0, \forall j$.

[i.e. Continuing the allocation procedure until possible all allocation will be completed].

4.3 Experiments and Discussions:

For the justification and effectiveness of the proposed allocation procedure, we have considered a typical example given in Table 4.1.

Table 4.1: A typical example of TP

	D_1	D_2	D_3	D_4	Supply
O_1	0	0	4	5	20
O_2	1	4	2	15	25
O_3	3	2	1	4	10
O_4	4	5	6	3	10
Demand	5	10	30	20	

4.3.1 Solution by LCM approach

As the allocation procedure is based on Least Cost Matrix (LCM) method, so for the comparison between LCM and WOC based LCM approach, at first we will solve the problem with LCM method and then by the proposed WOC-LCM approach. It is observed in the cost matrix of the TP (see Table 4.1) that there are two minimal cost cells namely C_{11} and C_{12}, so we may choose arbitrarily one of the two cells for allocation. Let us choose cell C_{12} and allocate *min* {20, 10} = 10 to the cell and cross out all cells contained in the column 2 as demand of the destination D_2 is satisfied. Accordingly let us update the amount of supply (= 20 – 10) to the origin O_1. After allocation to the cell C_{12}, the next step of allocation is to cell C_{11} as it contains the lowest cost of the remained cost matrix. Since LCM approach is very simple and its steps of allocations are directly control by the cost matrix, so we have skipped all the intermediate steps and displayed the final scenario after all allocations procedure completed in Table 4.2. In the Table the cell cost is displayed on the upper side each cell. On the other hand possible amount of cost corresponding to each cell is shown in the lower left corner and number steps of flow is shown on the lower right corner of each cell. After continuing the process according to the rule of LCM method we have obtained the final solution of the given TP.

Table 4.2: Final solution of the given TP problem obtained by the LCM method

	D_1	D_2	D_3	D_4	Supply
O_1	0 5 ❷	0 10 ❶	4 ×	5 5 ❻	20,10,5
O_2	1 ×	4 ×	2 20 ❹	15 5 ❼	25,5
O_3	3 ×	2 ×	1 10 ❸	4 ×	10
O_4	4 ×	5 ×	6 ×	3 10 ❺	10
Demand	5	10	30,20	20,10,5	

Therefore, the total cost of the TP obtained by the LCM method:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$=0\times5+0\times10+5\times5+2\times20+15\times5+1\times10+3\times10=\mathbf{180}$$

4.3.2 Solution by WOC-LCM approach

Now we will solve the problem by the virtual weighted cost based LCM procedure. According to the procedure of WOC-LCM, we first need to formulate the WOC matrix.

Solution: Step 1 (Formulation of WOC): Since there are two cells namely C_{11} and C_{12} having zero transportation cost. So we need to find *N* first as follows:

$$N = \max\{S_i, D_j\ ; \forall i, j\} = \{20, 25, 10, 10, 5, 10, 30, 20\} = 30$$

and since here $\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\ \} = \varphi$.

Therefore, the virtual weighted cost corresponding to the zero cost cells:

$$w_{c_{11}} = N \times min\,\{S_1, D_1\} = 30\times \min\,(5, 20)$$

$$=30 \times 5=150$$

$$w_{c_{12}} = N \times min\,\{S_1, D_2\} = 30\times \min\,(10, 20)$$

$$=30 \times 10=300.$$

And other weighted costs corresponding to each cell are given by the formula:

$$w_{c_{ij}} = \min \ \{S_i, D_j\} \times \frac{1}{c_{ij}}.$$

After finding all virtual weighted costs we have the WOC matrix which is in the Table 4.3.

Table 4.3: Weighted Opportunity Cost matrix

	D_1	D_2	D_3	D_4	Supply
O_1	150	300	20/4	20/5	20
O_2	5/1	10/4	25/2	20/15	25
O_3	5/3	10/2	10/1	10/4	10
O_4	5/4	10/5	10/6	10/3	10
Demand	5	10	30	20	

Now for hand calculation, we will incorporate this WOC into the given transportation Tableau (TT). After insertion, we have the WOC-TT which is displayed in the Table 4.4. In the WOC-TT each cell cost is shown on the upper right corner and corresponding virtual weighted cost is shown to the upper left corner of each cell.

Table 4.4: Weighted Opportunity Cost based Transportation tableau (WOC-TT)

	D_1		D_2		D_3		D_4		Supply
O_1	150	0	**300**	**0**	20/4	4	20/5	5	20
O_2	5/1	1	10/4	4	25/2	2	20/15	15	25
O_3	5/3	3	10/2	2	10/1	1	10/4	4	10
O_4	5/4	4	10/5	5	10/6	6	10/3	3	10
Demand	5		10		30		20		

Now we have to allocate to the cell according to the rule of LCM method but flow of allocation will be controlled according to the WOC matrix. That is, we will first allocate to the cell which contains the largest **weighted cost** rather than lowest cell cost. It is observed in the Table 4.4 that the cell C_{12} contain largest weighted cost namely 300, so we have to allocate first to the cell C_{12} which obviously *min* {20, 10} i.e. 10.

So after first allocation, the algorithm needs to update TT namely amount of supply, demand and cells. As the demand of the destination, D_2, is satisfied, so cross out all cells contained in the column 2 and also the algorithm update the amount of supply (= 20 – 10) correspond to the origin O_1. The amount of the allocation is shown in the lower left corner to the corresponding cell. The number of allocation step is indicated in the lower right corner of the cell. So after first allocation, the scenario of WOC-TT is displayed in the Table 4.5.

Table 4.5: After 1st allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
O_1	**150** 0	**300** 0 **10** ❶	20/4 4	20/5 5	~~20~~,10
O_2	5/1 1	**10/4** **4** ×	25/2 2	20/15 15	25
O_3	5/3 3	**10/2** **2** ×	10/1 1	10/4 4	10
O_4	5/4 4	**10/5** **5** ×	10/6 6	10/3 3	10
Demand	5	~~10~~	30	20	

After allocation to the cell C_{12}, the next step of allocation is to cell C_{11} as it contains the lowest cost of the remained (ignore column 2 as it satisfies all the demand) cost matrix (see Table 4.5). So after first allocation, it is observed in the reduced table 4.5 that the next largest virtual weighted cost is 150 corresponds to the cell C_{11}, so we need to allocate in this cell which is obviously *min* {10, 5} i.e. 5. After 2nd allocation to the cell C_{11}, algorithm needs to update corresponding row and column. As demand of D_1 is satisfied so the algorithm cross out all the cell containing in the column 1 and update the amount of available supply to the origin O_1 which is off course 5 (=10-5). Therefore after 2nd allocation the pictorial view of WOC-TT is shown in the Table 4.6.

Table 4.6: After 2nd allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
O_1	**150** 0 **5** ❷	**300** 0 **10** ❶	20/4 4	20/5 5	~~20~~,~~10~~,5
O_2	**5/1** **1** ×	**10/4** **4** ×	**25/2** 2	20/15 15	25
O_3	**5/3** **3** ×	**10/2** **2** ×	10/1 1	10/4 4	10
O_4	**5/4** **4** ×	**10/5** **5** ×	10/6 6	10/3 3	10
Demand	~~5~~	~~10~~	30	20	

Now, after second allocation, it is observed in Table 4.6 that in the reduced WOC-TT (ignore column 1 and 2 as they satisfy all corresponding demand) the remained largest virtual weighted cost is 25/2 corresponds to the cell C_{23}, so we need to allocate in this cell with amount *min* {25, 30} i.e. 25. After updating necessary supply, demand and cells we have the reduced WOC-TT and displayed in the Table 4.7.

Table 4.7: After 3rd allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
	150 0 **5** ❷	**300** 0 **10** ❶	20/4 4	20/5 5	~~20~~,~~10~~,5
O_2	**5/1** **1** ×	**10/4** **4** ×	**25/2** 2 **25** ❸	**20/15** **15** ×	~~25~~
O_3	**5/3** **3** ×	**10/2** **2** ×	**10/1** 1	10/4 4	10
O_4	**5/4** **4** ×	**10/5** **5** ×	10/6 6	10/3 3	10
Demand	~~5~~	~~10~~	~~30~~,5	20	

Similarly, we have to allocate step by step according to the WOC matrix. The step by step allocation procedures are displayed on the Tables 4.8 – 4.10 respectively and after completion of all allocation we have the Table 4.11.

Table 4.8: After 4th allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
O_1	**150** 0 **5** ❷	**300** 0 **10** ❶	**20/4** **4** ×	**20/5** 5	~~20~~,~~10~~,5
O_2	**5/1** **1** ×	**10/4** **4** ×	**25/2** 2 **25** ❸	**20/15** **15** ×	~~25~~
O_3	**5/3** **3** ×	**10/2** **2** ×	**10/1** 1 **5** ❹	10/4 4	~~10~~,5
O_4	**5/4** **4** ×	**10/5** **5** ×	**10/6** **6** ×	10/3 3	10
Demand	~~5~~	~~10~~	~~30~~,5	20	

Table 4.9: After 5th allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
O_1	**150** **0** **5** ❷	**300** 0 **10** ❶	**20/4** **4** ×	20/5 5 **5** ❺	~~20~~,~~10~~,~~5~~
O_2	**5/1** **1** ×	**10/4** **4** ×	**25/2** 2 **25** ❸	20/15 15 ×	~~25~~
O_3	**5/3** **3** ×	**10/2** **2** ×	**10/1** 1 **5** ❹	10/4 4	~~10~~,5
O_4	**5/4** **4** ×	**10/5** **5** ×	**10/6** **6** ×	**10/3** 3	10
Demand	~~5~~	~~10~~	~~30~~,5	~~20~~,15	

Table 4.10: After 6th allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
O_1	150 0 5 ❷	300 0 10 ❶	20/4 4 ×	20/5 5 5 ❺	~~20~~,~~10~~,~~5~~
O_2	5/1 1 ×	10/4 4 ×	25/2 2 25 ❸	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	10/1 1 5 ❹	10/4 4	~~10~~,5
O_4	5/4 4 ×	10/5 5 ×	10/6 6 ×	10/3 3 10 ❻	~~10~~
Demand	~~5~~	~~10~~	~~30~~,5	~~20~~,~~15~~,5	

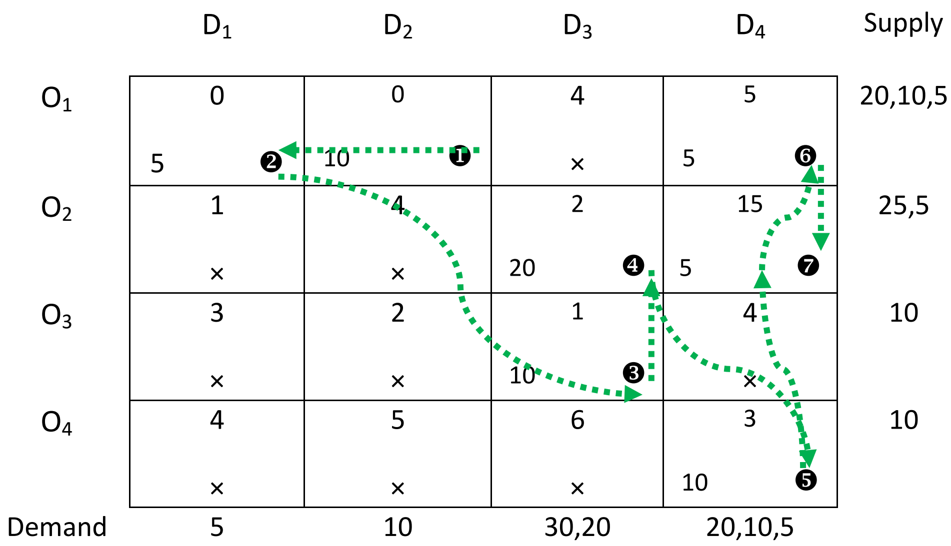
Table 4.11: After all allocation of WOC-LCM approach

	D_1	D_2	D_3	D_4	Supply
O_1	150 0 5 ❷	300 0 10 ❶	20/4 4 ×	20/5 5 5 ❺	~~20~~,~~10~~,~~5~~
O_2	5/1 1 ×	10/4 4 ×	25/2 2 25 ❸	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	10/1 1 5 ❹	10/4 4 5 ❼	~~10~~,~~5~~
O_4	5/4 4 ×	10/5 5 ×	10/6 6 ×	10/3 3 10 ❻	~~10~~
Demand	~~5~~	~~10~~	~~30~~,5	~~20~~,~~15~~,~~5~~	

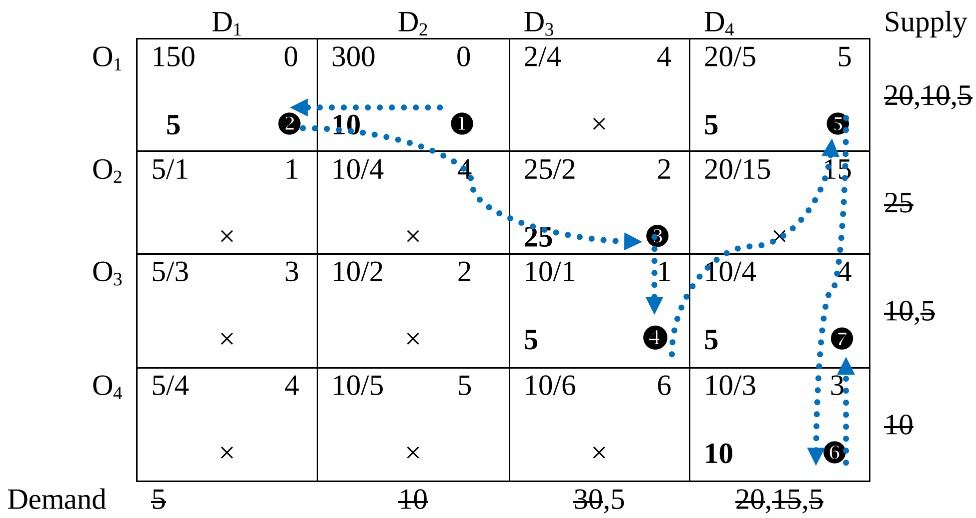
Therefore, the total cost of the proposed WOC -LCM method

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$=0\times5+0\times10+5\times5+2\times25+1\times5+4\times5+3\times10=\mathbf{130}$$



(a)



(b)

Figure 4.1 The Schematic views of the allocation procedures of the LCM and WOC-LCM approach

Now we would demonstrate the flow of allocations of the two approaches considered namely LCM approach and proposed WOC-LCM approach during allocation procedure. The schematic views of the allocation procedures of the two algorithms for the given TP are shown in the Figure 4.1. Figure 4.1 (a) indicates allocation procedure of LCM approach whereas Figure 4.1 (b) indicates allocation procedure of proposed WOC-LCM approach.

It is observed in the figures that the first two steps of allocation flow of both approaches are identical. But from the third steps, the allocation procedures of the two approaches are absolutely different due to the distribution indicator of the two approaches. It is remarked that after second step of allocation, the classical LCM approach considered cell C_{33} for next allocation whereas for the present of WOC matrix, the WOC-LCM approach considered cell C_{23} for third allocation as it contains larger virtual weight than that of cell C_{33}. Eventually the classical LCM bounds to consider the cell C_{24} for last allocation but this cell contains higher transportation cost whereas WOC based approach able to escape from this cell for allocation.

Now we have compared both the results obtained by the LCM method and the WOC based LCM method respectively. The comparison is shown in the Table 4.12. It is observed in the Table 4.12 that the proposed WOC based LCM approach outperforms the LCM approach.

Table 4.12: The comparison between LCM and WOC based LCM approach in TP

Method	**Total Cost**
LCM	180
WOC- LCM	**130**

4.4 Further Experiment and Discussions:

Now to compare the performance of the proposed WOC-LCM method to LCM method, we have to perform further experiments. For this experimental study, we have considered some instances given in the first column of the Table 4.13. For the comparison study, we have considered well-known approaches namely LCM method as the proposed method is formulated base upon this method. The experimental results are displayed in the Table 4.13.

It is observed in the Table 4.13 that out of 13 instances, the proposed WOC-LCM outperform in 8 instances namely Example No 4, 5, 6, 7,8, 9, 10 and 11 compare to LCM method. In the Example No 1, 2, 3 and 13 the results of all the two methods is identical. But only one case, namely Example No.12, the proposed method obtained worse results compared to that of LCM method**.** It is worthwhile to mention here that out of 13 instances in 5 instances the proposed method is able to find out optimal solutions.

Table 4.13 Comparison of LCM and WOC approaches in TP

Ex. No	**Problem**	**LCM**	**WOC-LCM**	**Optimal Value**
1	C_{ij}: {(2,4,6,8); (4,6,8,10); (6,8,10,12); (8,10,12,14)} S: {200,140,90,30} D: {60,90,130,180}	**3520**	**3520**	**3520**
2	C_{ij}: {(2,4,6,8); (4,6,8,10); (6,8,10,12); (8,10,12,14)} S: {140,200,90,30} D: {60,90,130,180}	**3640**	**3640**	**3640**
3	C_{ij}: {(1,2,3); (2,3,4); (3,4,5); (5,5,6)} S: {4,5,6,7} D: {6,7,9}	**85**	**85**	**85**
4	C_{ij}: {(1,2,3,4,5); (2,5,9,4,3); (3,6,5,7,6); (4,2,4,6,7)} S: {20,20,15,15} D: {5,6,14,20,25}	284	**273**	258
5	C_{ij}: {(55,30,40,50,40,0); (35,30,100,45,60,0); (40,60,95,35,30,0)} S: {50,30,50} D: {25,10,20,30,15,30}	3900	**3825**	3475
6	C_{ij}: {(3,3,5); (6,5,4); (6,10,7)} S: {9,8,10} D: {7,12,8}	159	**131**	125

7	C_{ij}: {9,8,5,7; (4,6,8,7); (5,8,9,5)} S: {12,14,16} D: {8,18,13,3}	248	**240**	**240**
8	C_{ij}: {(6,3,5,4); (5,9,2,7); (5,7,8,6)} S: {22,15,8} D: {7,12,17,9}	150	**149**	**149**
9	C_{ij}: {(1,5,7,9); (2,6,10,15); (19,3,8,4); (13,1,5,6)} S: {200,50,20,30} D: {100,80,10,110}	1700	**1610**	1390
10	C_{ij}: {(21,16,23,13); (17,18,14,23); (32,27,18,41)} S: {11,13,19} D: {6,10,12,15}	922	**919**	796
11	C_{ij}: {(1,2,3,1,2,3,1,2,3); (0,3,2,0,1,2,1,2,1); (2,1,0,1,2,1,2,1,1);(1,1,1,2,2,2,2,2,1)} S: {15,5,12,18} D: {12,8,14,5,1,2,3,4,1}	50	**48**	40
12	C_{ij}: {(4,2,6,8); (10,3,8,6); (5,4,9,7)} S: {10,20,20} D: {10,5,20,15}	**320**	325	305
13	C_{ij}: {(10,0,20,11); (12,7,9,20); (0,14,16,18)} S: {20,25,15} D: {10,15,15,20}	480	480	460

From these primary experimental investigations, we may conclude that the performance of the proposed algorithm better than LCM method. But it is noted that solution obtained by the proposed method is IBFS and there is no any guaranty that the solution to be optimal.

4.5 Optimal Solution:

In this section we will test the optimality of the IBFS. If the solution is not optimal, then we need to apply other approach which optimizes the IBFS. An optimal solution is one where there is no other set of transportation routes that will further reduce the total transportation cost. To test the optimality and/or obtain optimal solution here we have considered the well-known method Modified Distribution Indicator (MODI) method.

4.5.1 Modified Distribution Indicator MODI method:

To obtain an optimal solution by making successive improvements to initial basic feasible solution until no further decrease in the transportation cost is possible. Thus, we have to evaluate each unoccupied cell in the transportation table in terms of an opportunity of reducing total transportation cost. An unoccupied cell with the largest negative opportunity cost is selected to include in the new set of transportation routes (allocation). This value indicates the per unit cost reduction that can be achieved by raising the shipment allocation in the unoccupied cell from its present level of zero. This is also known as an incoming cell (or variable). The outgoing cell (or variable) in the current solution is the occupied cell (basic variable) in the unique closed path (loop) whose allocation will become zero first as more units are allocated to the unoccupied cell with largest negative opportunity cost. That is, the current solution cannot be improved further. This is the optimal solution.

4.5.2 An Optimal solution by using (MODI) method:

To find an optimal solution and/or to test the optimality of an initial basic feasible solution obtained by the proposed method (or any such approach) we have considered another **Example 4.2** of a transportation problem.

Example 4.2: The transportation tableau of a transportation problem is shown in the Table 4.14

Table 4.14: Typical example of TP

	D_1	D_2	D_3	D_4	Supply
O_1	8	9	3	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
Demand	4	3	4	4	

Solution: For test the optimality of an IBFS, we first find out the IBFS of the given TP by the proposed WOC-LCM method. The pictorial view of the final solution (IBFS) obtained by the WOC-LCM method is shown in the Table 4.15.

Table 4.15: Solution of the given TP by using proposed algorithm WOC-LCM.

	D_1	D_2	D_3	D_4	Supply
O_1	3/8 8 ×	3/9 9 ×	3/3 3 ×	3/1 1 **3**	~~3~~
O_2	4/10 10 ×	3/8 8 **2**	4/5 5 **4**	4/4 4 **1**	~~7~~,~~6~~,~~2~~
O_3	4/7 7 **4**	3/6 6 **1**	4/6 6 ×	4/8 8 ×	~~5~~,~~1~~
Demand	~~4~~	~~3~~,~~2~~	4	~~4~~,~~1~~	

The basic variables are 6 (= 4+3-1) and the set of basic cells do not contains a loop. So the initial basic feasible solution obtained by the proposed method is

$x_{14} = 3$, $x_{22} = 2$, $x_{23} = 4$, $x_{24} = 1$, $x_{31} = 4$ and $x_{32} = 1$

Therefore, the total transportation cost = $1\times 3 + 8 \times 2 + 5 \times 4 + 4 \times 1 + 7 \times 4 + 6 \times 1$

= 77

Now by using MODI algorithm we will find out the optimal solution of the problem in which we will use this IBFS obtained by the proposed method. Therefore, we have to calculate the cell evaluations corresponding to all non-basic or unoccupied cells.

Here we find the values u_i and v_j using the relation $u_i + v_j = c_{ij}$ for all basic (occupied) cells.

For basic cell:

(1, 4): $u_1 + v_4 = 1 \cdots (1)$

(2, 2): $u_2 + v_2 = 8 \cdots (2)$

(2, 3): $u_2 + v_3 = 5 \cdots (3)$

(2, 4): $u_2 + v_4 = 4 \cdots (4)$

(3, 1): $u_3 + v_1 = 7 \cdots (5)$

(3, 2): $u_3 + v_2 = 6 \cdots (6)$

Let arbitrary $u_2 = 0$ as second row contains the maximum number of occupied cells.

$\therefore$ (3) $\Rightarrow v_3 = 5$

(4) $\Rightarrow v_4 = 4$

(5) $\Rightarrow v_1 = 9$

(6) $\Rightarrow u_3 = -2$

(1) $\Rightarrow u_1 = -3$

(2) $\Rightarrow v_2 = 8$

$\therefore\ u_1 = -3, u_2 = 0, u_3 = -2, v_1 = 9, v_2 = 8, v_3 = 5$ and $v_4 = 4$

Which are displayed right side and below the Table 4.16.

Table 4.16: Optimal Solution of TP

	D_1	D_2	D_3	D_4	u_i
O_1	-3 9	-3 8	-1 3	**3** 1	-3
O_2	-1 10	**2** 8	**4** 5	**1** 4	0
O_3	**4** 7	**1** 6	-3 6	-6 8	-2
v_j	9	8	5	4	

Cell evaluation for unoccupied cells: $\Delta_{ij} = (u_i + v_j) - c_{ij}$

For cell:

(1, 1): $\Delta_{11} = (u_1 + v_1) - c_{11} = -3 + 9 - 9 = -3$

(1, 2): $\Delta_{12} = (u_1 + v_2) - c_{12} = -3 + 8 - 8 = -3$

(1, 3): $\Delta_{13} = (u_1 + v_3) - c_{13} = -3 + 5 - 3 = -1$

(2, 1): $\Delta_{21} = (u_2 + v_1) - c_{21} = 0 + 9 - 10 = -1$

(3, 3): $\Delta_{33} = (u_3 + v_3) - c_{33} = -2 + 5 - 6 = -3$

(3, 4): $\Delta_{34} = (u_3 + v_4) - c_{34} = -2 + 4 - 8 = -6$

Displayed these values of cell evaluation for unoccupied cells on the proposed Weighted Opportunity Cost matrix based Least Cost Method (WOC-LCM) of the corresponding cell. Here all $\Delta_{ij} \leq 0$ i.e. all the cell evaluation is negative the solution is optimum and optimum solution is

$x_{14}=3, x_{22}=2, x_{23}=4, x_{24}=1, x_{31}=4$ and $x_{32}=1$

and minimum cost $z = 77$ units.

It is worthwhile to remark that, the result obtained by the proposed algorithm is optimum which is 77. On the other the IBFS of the LCM method is 79 and obviously these are not optimum.

CHANPER V

Conclusion

Transportation Problem is a multi-disciplinary field and has been playing very important role in business arena. The main task in TP is to minimize transportation cost as well as other relevant issues so that the profit is maximized. Researchers are continuously hunting for finding better transportation algorithms. Regarding the methods of finding initial basic feasible solution, much of the research works concern with the transportation cost entries, and/or the manipulation of cost entries to find out the flow of allocations. To develop the indicator of flow of allocation, researchers are dedicated to the manipulation of cost matrix whatever be the structure of supply entries and demand entries. Besides LCM method which is very simple and flow of allocations are controlled by its own cost matrix directly does not need to formulate any further DI matrix. The strategy of allocation is simple: least cost prefer first for the allocation, whatever be the structure of supply/demand entries.

But it is observed that supply and demand play a vital role in real market. Exploiting this idea, we have formulated a weighted opportunity cost (WOC) matrix in which supply and demand entries act as a weight factor upon transportation cost entries. After successful formulation of WOC matrix we have intended to develop an effective algorithm for solving TP. Now exploiting the strategy of LCM approach we have developed a new algorithm named WOC-LCM in which flow of allocation is controlled by the WOC with the strategy – cost cell with largest virtual weighted cost prefers first.

Several experiments have been carried out to investigate the performance of the proposed algorithm. Experimental results reveal that the proposed method is effective as well as efficient to find out the IBFS of TP. Moreover, sometime the proposed method is able to find out optimal solution too. On the average, exception of few instances the proposed method has performed better than LCM methods to find out IBFS of TP.

Our main contribution of this research is that we have incorporated a new and unique idea i.e. supply/demand may play a significant role in modeling the TP. As it is a new way to think about solving TP, we hope by performing further intensive research works, some excellent and fruitful outputs might come out.

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