STUDY OF THE YAW STABILITY OF A DOWNWIND HORIZONTAL AXIS WIND TURBINE

BY

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ABSTRACT

Present thesis discuss a method to calculate the overall design, performance and structural analysis of a horizontal axis wind turbine. For a particular wind velocity, the optimum rotor configuration for twist and chord is determined using the momentum theory and the blade element theory, assuming zero drag, no coning and no tilting angle.

After determining the optimal aerodynamic shape of the blade, the forces and moments on blades and tower top are calculated. At this stage, the equations may be extended to include the effects of coning and tilting angles, and the effect of several wind conditions such as wind gradient, wind shift and tower shadow. In this thesis, effects of coning and tilting angles are not considered. But the analysis of stability by determining the forces and moments acting on a horizontal axis wind turbine when yawed to non-axial flow is presented.

The effect of tower blockage on the stability at various yaw angle is also considered. Three types of blade shape such as, Optimum-chord Optimum-twist, Linear-chord Linear-twist and Linear-chord Zero-twist are considered .

CONTENTS

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LIST of SYMBOLS

a	axial interference factor.
á	tangential interference factor.
A	turbine disc area, πR^2.
A_2	wake cross sectional area.
B	number of blades.
B_f	tower blockage factor.
C	chord of the blade.
C_D	blade drag coefficient, $\dfrac{dD}{1/2\,\rho C W^2 dr}$.
C_L	blade lift coefficient, $\dfrac{dL}{1/2\,\rho C W^2 dr}$.
C_{Ld}	design lift coefficient.
C_{mp}	pitching moment coefficient, $\dfrac{M_P}{1/2\,\rho A V_\infty^2 R}$.
C_{mz}	yawing moment coefficient, $\dfrac{M_{yaw}}{1/2\,\rho A V_\infty^2 R}$.
C_P	power coefficient, $\dfrac{P}{1/2\,\rho A V_\infty^3}$.
C_Q	torque coefficient, $\dfrac{Q}{1/2\,\rho A V_\infty^2 R}$.
C_T	thrust coefficient, $\dfrac{T}{1/2\,\rho A V_\infty^2 R}$.
D	drag force.
dA	blade element area, Cdr.
dC_P	elemental power coefficient, $\dfrac{dP}{1/2\,\rho A V_\infty^3}$.
dC_Q	elemental torque coefficient, $\dfrac{dQ}{1/2\,\rho A V_\infty^2 R}$.
dC_T	elemental thrust coefficient, $\dfrac{dT}{1/2\,\rho A V_\infty^2 R}$.
dD	blade element drag force.
dF_n	blade element normal force.
dF_t	blade element tangential force.
dP	blade element power.
dL	blade element lift force.

dQ blade element torque.
dT blade element thrust.
F Prandtl's loss factor.
F_{hub} hub loss factor.
F_{tip} tip loss factor.
F_{Xo} force acting along X-direction in S_o coordinate system.
F_{X3} force acting along X-direction in S_3 coordinate system.
F_{Yo} force acting along Y-direction in S_o coordinate system.
F_{Y3} force acting along Y-direction in S_3 coordinate system.
F_{Zo} force acting along Z-direction in S_o coordinate system.
F_{Z3} force acting along Z-direction in S_3 coordinate system.
g acceleration due to gravity.
H hub height from ground level.
K_T transformation matrix for tilting.
K_β transformation matrix for coning.
K_θ transformation matrix for azimuth.
L lift force.
M_P pitching moment.
M_X flapwise bending moment.
M_Y edgewise bending moment.
M_{Yaw} yawing moment.
P turbine power.
P_e amount of power to be extracted.
P^+ pressure immediately in front of the rotor.
P^- pressure immediately behind the rotor.
r local blade radius.
r_{hub} hub radius.
R rotor radius.
R_L radial distance along the blade.
Q torque.
Q_{st} starting torque.
S_o fixed reference coordinate system.
S_1 coordinate system considering tilt angle.
S_2 coordinate system considering blade azimuth .
S_3 coordinate system considering blade coning.
T thrust force.
U wind speed through the turbine.
V wake velocity behind the rotor.
$\bar{V}$ average wind velocity.
V_d design wind velocity.
V_R wind velocity at the centre line of the rotor.
$V_{Ref.}$ reference wind velocity.
V_{So} wind velocity correspoding to S_o coordinate system.
V_{S3} wind velocity correspoding to S_3 coordinate system.
V_t tangential wind velocity.
V_∞ undisturbed wind velocity.
$V_{\infty o}$ local wind velocity considering wind shear.
$V_{\theta k}$ instantaneous wind velocity correspoding to an azimuthal point.
W relative wind velocity.
X_o distance along X-direction in S_o coordinate system.
Y_o distance along Y-direction in S_o coordinate system.
Z_o distance along Z-direction in S_o coordinate system.
Z height of a particular point from the ground level.
$Z_{Ref.}$ height of a reference point from the ground level.

GREEK ALPHABETS

α	angle of attack.
α_d	design angle of attack.
α_T	tilt angle.
β	coning angle.
β_T	blade twist angle.
γ^*	yawing angle.
η	power law exponent.
η_{eff}	efficiency of a windmill.
θ_k	blade azimuth angle.
θ_P	blade pitch angle.
λ	tip speed ratio.
λ_d	design tip speed ratio.
λ_r	local tip speed ratio.
ν_a	induced axial velocity.
ν_{S3}	induced velocity in S_3 coordinate system.
ν_t	induced tangential velocity.
ν_v	induced vertical velocity.
ρ	air density.
σ	solidity, $\dfrac{BC}{2\pi r}$
ϕ	angle of relative wind velocity.
ω	wake rotational velocity.
Ω	angular velocity of the rotor.

SUBSCRIPTS

a	axial.
d	design.
D	drag.
eff.	efficiency.
g	gravity.
L	lift.
max.	maximum.
min.	minimum.
n	normal.
P	power.
Q	torque.
r	local.
ref.	reference.
St	starting.
T	tilt,twist and thrust.
t	tangential.
v	vertical.
ω	wake.
β	coning.
θ	azimuth.

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.0 General

In the present world, energy crisis is the vital problem. There are many sources of energy such as gas, petroleum oils, minerals, solar, tidal,wind power etc. The wind power is one of the cheapest source of energy. Wind energy can be converted into mechanical energy through wind turbine. The people of twentieth century interested to get energy from wind, so they are looking for different type of wind turbines at different countries of the world. Environmental pollution and public health hazard due to energy conversion may be avoided, if we extract energy from wind power.

Various studies have indicated that wind energy has the potential to make significant contributions to the national needs. Therefore, our nation is interested to new cheapest energy sources and to justify their parcticability.

1.1 Historical Background

From the ancient period different types of windmill have been designed and constructed to extract power from the wind. The earliest type of windmill known as Persian millwright built in 644 A.D. and windmills in Seistan, Persia 915 A.D. [13] were used for moving water, consisted of several sails that rotated on a vertical axis.

By the eleventh century A.D., windmills were in extensive use in the Middle East and were brought to Europe in the thirteenth century by the returning Crusaders. Many horizontal axis windmills had been constructed for grinding grains and lifting water in that period in Europe.

In the fourteenth century, the Dutch had improved the design of windmills and used for draining the marshe and lakes of Rhine river delta. The first oil mill was built in Holland in 1582, and the first paper mill was built in 1586. Sawmills were introduced to process timber, at the end of sixteenth century. About 9000 windmills were used in Netherlands for different purposes in the middle of nineteenth century. The Dutch improved the design of windmills particularly in the design of rotors. In the sixteenth century, the premitive jib sails on wooden booms had been used. The sails were typically constructed from sailcloth stretched over a wooden frame. But in modern designs sheet metal replaced the sailcloth. In the twentieth century, only 2500 windmills were still in operation in the Netherlands. In 1960, fewer than 1000 were still in working condition.

Since the mid-nineteenth century, many small multi-bladed windmills, power output capacity less than 1 hp each in an average wind speed, have been built and used in the United States to pump water, generate electricity and perform similar functions.

During the early part of the twentieth century, two quite different vertical types windmill were developed. One of the two, known as the Savonius rotor was formed by cutting a cylinder into two semi-cylindrical surfaces,moving these surfaces sideways along the cutting plane to form a rotor with cross-section in the form of the letter "S", placing a shaft in the centre of the rotor, and closing the end surfaces with circular end plates as shown in the Figure 1.1.1. The other vertical windmill design, patented in 1927 by George Darrieus consisted of two thin airfoils with one end mounted in the lower end of a vertical shaft and the other end mounted on the upper end of the same shaft,as shown in the Figure 1.1.2.

Palmer Putnam had constructed a windmill with the help of the S. Morgan Smith company of New York, Pensylvania using NACA 4418 as shown in the Figure 1.1.3 and operated the plant in the early 1940. The two bladed, 175 ft. diameter, propeller type rotor, weighed 16 tons and operated at a constant rotational speed of 28 rpm to produce up to 1.25 Megawatts (MW) of power during the period from 1941 to 1945.

At the end of nineteenth century, about 2500 windmills were in operation, supplying a total power of about 40,000 hp or 30 MW. During the world war-II, the Danes developed and operated a number of new types of large scale wind machines to produce electricity. The number of these machines were increased from 16 in the summer of 1940, to 88 in the beginning of 1944. After the world war-II the number of operating machines started to decrease, and at the end of 1947, it dropped to 57. The decrease continued up to 1950, as a result production of electricity decreases and windmills again returned for experimental work, with units rated 12 kW, 45 kW and 200 kW in Denmark. The 200 kW Gedser windmill, which was the latest in this series, was operated untill 1968. After the energy crisis of 1973, the 200 kW Gedser windmill was refurbished, and in 1977 it was again put back into service. In 1931, the Russians built an advanced 100 kW wnid turbine near Yalta on the Black sea. The annual output was found about 280,000 kWhr.

In the 1950,the Enfield cable company built a wind-powered generator as shown in the Figure 1.1.4. This machine was operated at St. Albans,England,and later in Algeria.It was designed for 100 kW output of A.C. power in a 13.41 m/sec wind speed. It was an interesting design rather than conventional design.It used air instead of gears to transmit the propeller power to the generator.

The propeller blades were hollow,and when it rotated, it acted as centrifugal pumps. The air entered into the ports in the lower part of the tower, passed through an air turbine which turned the electric generator, went up through the tower; and went out through the hollow tips of the blades. The efficiency of this unit was low compared to conventional horizontal axis wind turbine. The main advantage of this system is that the power generating equipment is not supported aloft.

The French built and operated several large wind powered electric generators in the period from 1958 to 1966. These included three horizontal-axis units,each consists of three propeller type blades. The smaller unit had a 70 ft. diameter rotor operated at 56 r.p.m. The largest one was rated at 1,000 kW in wind speed of 18.25 m/sec and weighed 96 tons, excluding the tower.

In Germany the number of windmill was in use about 18,000 in 1895, 17,000 in 1907, 11,400 in 1914 and between 4,000 to 5,000 in 1933.The Germans introduced a number of improvements in the design of wind-powered generator. The largest unit generated 100 kW in 8 m/sec of wind speed. These units operated sucessfully for more than 4,000 hrs. during the period from 1957 to 1968.

Recently, large wind energy conversion systems have been built and tested in a number of countries around the world. But the problem was the installation cost per kilowatt which was too high when compared with the costs of other methods of producing electric power. Todays increasing cost of fuel, coupled with energy crisis, the nation is forced to reexamine the wind energy as a cheap source of power.

1.2 Motive of this thesis

Due to recent world wide energy crisis our nation is interested to search a new energy source rather than conventional energy source i.e. petroleum oil, mineral fuels, which may be solar and wind energy.

The availability of faster and more economical digital computer has encouraged to do research on computational basis. Aerodynamic and structural analysis are important but in most existing design only aerodynamic design is considered. Most of these programs do not considered important effect of coning, tilting and azimuth. Again, there is no such program which considers the effect of yawing angle and tower shadow for a downwind horizontal axis wind turbine at different wind conditions. It is obvious that this lack of adequate method results in both uneconomical and dangerous situation. It is dangerous because structural safety is not always guaranted and designs without consideration of all important parameters are not safe.

In this work, only aerodynamic performance of the turbine is analysed. The obtained results are forces, moments, thrusts, torque and power. The present analysis is based on the combination of the momentum theory and blade element theory, which is known as strip theory. Therefore, the design &

performance and stability of a horizontal axis wind turbine is carried out by using a simplified method, which covers all the important parameters that are necessary for a best design.

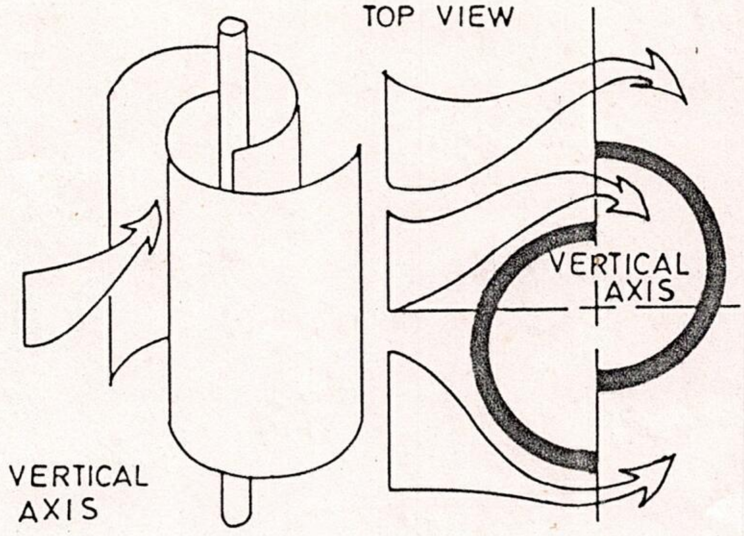

FIGURE 1.1.1: SAVONIUS ROTOR

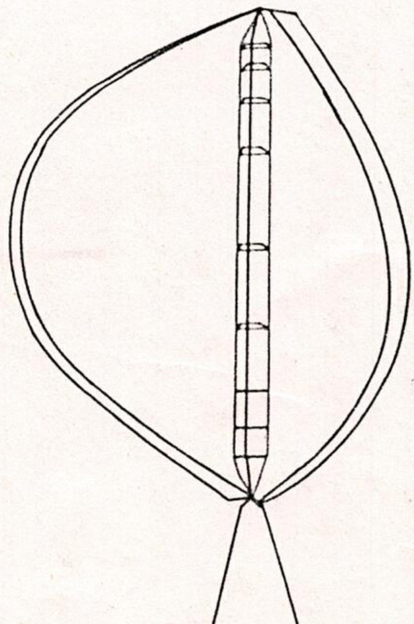

FIGURE 1.1.2: DARRIEUS ROTOR

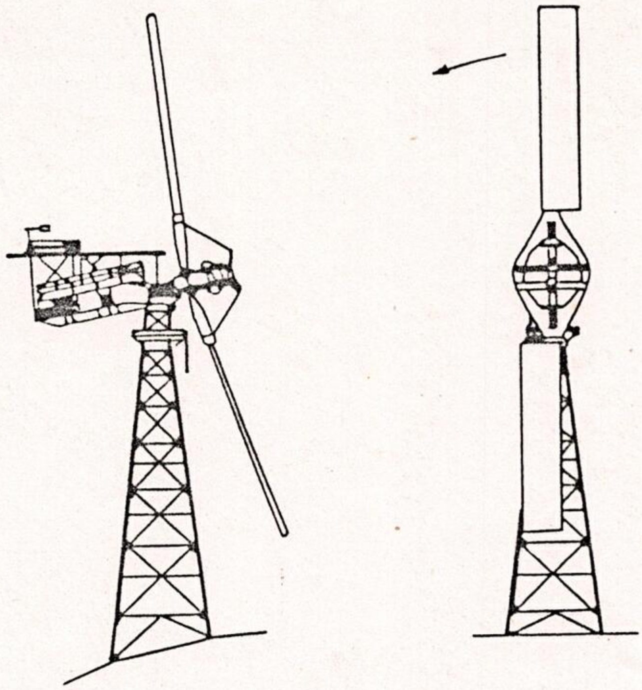

FIGURE 1.1.3 : SMITH – PUTNAM WIND TURBINE

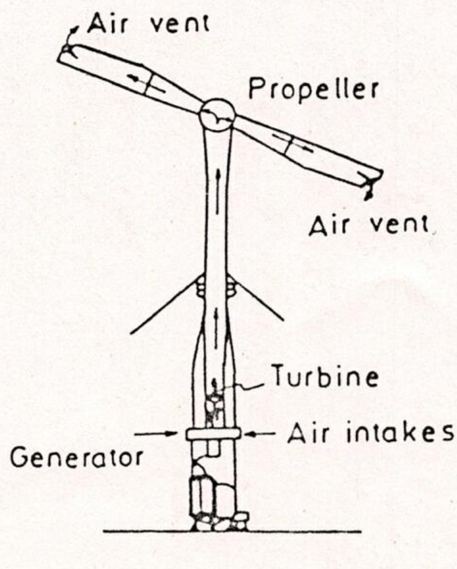

FIGURE 1.1.4 : ENFIELD – ANDREAU WIND ENERGY CONVERSION SYSTEM

CHAPTER II

LITERATURE SURVEY

2.0 General

The research on the wind turbine is very little and a few literature has been published on structural and aerodynamical design for stability problem and for the effect of tower shadow of the horizontal axis wind turbine. In order to visualize actual flow field and much of the general behavior, propeller technology has been used. Because of the restricted interest and financial support, wind power studies is very recent.Prototype experiment is very difficult, owing to the size of the rotors and steadiness of the wind over sufficiently long periods.Model experiments are possible but difficult due to wind tunnel walls blockage. In the following chapter the review of the existing literature which forms the foundation of the present analysis is presented.

2.1 Review of Existing Literature

The horizontal axis wind turbine is to be cosidered as a airscrew that extracts energy from the driving air and converts it into mechanical energy. But a propeller delivers energy into the air which extracts from the another source of energy. Since there is a similarity between the propeller and the wind turbine, it is possible to use same theoretical development for the performance analysis. The propeller theory was developed based on two different independent methods of approach, one of which is called as momentum theory approach and the other is blade element theory approach.

W.J.M.Rankine [6], developed and described the axial momentum theory in 1865 and it was improved later by R.E.Froude [12]. The basis of the theory is to determine the forces acting on the rotor to produce the motion of the fluid. The theory predicts the ideal efficiency and flow velocity, but it gives no imformation about the blade shape, which is necessary to generate wake effects later were included by A.Betz [41] in this theory.

The blade element theory was first originated by W.Froude [11] in 1878 and later developed by S.Drzewiecki [8]. The approach of blade element theory is opposite to that of momentum theory. The blade element theory determines, the forces produced by the blades as a result of the fluid motion. It was hampered in its original development by lack of knowledge of sectional aerodynamics and mutual interference blades.

Modern propeller theory has developed from the concept of free vortices being shed from the rotating blades. These vortices define a slipstream and generate induced velocities. The theory can be attributed to the work of Lanchester [42] and Flamm [43] for the original concept. Later, Joukowski [7] introduced the induced velocity analysis and A.Betz [6] introduced optimization concept. L. Prandtl [29] and Goldstein [37] developed seperately the circulation distribution or tip loss analysis and H.Glauert [15,16,17], E. Pistolesti [35] and S.Kawada [26], for general improvements. Recently, R.E.Wilson, Stel N. Walker and P.B.S. Lissaman [45] have further analysed the aerodynamic performance of wind turbines. They have introduced a new method to apply tip loss which is sometimes referred as the linear method. This method is based on the assumption that the axial and tangential induced velocities are localised at the blade and only a fraction of these occur in the plane of the rotor. The theory has been referred by a number of names; vortex theory, modified blade element thoery and strip theory.

The strip theory used extensively for the performance analysis of a horizontal axis wind turbine and helicopter rotors. The technique which assumes local two dimensional flow at each radial rotor station is a design analysis approach in which the airfoil sectional aerodynamics, chord and pitch angle are required in order to calculate the forces and torque.

The application limit of the momentum theory is the state where reverse flow begins to occur downstream of the rotor. The operating states of a wind turbine can be classified theoretically into four categories depends on interference factor [47].

For $a<0$, the regime is called, the propeller state. In this state the rotor is acting as a propeller accelerating the flow, with thrust opposing flow and power is to be added. The thrust coefficient and the power coefficient are then negative.

For $0 <a<1$, the regime is called the windmill state. The windmill state is divided into two regime namely, windmill brake state ($0<a<0.5$) and turbulent wake state ($0.5<a<1$). Both are for decelerating flow with positive thrust coefficient and power coefficient and power is extracted from the flow. At $a=0.5$, thrust coefficient is maximum and at $a=1/3$, power coefficient is maximum. At $a> 0.5$, the wake velocity is zero and streamlines cease to exist. In the turbulent wake state, the simple theory would require flow reversal in the far wake, velocity is zero somewhere between disk and infinity and the rotor operates continuously in this anomalous state, characterized by large recirculating flows and high turbulence.

For $a>1$, the rotor enters the propeller brake state, with power being added to the flow to create downwind thrust, corresponding to reversing propeller thrust on loading. For values of a, slightly greater than unity, the flow regime is called the vortex ring state, experienced by helicopter rotors during part-power descent. Airplane propellers pass through this state to that of the brake state with reversing thrust, as shown in the Figure 2.1.1.

The states mentioned above can occur simultaneously at different positions of the blade. In the helicoptor analysis, several experimental studies have been made on the vortex ring state and on the turbulent wake state. Lock [30,31] conducted wind tunnel tests using two model rotors and Glauert [18], defined a characteristic curve utilizing the data of Lock and its approximate formula is given by Johnson [23]. Wilson [46],has suggested linear algebraic expressions for the local thrust at high tip speed ratios where a wind turbine may operate in the vortex ring state. There is very little published literature on the performance of a wind turbine operating in the turbulent wake state. Yamane [52] has introduced a performance prediction method for windmill in the turbulent wake state utilizing the emperical characteristic curve of Glauert in combination with the blade element theory. Recently, Anderson [1], has published the results of a vortex-wake analysis of a horizontal axis wind turbine and he compared his results with those obtained from strip theory. He suggests unless information is required about the wake, it is satisfactory to use the blade element theory with a suitable model for tip loss. Viterna [19] has suggested an improved method to calculate the aerodynamic performance of a horizontal axis wind turbine at tip speed ratios where aerodynamic stall can occur as the blade experiences high angle of attack.

Walker [50] has developed a method to determine the blade shapes for maximum power. According to his method, the blade chord and twist angle are continuously varied at each radial station until the elemental power coefficient has been maximized. This is obtained when every radial element of the blade is operating at the airfoil's maximum lift to drag ratio. This results in the lift coefficient and angle of attack being identical at each radial element. Anderson [2] has compared near-optimum and optimum blade shapes for turbines operating at both constant tip speed ratio and constant rotational speed. Shepherd [38], has suggested a simplified method for design and performance analysis of a horizontal axis wind turbine which can be carried out on a hand held calculator by elimination of iteration process. It is based on the use of the ideal and optimised analysis to determine the blade geometry. It requires only fixed values of the axial induction factors and corresponding optimised rotational induction factors.

Jansen [24] worked on the theories that form the basis for calculation of the design and the behaviour of a windmill. A modification of the Prandtl tip loss model is derived. Due to this modification, relatively heavy loading of the windmill rotor,takes into account. It is argumented that, in contrast with propeller design, a maximum energy extraction may be achieved by enlarging the chords of the blades near the tips. He concluded that with simple materials high power coefficients are possible. Large horizontal axis wind turbines must be designed for optimum performance with minimum maintenance and weight. It should be stable under different wind conditions. Ormiston [33] reported a qualitative discussion of the effects of size, number of blades, hub configuration and type of control system on the turbine dynamic characteristics. Ormiston [34] considers the basic flapping response of a wind turbine blade using elementary analytical techniques. Various other effects such as cross wind, rotor shaft and yaw precession were also treated in a simple manner.

Powells [36,51] investigated the effects of tower shadow on a downwind two-bladed horizontal axis wind turbine. A rotor aero-elastic simulation is used to predict the blade response to tower shadow, and subsequently to estimate increased blade fatigue damage. He suggested to reduce the effect of tower shadow by making the aerodynamically smooth tower, thereby reducing the flow disturbance. Due to change in wind direction, it is necessary to have an aerodynamic tower fairing which is free to move in yaw with the turbine nacelle.Since an effective fairing has been easily and cheaply built,and since it encounters a far easier regime than the blades of the machine, it follows that this may well be a worthwhile addition for small and medium size downwind machines. By reducing both the strains on the blades can be considerably moderate. The reduction in the noise lavels observed would help to make such machines more environmentally acceptable as well.

Islam M.Q. [21] presented a procedure for the aerodynamic design and structural analysis of horizontal axis wind turbines using simplified methods. In the first part of this programe,optimum rotor configuration is determined, using the momentum and blade element theories. The equations are extended to include the effects of wind shift, wind gradient, blade azimuth, and tilting angle. Forces and moments are then calculated and compared with the results of existing wind turbines.

2.2 Scope of the Thesis

This thesis discussed a method for determining aerodynamic design, problems of stability and effect of tower shadow of the horizontal axis wind turbine due to various wind conditions. In chapter one, a general outline concerning wind turbine, a historical background of different types of wind turbine and motive behind the present work is discussed.

In chapter two, a review of the existing literature is discussed.

In chapter three, axial momentum theory and blade element theory including the effect of wake rotation is presented. Modifications of these theories due to various losses are also presented to make these theories adequate for performance analysis of wind turbines. With the help of these theories, expressions for maximum power extraction is also derived.

In the chapter four, the existing theories are modified further for various important effects such as coning angle, tilting angle, yawing angle and blade azimuth angle. Performance calculations are performed for each blade segment taking the relative wind speed and angle of attack axially axisymmetric. Also the expressions for various forces and moments that are responsible for unstability are derived.

In the chapter five, the selection criteria and investigation techniques of various design parameters are presented. The procedure of calculation of blade configurations for optimum performance is also presented in this chapter.

Chapter six discussed with the problems of unstablity of horizontal axis wind turbine under the influence of yaw. Equations of tower top forces and moments are derived here.

Chapter seven deals with the results obtained and the general dicussions.

In the chapter eight conclusions and recommendations are presented.

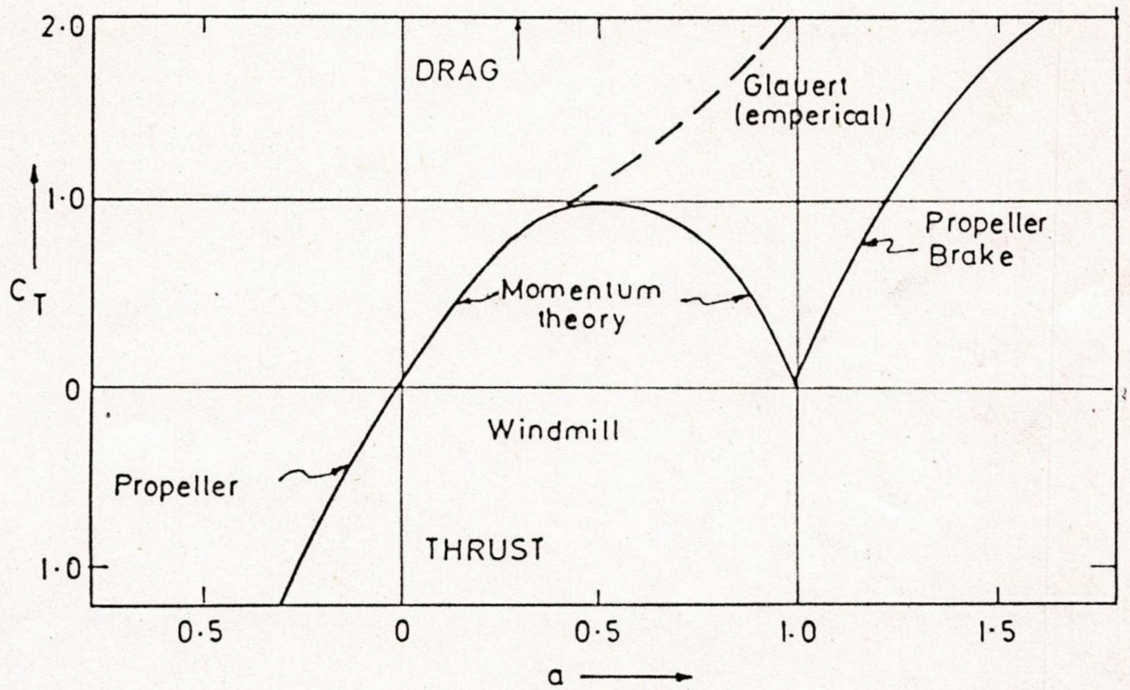

Fig. 2·1·1 Various Rotor States

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CHAPTER III

EXISTING THEORIES

3.0 General

Wind turbine is a device that extract energy from the flowing air and converts the extract energy into mechanical energy which later transformed into other forms of energy. For performance analysis of the wind turbine the flow of air is considered as steady flow and the influence of turbulence of the atmospheric boundary layer is neglected.

Most existing theoretical models are based on the strip theory. The basic theoretical development of strip theory is discussed in the present chapter. Effects of wake rotation, tip and hub losses and expressions for maximum power are also presented in this chapter.

3.1 Axial Momentum Theory

The axial momentum theory was presented by Rankine in 1865 and later modified by Froude [12]. The theory determines the forces, acting on the rotor to create motion of the fluid. The theory is useful to predict the ideal efficiency of a wind turbine. For the maximum possible output of a wind turbine the assumptions underlying the axial momentum theory are:

1. The fluid is inviscid and incompressible.
2. Infinite number of blades.
3. Flow is entirely axial with no rotational motion.
4. Thrust loading is uniform over the disc.
5. The flow is homogenous.
6. Static pressure far ahead and far behind the rotor are equal to the undisturbed ambient static pressure ($P_2=P_\infty$).

Considering the control volume in Figure 3.1.1, where the upstream and downstream control volume planes are infinitely far away from the turbine disc plane. The air approach velocity V_∞, decelerated to velocity U, at the turbine disc and exit velocity become V, at the downstream plane. The conservation of mass can be expressed as

$$\rho A_1 V_\infty = \rho A U = \rho A_2 V \qquad (3.1.1)$$

where,

V_∞ = Undisturbed wind velocity.
U = Wind velocity through the rotor.
V = Wind velocity far behind the rotor.
A = Turbine disc area.
A_1 = Cross sectional area of incoming wind.
A_2 = Wake cross sectional area.
ρ = Mass density of air.

The thrust force T on the rotor is given by the change of momentum of the flow,

$$T = \dot{m}(V_\infty - V) = \rho A_1 V_\infty^2 - \rho A_2 V^2 \qquad (3.1.2)$$

From the equation (3.1.1) and (3.1.2) we have,

$$T = \rho AU(V_\infty - V) \qquad (3.1.3)$$

The thrust on the rotor can also be expressed from the pressure difference over the rotor area,

$$T = A(P^+ - P^-) \qquad (3.1.4)$$

where,

P^+= Pressure immediately in front of the rotor.

P^-= pressure immediately behind the rotor.

Now applying Bernoulli's equation between the upstream plane and the rotor plane, we have

$$P_\infty + \frac{1}{2}\rho V_\infty^2 = P^+ + \frac{1}{2}\rho U^2 \qquad (3.1.5)$$

Again applying Bernoulli's equation between the rotor plane and downstream plane, we get

$$P_\infty + \frac{1}{2}\rho V^2 = P^- + \frac{1}{2}\rho U^2 \qquad (3.1.6)$$

Substracting equation (3.1.6) from equation (3.1.5), we get

$$P^+ - P^- = \frac{1}{2}\rho(V_\infty^2 - V^2) \qquad (3.1.7)$$

The expression for thrust from equation (3.1.4) becomes,

$$T = \frac{1}{2}\rho A(V_\infty^2 - V^2) \qquad (3.1.8)$$

Equating the equation (3.1.8) with (3.1.3),

$$\frac{1}{2}\rho A(V_\infty^2 - V^2) = \rho AU(V_\infty - V)$$

$$\text{or,} \quad U = \frac{V_\infty + V}{2} \qquad (3.1.9)$$

Above results show that the velocity through the turbine rotor is equal to the average of the wind velocity ahead of the turbine and wake velocity behind the turbine.

The velocity at the rotor U is often defined in terms of an axial inetrference factor a as,

$$U = V_\infty (1-a) \tag{3.1.10}$$

Balancing equations (3.1.9) and (3.1.10), the wake velocity can be expressed as,

$$V = V_\infty (1-2a) \tag{3.1.11}$$

The change in kinetic energy of the mass flowing through the rotor area is the power absorbed by the rotor,

$$P = \dot{m}\Delta KE = \frac{1}{2} \rho A U (V_\infty^2 - V^2) \tag{3.1.12}$$

With equations (3.1.10) and (3.1.11) the expression for power becomes,

$$P = 2 \rho A V_\infty^3 a(1-a)^2 \tag{3.1.13}$$

Power coefficient C_p is given by

$$C_p = \frac{P}{1/2 \rho A V_\infty^3}$$

$$= \frac{2 \quad A V_\infty^3 a(1-a)^2}{1/2 \rho V_\infty^3 A}$$

$$= 4a(1-a)^2 \tag{3.1.14}$$

Differentiating equation (3.1.13) with respect to a, and equating to zero, maximum power is found,

$$\frac{dp}{da} = 2 \rho A \ V_\infty^3 (1-4a+3a^2) = 0$$

Which gives an optimum interference factor,

$$a = 1/3.$$

Putting a=1/3, in equation (3.1.13), maximum power becomes,

$$P_{max.} = \frac{16}{27} \left(\frac{1}{2} \rho A V_\infty^3 \right) \tag{3.1.15}$$

Again putting a=1/3, in equation (3.1.14) maximum power coefficient can be obtained,

$$(C_p)_{max.} = \frac{16}{27} = 0.593$$

The factor 16/27 is known as Betz coefficient [55] and represents maximum fraction of power which an ideal rotor can extract from the flow.

The fraction is related to the power of an undisturbed flow through an area A, whereas in reality the mass flow rate through A is not AV_∞, it will be AU. Hence the efficiency for maximum power can be written as,

$$\eta_{eff.} = \frac{P_{max.}}{1/2 \rho A U V_\infty^2} = \frac{16}{27} \times \frac{3}{2} = \frac{8}{9}.$$

This modelling does not take into account additional effects of wake rotation. As the initial stream is not rotational, interaction with a rotating windmill will cause to rotate in opposite direction.

The produced torque implies tangential forces and thus momemtum changes in tangential direction, the flow is entirely axial is acceptable only for very high speed turbines. As the produced torque will be higher, the tangential momentum in the downstream will also be higher. This is the first reason, than the other losses that occur in reality, why the value of $(C_p)_{max.}=0.593$ cannot be achieved in a real construction[22].

3.2 Effect of Wake Rotation on Momentum Theory

To include the effect of wake rotation in the momentum theory, the following assumptions are considered,

1. At the upstream of the rotor, the flow is entirely axial and at the downstream the flow rotates with an angular velocity , but the flow remains irrotational.

2. The angular velocity of the flow at the downstream , is considered to be small compared to the angular velocity Ω of the rotor.

3. Pressure in the wake is equal to the free stream pressure. Writing the energy equation, illustrated in the Figure 3.2.1.

$$\text{K.E.translational } (V_\infty) = \text{Power extracted} + \text{K.E.translational}(U) + \text{K.E.rotational}(\omega) \text{ [48].} \tag{3.2.1}$$

The above equation shows that the rotational kinetic energy reduces the power, which is extracted by the turbine. The wake rotation is opposite to the rotation of the rotor. Since the power is equal to the product of the torque Q, acting on the rotor and the angular velocity Ω, maximum power is available with high angular velocity and low torque, since high torque will result large wake rotational energy. The angular velocity of the wake and the angular velocity of the rotor Ω, are related by an angular interference factor á,

$$á = \frac{\text{Angular velocity of the wake}}{\text{Twice the angular velocity of the rotor}} = \frac{\omega}{2\Omega} \qquad (3.2.2)$$

The annular ring through which a blade element will pass, is illustrated in the Figure 3.2.2.

Using the relation for momentum flux through the ring the axial thrust force dT can be expressed as,

$$dT = d\dot{m}(V_\infty - V) = \rho\, dAU(V_\infty - V) \qquad (3.2.3)$$

Using the equation (3.1.10) and (3.1.11)

$$U = V_\infty\,(1-a) \qquad (3.1.10)$$

and $$V = V_\infty\,(1-2a) \qquad (3.1.11)$$

And expressing the area of the annular ring dA as,

$$dA = 2\pi r dr \qquad (3.2.4)$$

The expression for the thrust becomes,

$$dT = 4\pi r \rho V_\infty^2\; a(1-a)\; dr \qquad (3.2.5)$$

The thrust force may also be calculated from the pressure difference over the blades by applying Bernoulli's equation. Since the relative angular velocity changes from Ω to $(\Omega+\omega)$, while the axial components of the velocity remains unchanged. Bernoulli's equation gives,

$$P^+ - P^- = \frac{1}{2}\rho(\Omega + \omega)^2 r^2 - \frac{1}{2}\rho\Omega^2 r^2$$

or, $$P^+ - P^- = \rho\left(\Omega + \frac{\omega}{2}\right)\omega r^2$$

The resulting thrust on the annular element is given by,

$$dT = (P^+ - P^-)dA$$

or, $$dT = \rho\left(\Omega + \frac{\omega}{2}\right)\omega r^2\; 2\pi\; r\; dr$$

Using equation (3.2.2),

$$dT = 4á\;(1+á)\;\frac{1}{2}\rho\Omega^2\; r^2\; 2\pi\; r\; dr \qquad (3.2.6)$$

Equating equation (3.2.5) and (3.2.6),

$$\frac{a(1-a)}{á(1+á)} = \frac{\Omega^2 r^2}{V_\infty^2} = \lambda_r^2 \tag{3.2.7}$$

Where, λ_r is called as the local tip speed ratio which is given by,

$$\lambda_r = \frac{r\Omega}{V_\infty} \tag{3.2.8}$$

An expression for the torque acting on the rotor can be derived by considering the change in angular momentum flux dQ through the annular ring.

$$dQ = d\dot{m}\ V_t\ r$$

or, $$dQ = \omega\, dA\ U \rho\, r\ r$$

where, V_t is the tangential wind velocity.

Inserting equations (3.1.10), (3.2.2) and(3.2.4), the expression for the torque acting on the annular ring is given by,

$$dQ = 4\pi\ r^3 \rho\, V_\infty\ á(1-a)\ \Omega\ dr \tag{3.2.9}$$

Again, the power generated through the annular ring is equal to $dP = \Omega . dQ$, so the total power becomes,

$$P = \int_0^R \Omega . dQ \tag{3.2.10}$$

The tip speed ratio,

$$\lambda = \frac{R\Omega}{V_\infty} \tag{3.2.11}$$

Now, the equation of total power from the equations (3.2.9) and (3.2.10) is,

$$P = \int_0^R 4\pi r^3 \rho V_\infty\ á(1-a)\ \Omega^2\ dr$$

The above equation can be expressed in terms of the local tip speed ratio as,

$$P = \frac{1}{2} \rho A V_\infty^3\ \frac{8}{\lambda^2} \int_0^\lambda á(1-a)\ \lambda_r^3\ d\lambda_r \tag{3.2.12}$$

Where, A is the turbine swept area, $A = \pi R^2$.

The power coefficient is,

$$C_p = \frac{P}{1/2\, \rho A V_\infty^3}$$

Using the equation (3.2.12) power coefficient becomes,

$$C_p = \frac{8}{\lambda^2} \int_0 a(1-a)\, \lambda_r^3 \, d\lambda_r \qquad (3.2.13)$$

From the equation (3.2.7) angular interference factor,

$$\acute{a} = -\frac{1}{2} + \frac{1}{2}\left[1 + \frac{4}{(\lambda_r)^2}\, a(1-a)\right]^{1/2} \qquad (3.2.14)$$

Substuting the value of $\acute{a}$ in (3.2.13) and taking the derivative equal to zero, the relation between λ_r and a for maximum power becomes,

$$\lambda_r = \frac{(1-a)(4a-1)^2}{(1-3a)} \qquad (3.2.15)$$

Taking the derivative equal to zero of the equation (3.2.13) and from the derivative of (3.2.7), the relation between a and $\acute{a}$ is found (Appendix-D),

$$\acute{a} = \frac{(1-3a)}{(4a-1)} \qquad (3.2.16)$$

The above relation will be used later for design purposes.

3.3: **Blade Element Theory**

The blade element theory determines the forces acting on a differential element of a blade due to the motion of the fluid. By integrating over the length of the blade the performance of the entire rotor is calculated. The following assumptions are considered for the blade element theory:

1. There is no interference between adjacent blade elements along each blade.

2. The forces acting on a blade element are solely due to the lift and drag characteristics of the sectional profile of the element.

3. The pressure in the far wake is equal to thefree stream pressure.

The aerodynamic force components acting on the blade element are the lift force dL perpendicular to the resulting velocity vector and the drag force dD acting in the direction of the resulting velocity vector. The following expressions for the sectional lift and drag forces may be introduced:

$$dL = C_L \frac{1}{2} \rho W^2 C\, dr \tag{3.3.1}$$

$$dD = C_D \frac{1}{2} \rho W^2 C\, dr \tag{3.3.2}$$

The thrust and torque found by the blade element are,

$$dT = (dL \operatorname{Cos}\phi + dD \operatorname{Sin}\phi) \tag{3.3.3}$$

$$dQ = (dL \operatorname{Sin}\phi - dD \operatorname{Cos}\phi) r \tag{3.3.4}$$

Assuming that the rotor has B blades, the expressions for the thrust and torque becomes,

$$dT = BC \frac{1}{2} \rho W^2 (C_L \operatorname{Cos}\phi + C_D \operatorname{Sin}\phi) dr$$

or, $$dT = BC \frac{1}{2} \rho W^2 C_L \operatorname{Cos}\phi \left(1 + \frac{C_D}{C_L} \tan\phi\right) dr \tag{3.3.5}$$

and, $$dQ = BC \frac{1}{2} \rho W^2 (C_L \operatorname{Sin}\phi - C_D \operatorname{Cos}\phi) r.dr$$

or, $$dQ = BC \frac{1}{2} \rho W^2 C_L \operatorname{Sin}\phi \left(1 - \frac{C_D}{C_L} \frac{1}{\tan\phi}\right) r.dr \tag{3.3.6}$$

From the Figure 3.3.1, the expressions for relative velocity W,

$$W = \frac{V_\infty (1-a)}{\operatorname{Sin}\phi} = \frac{r\Omega\,(1+a')}{\operatorname{Cos}\phi} \tag{3.3.7}$$

From the equation (3.3.7) and trigonometric relation,

$$\tan\phi = \frac{(1-a) V_\infty}{r\Omega\,(1+a')} = \frac{(1-a)}{(1+a')} \times \frac{1}{\lambda_r} \tag{3.3.8}$$

and, $$\beta_T = \phi - \alpha \tag{3.3.9}$$

The local solidity ratio σ is

$$\sigma = \frac{B\,C}{2\pi r} \tag{3.3.10}$$

The equations of the blade element theory become,

$$dT = (1-a)^2 \frac{\sigma C_L \, Cos\phi}{Sin^2\phi}\left(1 + \frac{C_D}{C_L}\tan\phi\right)\frac{1}{2}\rho V_\infty^2 \, 2\pi r.dr \tag{3.3.11}$$

$$dQ = (1+\acute{a})^2 \frac{\sigma C_L Sin\phi}{Cos^2\phi}\left(1 - \frac{C_D}{C_L}\frac{1}{\tan\phi}\right)\frac{1}{2}\rho\,\Omega^2 r^3 \, 2\pi r.dr \tag{3.3.12}$$

3.4 Strip Theory

From the axial momentum theory and the blade element theory a series of relationships can be developed to calculate the performance of a wind turbine. By equating the thrust obtained from the momentum theory equation (3.2.5) to equation (3.3.11) of blade element theory for an annular element at a radius r, we have,

$$dT_{Momentum} = dT_{Blade\ Element}$$

or,

$$\frac{a}{(1-a)} = \frac{\sigma C_L \, Cos\phi}{4 \, Sin^2\phi}\left(1 + \frac{C_D}{C_L}\tan\phi\right) \tag{3.4.1}$$

Again, equating the angular momentum, determined from the axial momentum theory equation(3.2.9) and equation(3.3.12) of blade element theory, we get,

$$dQ_{Momentum} = dQ_{Blade\ Element}$$

$$\frac{\acute{a}}{(1+\acute{a})} = \frac{\sigma C_L}{4Cos\phi}\left(1 - \frac{C_D}{C_L}\frac{1}{\tan\phi}\right) \tag{3.4.2}$$

Equations (3.4.1) and (3.4.2) which determines axial and angular interference factor contain drag terms. It has been suggested in [46, 47] that the drag terms may be neglected in determining a and á. Because of the retarded air due to drag is confined to thin helical sheets in the wake and have little effects on the induced flow. Omitting the drag terms the induction factors a and á may be calculated with the following equations.

$$\frac{a}{1-a} = \frac{\sigma C_L \, Cos\phi}{4 \, Sin^2\phi} \tag{3.4.3}$$

and,

$$\frac{\acute{a}}{1+\acute{a}} = \frac{\sigma C_L}{4 \, Cos\phi} \tag{3.4.4}$$

Considering the equations (3.4.3) and (3.3.11), elemental thrust may be written as,

$$dT=4a(1-a)(1+\frac{C_D}{C_L}\tan\phi)\frac{1}{2}\rho V_\infty^2\,2\pi r\,dr \qquad (3.4.5)$$

From equations(3.4.4) and(3.3.12) we have elemental torque,

$$dQ=4\acute{a}(1-a)(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\frac{1}{2}\rho V_\infty\,\Omega\,2\pi r^3\,dr \qquad (3.4.6)$$

Elemental power is given by,

$$dP=\Omega.dQ$$

or,
$$dP=4\acute{a}(1-a)(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\frac{1}{2}\rho V_\infty\,\Omega^2\,2\pi\,r^3\,dr \qquad (3.4.7)$$

Introducing the local tip speed ratio λ_r with,

$$\lambda_r=\frac{\Omega r}{V_\infty} \qquad (3.2.8)$$

Equation of total thrust, torque and power become,

$$T=\frac{1}{2}\rho AV_\infty^2\frac{8}{\lambda^2}\int_0^\lambda a(1-a)(1+\frac{C_D}{C_L}\tan\phi)\lambda_r\,d\lambda_r \qquad (3.4.8)$$

$$Q=\frac{1}{2}\rho AV_\infty^2R\frac{8}{\lambda^3}\int_0^\lambda \acute{a}(1-a)(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})(\lambda_r)^3\,d\lambda_r \qquad (3.4.9)$$

and,

$$P=\frac{1}{2}\rho AV_\infty^3\frac{8}{\lambda^2}\int_0^\lambda \acute{a}(1-a)(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})(\lambda_r)^3\,d\lambda_r \qquad (3.4.10)$$

These equations are valid only for a wind turbine having infinite number of blades.

3.5 Tip and Hub Losses

In the preceeding sections the rotor was assumed to be possessing an infinite number of blades with infinitely small chord. Practically, the number of blades is finite. In the theory discussed early, the wind imparts a rotation to the rotor, therefore, dissipating some of its kinetic energy or velocity and creating a pressure difference between one side on the blade to the other side. At tip and hub, this pressure difference leads to secondary flow effects. The flow becomes three-dimensional and tries to equalize the pressure difference as shown in the Figure 3.5.1. This effect is more pronounced as one approaches the tip. As a result, the torque on the rotor is reduced and output power also reduced.

Several different existing models take into account this loss, discussed in reference[48]. The method suggested by Prandtl will be used here. The idea in Prandtl's method is to replace the system of vortices at the tip with a series of parallel planes for which the flow is more easily calculated. The correction factor suggested by Prandtl is,

$$F_{tip} = \frac{2}{\pi} \text{ arc Cos } e^{-f}$$

Where, $$f = \frac{B}{2} \frac{R-r}{R \text{ Sin}\phi}$$

For hub region f is defined as,

$$f = \frac{B}{2} \frac{r - r_{hub}}{r_{hub} \text{ Sin}\phi}$$

Hence, the correction factor F for total losses will be,

$$F = F_{tip} \times F_{hub} \tag{3.5.1}$$

The loss factor F may be introduced in several ways for the rotor performance calculations. In the method adopted by Wilson and Lissaman [47], the induction factors a and á are multiplied with F, and thus the axial and tangential velocities in the rotor plane are modified. Further assumption is that these corrections only involve in the momentum formulas. Therefore, the thrust and torque from momentum theory become,

$$dT = 4\pi \rho r V_\infty^2 aF(1-aF) \, dr \tag{3.5.2}$$

$$dQ = 4\pi \rho r^3 V_\infty \, a \, F(1-aF) \, \Omega \, dr \tag{3.5.3}$$

The expressions of the blade element theory remain unchanged.

$$dT = (1-a)^2 \frac{\sigma C_L \text{ Cos}\phi}{\text{Sin}^2\phi} \left(1 + \frac{C_D}{C_L} \tan\phi\right) \frac{1}{2} \rho V_\infty^2 \, 2\pi \, r.dr \tag{3.3.11}$$

and, $$dQ = (1+a)^2 \frac{\sigma C_L \text{Sin}\phi}{\text{Cos}^2\phi} \left(1 - \frac{C_D}{C_L} \frac{1}{\tan\phi}\right) \frac{1}{2} \rho \, \Omega^2 \, r^3 \, 2\pi r.dr \tag{3.3.12}$$

Equation (3.3.12) can also be written as,

$$dQ = (1-a)^2 \frac{\sigma C_L}{\text{Sin}\phi} \left(1 - \frac{C_D}{C_L} \frac{1}{\tan\phi}\right) \frac{1}{2} \rho V_\infty^2 \, 2\pi r^2.dr \tag{3.5.4}$$

Balancing the equation (3.5.2) with (3.3.11) one can obtains,

$$aF(1-aF)=\frac{\sigma C_L Cos\phi(1-a)^2}{4Sin^2\phi}(1+\frac{C_D}{C_L}\tan\phi) \tag{3.5.5}$$

and considering the equations (3.5.3) and (3.5.4),

$$\acute{a}F(1-aF)= (1-a)^2\frac{\sigma C_L}{4Sin\phi}(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}) \tag{3.5.6}$$

Neglecting the drag terms in equations (3.5.5) and (3.5.6),

$$aF(1-aF)=\frac{\sigma C_L Cos\phi(1-a)^2}{4Sin^2\phi} \tag{3.5.7}$$

$$\acute{a}F(1-aF)= (1-a)^2\frac{\sigma C_L}{4Sin\phi} \tag{3.5.8}$$

From the equations (3.5.7) and (3.5.8), the final expressions for elemental torque and thrust become,

$$dT= 4aF(1-aF)(1+\frac{C_D}{C_L}\tan\phi)\rho V_\infty^2 \pi r\, dr \tag{3.5.9}$$

and,

$$dQ= 4aF(1-aF)(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\rho V_\infty^2 \pi r^2\, dr \tag{3.5.10}$$

3.6 Equations for Total Thrust, Torque and Power Coefficient

Elemental thrust, torque and power coefficients are defined as,

$$dC_T= \frac{dT}{1/2\rho AV_\infty^2} \tag{3.6.1}$$

$$dC_Q= \frac{dQ}{1/2\rho AV_\infty^2 R} \tag{3.6.2}$$

and, $$dC_P= \frac{dP}{1/2\rho AV_\infty^3} = \frac{\Omega.dQ}{1/2\rho AV_\infty^3}$$

or, $$dC_P= \frac{dQR.\Omega}{1/2\rho AV_\infty^2 RV_\infty} = \lambda\cdot dC_Q \tag{3.6.3}$$

Considering the equations (3.5.9) and (3.6.1), elemental thrust coefficient can be written as,

$$dC_T = \frac{8}{R^2} aF(1-aF)(1+ \frac{C_D}{C_L} \tan\phi)\, r dr \qquad (3.6.4)$$

Again from equations (3.5.10) and (3.6.2), elemental torque coefficient can be written as,

$$dC_Q = \frac{8}{R^3} \acute{a}F(1-aF)(1- \frac{C_D}{C_L} \frac{1}{\tan\phi})\, r^2 dr \qquad (3.6.5)$$

Elemental power coefficient can be found from the equations (3.6.3) and (3.6.5),

$$dC_P = \frac{8\Omega}{R^2 V_\infty} \acute{a}F(1-aF)(1- \frac{C_D}{C_L} \frac{1}{\tan\phi})\, r^2\, dr \qquad (3.6.6)$$

Finally, the total thrust, torque and power coefficients can be found by integrating along the radius R.

$$C_T = \frac{8}{R^2}\int_0^R aF(1-aF)(1+ \frac{C_D}{C_L} \tan\phi)\, r\, dr \qquad (3.6.7)$$

$$C_Q = \frac{8}{R^3}\int_0^R \acute{a}F(1-aF)(1- \frac{C_D}{C_L} \frac{1}{\tan\phi})\, r^2\, dr \qquad (3.6.8)$$

and,

$$C_P = \frac{8\Omega}{R^2 V_\infty}\int_0^R \acute{a}F(1-aF)(1- \frac{C_D}{C_L} \frac{1}{\tan\phi})\, r^2\, dr \qquad (3.6.9)$$

3.7 Expressions for Maximum Power

For maximum power output the relation between a and a may be expressed by the equation (3.2.16),

$$\acute{a} = \frac{(1-3a)}{(4a-1)} \qquad (3.2.16)$$

Introducing the equations of induction factors as follows,

$$\frac{a}{1-a} = \frac{\sigma C_L \cos\phi}{4 \sin^2\phi} \qquad (3.4.3)$$

and, $$\frac{\acute{a}}{1+\acute{a}} = \frac{\sigma C_L}{4 \cos\phi} \qquad (3.4.4)$$

From the above equations (3.2.16), (3.4.3) and (3.4.4), the following expression is found,

$$\sigma C_L = 4(1-\cos\phi) \tag{3.7.1}$$

Considering the local solidity σ as,

$$\sigma = \frac{B\ C}{2\pi r} \tag{3.3.10}$$

From equations (3.7.1) and (3.3.10), we have

$$C = \frac{8\pi r}{BC_L}(1-\cos\phi) \tag{3.7.2}$$

Local tip speed ratio $_r$ is given by,

$$\lambda_r = \frac{r\Omega}{V_\infty} \tag{3.2.8}$$

From the equation (3.3.8),

$$\tan\phi = \frac{(1-a)V_\infty}{r\Omega(1+a')} = \frac{(1-a)}{(1+a')} \times \frac{1}{\lambda_r} \tag{3.3.8}$$

Now, replacing the values of (1-a) and (1+á) from equations (3.4.3) and (3.4.4), and putting the value of σC_L from equation (3.7.1), the following relation can be deduced,

$$\lambda_r = \frac{\sin\phi(2\cos\phi-1)}{(1-\cos\phi)(2\cos\phi+1)} \tag{3.7.3}$$

and this can be reduced to,

$$\phi = \frac{2}{3}\arctan\frac{1}{\lambda_r} \tag{3.7.4}$$

Equation of the blade twist angle can be written as,

$$\beta_T = \phi - \alpha \tag{3.3.9}$$

equations (3.7.2), (3.2.8), (3.7.4) and (3.3.9) will be required to calculate the blade configuration later.

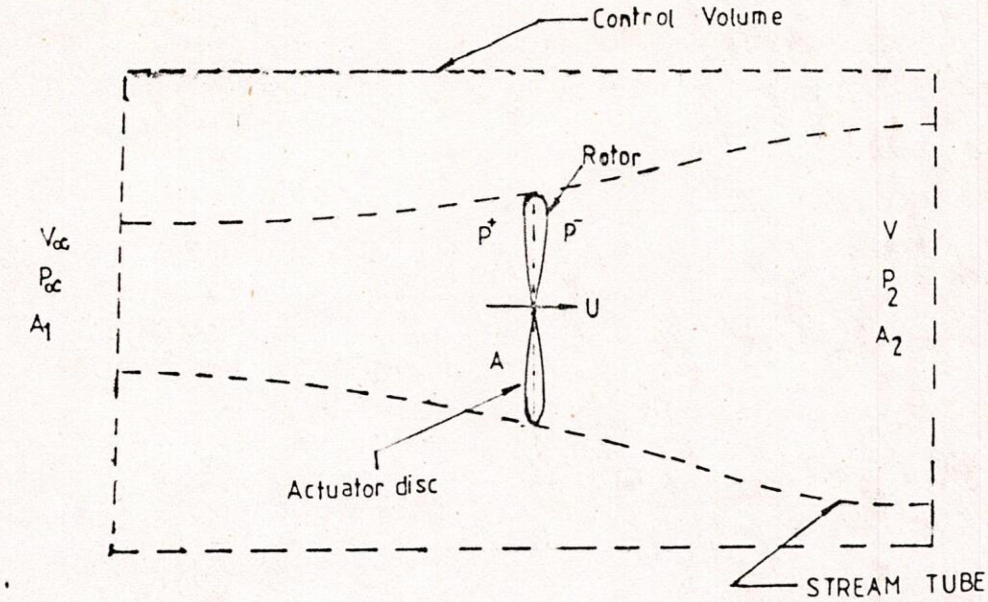

Fig.3.1.1 : Wind Turbine Stream Tube.

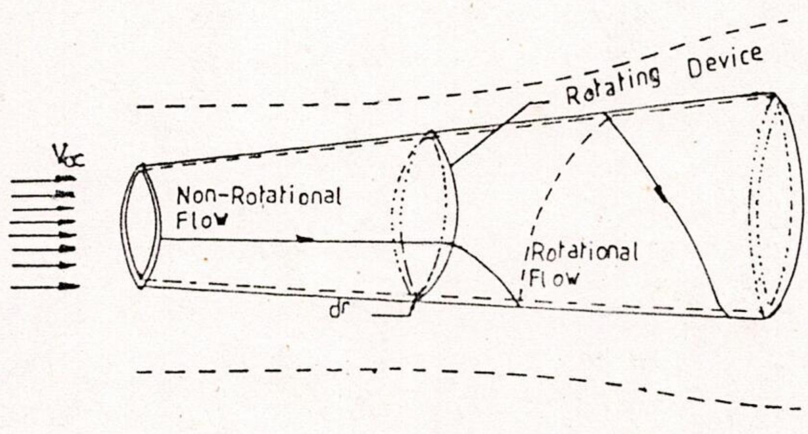

Fig. 3.2.1 : Streamtube Showing Wake Rotation.

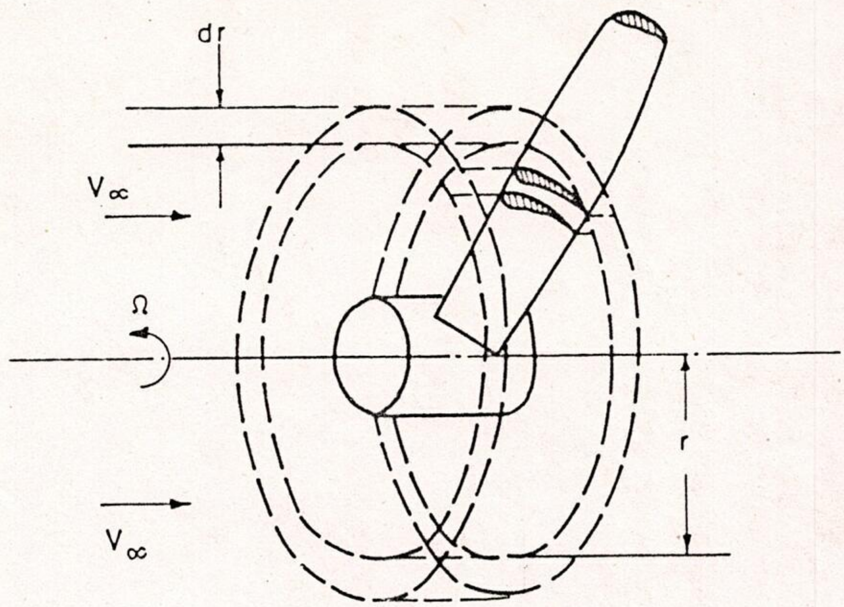

FIGURE 3.2.2: BLADE ELEMENT ANNULAR RING

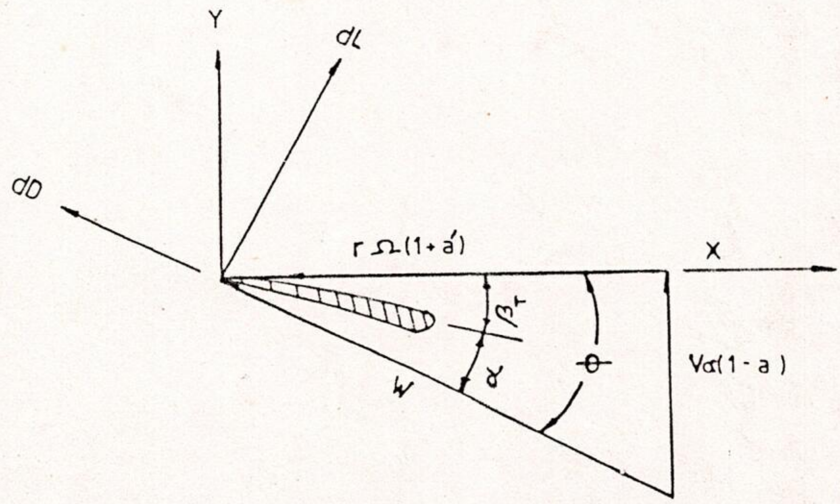

Fig.3.3.1 : Blade Element Velocity Diagram.

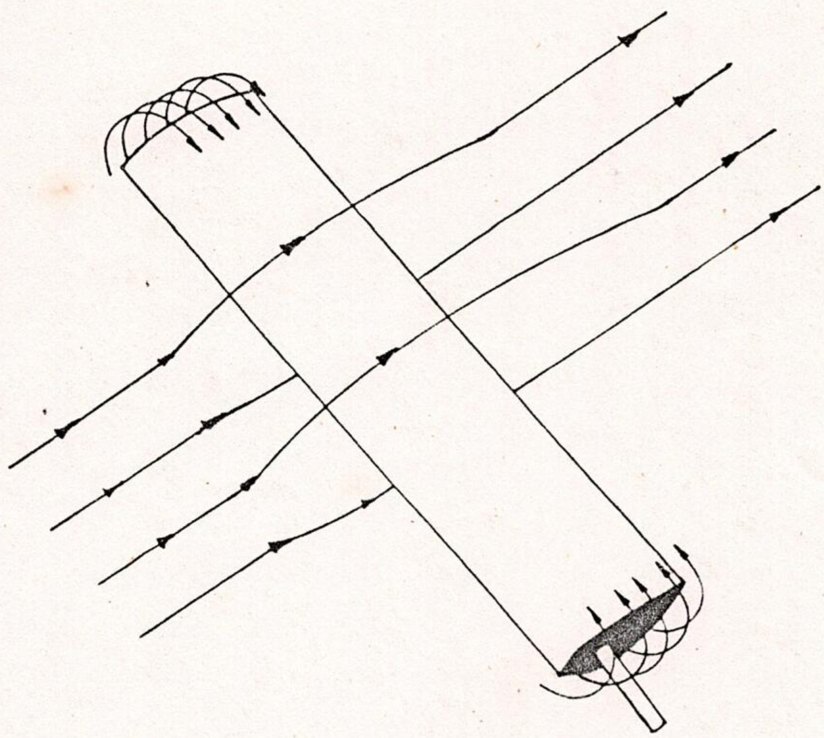

Fig. 3.5.1 : Tip and Hub Losses Flow Diagram [48].

CHAPTER IV

MODIFICATION OF EXISTING THEORIES

4.0 General

In the preceeding chapter, the assumption is that the flow of stream is steady uniform. For performance analysis each individual streamtubes was taken as an annuli without interacting each other.

The velocity field of the wind may be non-uniform with time as well as spatial distribution. This means that effect of wind shear, wind shift, tower shadow, coning and tilting on performance must be considered. Angle of attack α and wind speed W are different from the previously used value for each blade segment and no longer axially symmetric. Which indicates that the wind velocity and the induction factors are not constant inside a annular streamtube, but they depend on the azimuthal position of the blade. The individual streamtubes can no longer be taken as annuli but must be of elemental area dA, perpendicular to the wind direction.

In the following chapter, the performance of a horizontal axis wind turbine is done in a quasi steady manner, it means lift, drag, axial and tangential induced velocities are calculated for several azimuthal position of the rotor. The procedure used is similar to the case a uniform wind velocity along the rotor axis and is used on a combination of moentum and blade element theory.

4.1: Velocity Components at the Blade at the Different Frame of Reference

For the aerodynamic and structural analysis the velocity components of the air flow relative to any point on the blade and the induced velocity components required to determine. Different frame of reference are considered to include the effects of wind shift, tilting, coning and azimuth. These frame of reference are shown in the Figures 4.1.1 to 4.1.4. A referance frame S_o, is fixed at the top of the tower of a wind turbine with Z_o is the vertical axis and (X_o, Y_o) forming horizontal plane. A second non-rotating frame S_1 is fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance Y and a rotation of tilting angle α_T around the X_o axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally, a local reference frame S_3 is attached to a particular point of the blade at a distance r from the hub and is rotated over a coning angle β. The relationships between the reference frames can be expressed as,

$$S_1=[K_T]S_o$$

$$S_2=[K_\theta]S_1$$

$$S_3=[K_\beta]S_2$$

and inversely,

$$S_0=[K_T]^T S_1$$

$$S_1=[K_\theta]^T S_2$$

$$S_2=[K_\beta]^T S_3$$

the superscript T indicates the transposed matrix. The transformation matrices are,

for tilting, $K_T=\begin{vmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\alpha_T & -\mathrm{Sin}\alpha_T \\ 0 & \mathrm{Sin}\alpha_T & \mathrm{Cos}\alpha_T \end{vmatrix}$

for azimuth, $K_\theta=\begin{vmatrix} \mathrm{Cos}\theta_k & 0 & \mathrm{Sin}\theta_k \\ 0 & 1 & 0 \\ -\mathrm{Sin}\theta_k & 0 & \mathrm{Cos}\theta_k \end{vmatrix}$

for coning, $K_\beta=\begin{vmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\beta & -\mathrm{Sin}\beta \\ 0 & \mathrm{Sin}\beta & \mathrm{Cos}\beta \end{vmatrix}$

In the fixed frame of reference S_o, considering the wind shift the wind velocity can be expressed as,

$$\overleftarrow{V}_{s_o}=V_\infty\begin{vmatrix} \mathrm{Cos}\,\gamma \\ \mathrm{Sin}\,\gamma \\ 0 \end{vmatrix} \qquad \gamma = 90-\gamma^* \qquad (4.1.1)$$

Where, γ^* is the wind shift angle in relation to the nacelle axis. The wind velocity components in the S_3 frame of reference is,

$$\overleftarrow{V}_{s3}=[K_\beta]^T\,[K_\theta]^T\,[K_T]^T\,\overleftarrow{V}_{so} \qquad (4.1.2)$$

or, $V_{s3}=[K_\beta]^T\,[K_\theta]^T\,[K_T]^T\begin{vmatrix} \mathrm{Cos}\,\gamma \\ \mathrm{Sin}\,\gamma \\ 0 \end{vmatrix}V_\infty$

Which can be expressed in the following form,

$$\overleftarrow{V}_{s3}=V_{\infty}\begin{vmatrix} \cos\gamma\ \cos\theta_k+\sin\gamma\ \sin\theta_k\sin\alpha_T \\ \sin\gamma\cos\alpha_T\cos\beta+\cos\gamma\ \sin\beta\ \sin\theta_k-\sin\gamma\sin\alpha_T\cos\theta_k\sin\beta \\ -\sin\gamma\ \sin\beta\cos\alpha_T+\cos\gamma\sin\theta_k\cos\beta-\sin\gamma\cos\theta_k\sin\alpha_T\cos\beta \end{vmatrix} \quad (4.1.3)$$

4.2 Effect of Wind Shear

Wind flow is seriously affected by friction near the earth surface. Because of this friction, a boundary layer is developed and wind velocity gradient with altitude is created. The wind shear depends on the wind direction, the wind velocity and stability of the atmosphere. Surface roughness vary from coast (smooth) to city centre (rough). Velocity profile will depend upon the nature of the terrain.

Atmospheric boundary layer is considered two-dimensional for the present work, though it is three-dimensional and complicated. The velocity of the wind increases with the height up to a level where the friction is neglected. The level is called the gradient height Z_g and the velocity at gradient height is called the gradient velocity V_g. The distance between the surface of the earth and the gradient height is the thickness of the boundary layer.

The variation of the wind velocity with the height can be expressed analyticaly by a power law as,

$$\frac{V_{\infty o}}{V_{Ref}} = \left(\frac{Z}{Z_{Ref}}\right)^{\eta} \qquad 0.15<\eta<0.45 \qquad (4.2.1)$$

Where,

$V_{\infty o}$ =Wind velocity at the height,Z
V_{Ref}=Reference wind velocity.
Z_{Ref}=Reference height.
η =Power law exponent.

The main source of fluctuating aerodynamic load is the wind shear which is once per revolution for a wind turbine. The instantaneous wind velocity approaching an upwind or a downwind rotor corresponding to a particular point of the blade can be deduced as follows

Radial distance of a particular point on the blade can be expressed in S_o co-ordinate system as,

$$\begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} = [K_T][K_\theta][K_\beta] \begin{vmatrix} 0 \\ 0 \\ R_L \end{vmatrix} \qquad (4.2.2)$$

Where, R_L is the radial distance of a point from the root of the blade. This leads to the following expression-

$$\begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} = \begin{vmatrix} R_L Cos\beta Sin\theta_k \\ -R_L Sin\beta Cos\alpha_T - R_L Cos\beta Cos\theta_k Sin\alpha_T \\ -R_L Sin\beta Sin\alpha_T + R_L Cos\beta Cos\theta_k Cos\alpha_T \end{vmatrix} \tag{4.2.3}$$

Now, the height of a particular point of the blade from the ground level is written as,

$$Z = Z_{Ref} + Z_o$$

or, $$Z = Z_{Ref} + R_L(-Sin\beta Sin\alpha_T + Cos\beta Cos\theta_k Cos\alpha_T) \tag{4.2.4}$$

Considering Z_{Ref} as the hub height of a wind turbine from the ground level and putting the value of Z in equation (4.2.1), one finds,

$$V_{\infty o} = V_{Ref}[\{H + R_L(-Sin\beta Sin\alpha_T + Cos\beta Cos\theta_k Cos\alpha_T)\}/H]^{n} \tag{4.2.5}$$

Now the equation for wind velocity approaching at a particular point of the Kth blade correspoding to an azimuthal angle θ and wind shift angle γ, can be expressed as

$$V_\theta = V_{Ref} Sin\gamma[1 - R_L/H(Sin\beta Sin\alpha_T - Cos\beta Cos\theta_k Cos\alpha_T)]^{n} \tag{4.2.6}$$

Where,

V_{Ref}=Reference wind velocity at the centre line of the rotor.

H =Hub height from the ground level.

At hub height if the reference wind velocity is V_∞, above equation becomes

$$V_{\theta k} = V_\infty Sin\gamma[1 - R_L/H(Sin\beta Sin\alpha_T - Cos\beta Cos\theta_k Cos\alpha_T)]^{n} \tag{4.2.7}$$

This equation may be used for structural analysis.

The vertical wind gradient will induce forces and moments as shown in the Figure 4.2.1. The largest of these are torque variation and the pitching moment.

4.3 Momentum Theory

Radial elemental area dA subtended by a small angle dθ in the plane of the rotor is shown in the Figure 4.3.1.
Introducing the angular interference factor á from equation (3.2.2),

$$\acute{a} = \frac{\omega}{2\Omega} \tag{3.2.2}$$

and using the small segment in the above Figure the thrust can be written as,

$$dT = d\dot{m}(V_\infty - V)$$

or, $dT = \rho U dA (V_\infty - V)$ (4.3.1)

Equation (3.1.10) is expressed as,

$$U = V_\infty (1-a) \quad (3.1.10)$$

Considering the wind shear, wind shift and tilting, it may be rewritten as,

$$U = V_{\infty o} \cos\alpha_T \sin\gamma (1-a) \quad (4.3.2)$$

Equation (3.1.11) can be expressed as,

$$V_\infty - V = 2aV_\infty \quad (3.1.11)$$

With the addition of wind shear, yawing and tilting, this equation becomes,

$$V_\infty - V = 2aV_{\infty o} \cos\alpha_T \sin\gamma \quad (4.3.3)$$

Now for a coned blade the expression for differential area dA can be written as,

$$dA = r\cos^2\beta\, d\theta\, dr \quad (4.3.4)$$

The expression for thrust becomes,

$$dT = 2\rho r \cos^2\beta \cos^2\alpha_T \sin^2\gamma V_{\infty o}^2\ a(1-a) dr d\theta \quad (4.3.5)$$

Due to the change of momentum in the air in tangential direction tangential force acting upon the elemental area dA,

$$dF_t = d\dot{m}\, \omega\, r\cos\beta$$

or, $dF_t = \rho U dA r \omega \cos\beta$ (4.3.6)

Taking the equations (3.2.2), (4.3.2) and (4.3.4) the tangential force is expressed as,

$$dF_t = 2\rho \acute{a}\Omega\ r^2 \cos^3\beta (1-a) V_{\infty o} \cos\alpha_T \sin\gamma\, dr d\theta \quad (4.3.7)$$

The torque equation can be written as,

$$dQ = dF_t r \cos\beta$$

Putting the value of dF_t, from the equation (4.3.7) we have,

$$dQ = 2\rho r^3 \acute{a} (1-a) V_{\infty o} \cos\alpha_T \sin\gamma \cos^4\beta\ \Omega\ dr d\theta \quad (4.3.8)$$

4.4 Blade Element Theory

The velocity components acting on a blade element rotating at a radius r are shown in the Figure 4.4.1. Now, introducing the induced velocity ν in S_3 co-ordinate system as,

$$\hat{v}_{s3}=\begin{vmatrix} v_t \\ v_a Cos\beta Cos\alpha_T \\ v_v \end{vmatrix} \tag{4.4.1}$$

Where,

v_t=Tangential component of induced velocity.
v_a=Axial component of induced velocity.
v_v=Vertical component of induced velocity.

Equation (4.4.1) may be written as,

$$\hat{v}_{s3}=\begin{vmatrix} \Omega r\ Cos\beta\ \acute{a} \\ V_\infty a Cos\beta Cos\alpha_T Sin\gamma \\ v_v \end{vmatrix} \tag{4.4.2}$$

The rotational motion of the blade will add a velocity component $\Omega r\ Cos\beta$ in the total velocity vector relative to blade and this component can be written in S_3 co-ordinate system as,

$$\vec{v}_{b1}=\begin{vmatrix} \Omega r Cos\beta \\ 0 \\ 0 \end{vmatrix} \tag{4.4.3}$$

With the effect of wind shear the components of the relative velocity W can be expressed from equation (4.1.3), (4.4.2) and (4.4.3) as,

$$W_X=V_{\infty o}Cos\gamma Cos\theta_K+V_{\infty o}Sin\gamma Sin\theta_K Sin\alpha_T-\ \Omega r Cos\beta(1+a) \tag{4.4.4}$$

and

$$W_Y=V_{\infty o}[Sin\gamma Cos\alpha_T Cos\beta(1-a)+Sin\beta Sin\theta_K Cos\ \gamma \\ -Sin\gamma Sin\beta Sin\alpha_T Cos\theta_K] \tag{4.4.5}$$

The local angle of attack α is defined as,

$$\alpha\ =\phi-\beta_T=\tan^{-1}\frac{W_Y}{W_X}\ -\ \beta_T \tag{4.4.6}$$

and the relative velocity,

$$W=(W_X{}^2+W_Y{}^2)^{1/2} \tag{4.4.7}$$

The aerodynamic force components acting on the blade element are the lift force dL perpendicular to the resulting velocity vector and drag force dD acting along the direction of the resulting velocity vector. The following expressions are used for the sectional lift and drag forces.

$$dL=\ \frac{1}{2}C_L\,\rho W^2 dA \tag{4.4.8}$$

$$dD = \frac{1}{2} C_D \rho W^2 dA \tag{4.4.9}$$

Because we are assuming an infinite number of blades, the differential area dA is given as,

$$dA = \frac{BCd\theta dr}{2\pi} \cos\beta \tag{4.4.10}$$

So, the elemental lift and drag forces are,

$$dL = \frac{1}{2} C_L \rho W^2 \frac{BCd\theta dr}{2\pi} \cos\beta \tag{4.4.11}$$

$$dD = \frac{1}{2} C_D \rho W^2 \frac{BCd\theta dr}{2\pi} \cos\beta \tag{4.4.12}$$

The thrust and torque of the blade element are,

$$dT = dL\cos\phi + dD\sin\phi$$

or, $$dT = \cos\phi(dL + dD\tan\phi) \tag{4.4.13}$$

and, $$dQ = (dL\sin\phi - dD\cos\phi)r$$

or, $$dQ = \sin\phi\left(dL - \frac{dD}{\tan\phi}\right)r \tag{4.4.14}$$

Considering the equations (4.4.11) and (4.4.12) the thrust and torque eqution are,

$$dT = \frac{1}{2} \rho W^2 \cos\phi(C_L + C_D\tan\phi) \frac{BCd\theta dr}{2\pi} \cos\beta \tag{4.4.15}$$

and, $$dQ = \frac{1}{2} \rho W^2 \sin\phi\left(C_L - \frac{C_D}{\tan\phi}\right) \frac{BCrd\theta dr}{2\pi} \cos\beta \tag{4.4.16}$$

Equations for thrust and torque co-efficients are,

$$dC_T = \frac{dT}{1/2\rho A V_\infty^2} = \left(\frac{W}{V_\infty}\right)^2 \frac{\sigma r}{\pi R^2} \cos\phi(C_L + C_D\tan\phi)\cos\beta dr d\theta$$

or, $$dC_T = \left(\frac{W}{V_\infty}\right)^2 \frac{\sigma C_L \cos\phi}{\pi R^2} \left(1 + \frac{C_D}{C_L}\tan\phi\right)\cos\beta r dr d\theta \tag{4.4.17}$$

again,

$$dC_Q = \frac{dQ}{1/2\rho A V_\infty^2 R} = \left(\frac{W}{V_\infty}\right)^2 \frac{\sigma r^2}{\pi R^3} \sin\phi\left(C_L - \frac{C_D}{\tan\phi}\right) \cos\beta \, dr \, d\theta$$

$$\text{or,} \quad dC_Q = \left(\frac{W}{V_\infty}\right)^2 \frac{\sigma C_L \sin\phi}{\pi R^3} \left(1 - \frac{C_D}{C_L} \frac{1}{\tan\phi}\right) \cos\beta \; r^2 \, dr \, d\theta \qquad (4.4.18)$$

Therefore, elemental power coefficient is,

$$dC_p = \frac{dP}{1\backslash 2 \rho A V_\infty^3} = \frac{\Omega dQ}{1\backslash 2 \rho A V_\infty^3}$$

$$\text{or,} \quad dC_p = \frac{\Omega R dQ}{1\backslash 2 \rho A V_\infty^3 R} = dC_Q \cdot \lambda \qquad (4.4.19)$$

4.5 Strip Theory

To calculate the interference factor a and a′ equations for thrust and torque from the momentum theory and the blade element theory are to be equated.

$$dT_{\text{Blade Element}} = dT_{\text{Momentum}}$$

$$dQ_{\text{Blade Element}} = dQ_{\text{Momentum}}$$

Equating the equations (4.3.5) and (4.4.15) and introducing the tip loss factor F we have,

$$a(1-aF) = \frac{\sigma W^2 \cos\phi (C_L + C_D \tan\phi)}{8 \cos\beta \cos^2\alpha_T \sin^2\gamma V^2_{\infty o} \; F} \qquad (4.5.1)$$

Again equating the equations (4.3.8)and (4.4.16) and considering tip loss factor F we have,

$$\acute{a}(1-aF) = \frac{\sigma W^2 \sin\phi (C_L - C_D/\tan\phi)}{8 r \cos\beta^3 \cos\alpha_T \sin\gamma V_{\infty o} F \; \Omega} \qquad (4.5.2)$$

According to reference[47] the drag terms should be neglected in the calculations of a and á, then the equations (4.5.1) and (4.5.2) become,

$$a(1-aF) = \frac{\sigma W^2 \cos\phi C_L}{8 \cos\beta \cos^2\alpha_T \sin^2\gamma V_{\infty o}^2 \; F} \qquad (4.5.3)$$

and,

$$\acute{a}(1-aF) = \frac{\sigma \; W^2 \sin\phi C_L}{8 r \cos\beta^3 \cos\alpha_T \sin\gamma V_{\infty o} F \; \Omega} \qquad (4.5.4)$$

4.6 Equations for Thrust, Torque and Power Coefficient

From the equations (4.4.17) and (4.5.3) the elementary thrust coefficient can be written as,

$$dC_T = \frac{8}{\pi R^2}\left(\frac{V_{\infty o}}{V_\infty}\right)^2 aF(1-aF)\cos\beta^2\cos^2\alpha_T\sin^2\gamma\left(1+\frac{C_D}{C_L}\tan\phi\right) r\,dr\,d\theta \quad (4.6.1)$$

Therefore, the total thrust coefficient is,

$$C_T = \frac{8}{\pi R^2}\cos\beta^2\cos^2\alpha_T\sin^2\gamma\int_0^{2\pi}\int_0^R\left(\frac{V_{\infty o}}{V_\infty}\right)^2 aF(1-aF)\left(1+\frac{C_D}{C_L}\tan\phi\right) r\,dr\,d\theta \quad (4.6.2)$$

Considering equations (4.4.18) and (4.5.4) the elemental torque coefficient can be expressed as,

$$dC_Q = \frac{8}{\pi R^3}\frac{V_{\infty o} r^3}{V_\infty^2} \acute{a}F(1-aF)\cos\beta^4\cos\alpha_T\sin\gamma\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right)\Omega\, dr\,d\theta \quad (4.6.3)$$

So, the total torque coefficient is,

$$C_Q = \frac{8\Omega}{\pi R^3 V_\infty^2}\cos\beta^4\cos\alpha_T\sin\gamma\int_0^{2\pi}\int_0^R V_{\infty o} r^3 \acute{a}F(1-aF)\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right) dr\,d\theta \quad (4.6.4)$$

From the equation (4.4.19) elemental power coefficient will be,

$$dC_P = dC_Q \, . \, \lambda \quad (4.4.19)$$

Hence the total power coefficient can be found as,

$$C_P = \frac{8\Omega^2}{\pi R^3 V_\infty^2}\cos\beta^4\cos\alpha_T\sin\gamma\int_0^{2\pi}\int_0^R V_{\infty o} r^3 \acute{a}F(1-aF)\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right) dr\,d\theta \quad (4.6.5)$$

4.7 Forces and Moments at the Different Frame of Reference

Four co-ordinate systems are required for forces and moments analysis. The forces are divided into two components. One acting in the plane of the rotor and another acting normal to the rotor plane. The co-ordinate systems are discussed and shown in Appendix A.

S_o= Initial system attached to tower top.

S_1= Inertial system attached to hub which is created by translation and a rotation over a tilting angle α_T.

S_2= Rotating frame at hub which is introduced by rotation of the reference frame S_1 over an azimuth angle θ.

S_3= Local reference frame S_3 is attached to a specific point of the blade at a distance r from the hub and is rotated over a coning angle β.

4.7.1 Forces

All the forces in the local S_3 co-ordinate system can be expressed as,

$$\overleftarrow{F}_{S3}=\begin{vmatrix} F_{X3} \\ F_{Y3} \\ F_{Z3} \end{vmatrix} \tag{4.7.1.1}$$

Considering the non-rotating system S_1 attached to the hub the equation of forces is

$$\overleftarrow{F}_{S1}=[K_\theta][K_\beta]F_{S3} \tag{4.7.1.2}$$

The folloing equation is obtained above equations (4.7.1.1) and (4.7.1.2),

$$\overleftarrow{F}_{S1}=\begin{vmatrix} F_{X3}\mathrm{Cos}\theta+F_{Y3}\mathrm{Sin}\theta\mathrm{Sin}\beta+F_{Z3}\mathrm{Sin}\theta\mathrm{Cos}\beta \\ F_{Y3}\mathrm{Cos}\beta \quad -F_{Z3}\mathrm{Sin}\beta \\ -F_{X3}\mathrm{Sin}\theta+F_{Y3}\mathrm{Cos}\theta\mathrm{Sin}\beta+F_{Z3}\mathrm{Cos}\theta\mathrm{Cos}\beta \end{vmatrix} \tag{4.7.1.3}$$

At the top of the tower, the forces become,

$$\overleftarrow{F}_{So}=[K_T]F_{S1} = [K_T][K_\theta][K_\beta]\overleftarrow{F}_{S3} \tag{4.7.1.4}$$

$$\overleftarrow{F}_{So}=\begin{vmatrix} F_{X3}\mathrm{Cos}\theta \quad +F_{Y3}\mathrm{Sin}\theta\mathrm{Sin}\beta+F_{Z3}\mathrm{Sin}\theta\mathrm{Cos}\beta \\ F_{X3}\mathrm{Sin}\alpha_T\mathrm{Sin}\theta+F_{Y3}(\mathrm{Cos}\beta\mathrm{Cos}\alpha_T-\mathrm{Sin}\alpha_T\mathrm{Sin}\beta\mathrm{Cos}\theta) \\ -F_{Z3}(\mathrm{Sin}\beta\mathrm{Cos}\alpha_T+\mathrm{Sin}\alpha_T\mathrm{Cos}\beta\mathrm{Cos}\theta) \\ -F_{X3}\mathrm{Cos}\alpha_T\mathrm{Sin}\theta+F_{Y3}(\mathrm{Cos}\beta\mathrm{Sin}\alpha_T+\mathrm{Cos}\alpha_T\mathrm{Sin}\beta\mathrm{Cos}\theta) \\ +F_{Z3}(\mathrm{Cos}\beta\mathrm{Cos}\alpha_T\mathrm{Cos}\theta-\mathrm{Sin}\beta\mathrm{Sin}\alpha_T) \end{vmatrix}$$

(4.7.1.5)

4.7.2 Moments

In large wind turbine,running with low shaft speeds, blade bending loads are much more complicated than other loads. These bending loads are divided into two components, one arround the Z-axis known as flapwise and the other arround the Y-axis known as edgewise bending moment, as shown in the Figure 4.7.2.1. The flapwise bending moment produces stresses on the pressure and suction surfaces of the blade. While the edgewise bending moment produces stresses at the leading and trailing edges. For a downwind rotor, the flapwise and chordwise moments are dominated by the impulse applied to the blade each time it passes through the tower wake. It is customary to take flapwise bending moments positive which produce compression in the blade high pressure side. It means negative bending moments when an unconed rotor is extracting power from the wind. Edgewise bending moment is considered negative when tension is produced in the blade trailing edge, that is, when the rotor is extracting power from the wind.

At the blade attachment point, the expression for moment to a differential element can be written as,

$$\overleftarrow{dM}_{s3}=\overleftarrow{dF}_{s3} x \overleftarrow{r}_{s3} \tag{4.7.2.1}$$

This can be expressed as,

$$\overleftarrow{dM}_{s3}=\begin{vmatrix} i_3 & j_3 & k_3 \\ dF_{X3} & dF_{Y3} & dF_{Z3} \\ 0 & 0 & r_3 \end{vmatrix} \tag{4.7.2.2}$$

Where r_3 is the distance from the blade root along Z_3 direction. Above equation can be reduced as,

$$\begin{vmatrix} dM_{X3} \\ dM_{Y3} \\ dM_{Z3} \end{vmatrix} = \begin{vmatrix} r_3 dF_{Y3} \\ -r_3 dF_{X3} \\ 0 \end{vmatrix} \tag{4.7.2.3}$$

The equations for total moments in different directions for one blade are written as follows,

$$\text{Flapwise moment,} M_{X3}= \int_0^R r_3 \, dF_{Y3} \tag{4.7.2.4}$$

and,

$$\text{Edgewise moment,} M_{Y3}=-\int_0^R r_3 d \, F_{X3} \tag{4.7.2.5}$$

At the tower top corresponding to S_o co-ordinate system the moment can be expressed as,

$$\overleftarrow{M}_{so}=\overleftarrow{F}_{so} x \overleftarrow{r}_{so} \tag{4.7.2.6}$$

The following equation may be obtained from the above equation,

$$\begin{vmatrix} M_{Xo} \\ M_{Yo} \\ M_{Zo} \end{vmatrix} = \begin{vmatrix} i_o & j_o & k_o \\ F_{Xo} & F_{Yo} & F_{Zo} \\ X_o & Y_o & Z_o \end{vmatrix} \tag{4.7.2.7}$$

Where, X_o, Y_o and Z_o are the moments arm in the respective coordinate system. Equation (4.7.2.7) can be reduced as,

$$\begin{vmatrix} M_{Xo} \\ M_{Yo} \\ M_{Zo} \end{vmatrix} = \begin{vmatrix} Z_oF_{Yo}-Y_oF_{Zo} \\ X_oF_{Zo}-Z_oF_{Xo} \\ Y_oF_{Xo}-X_oF_{Yo} \end{vmatrix} \tag{4.7.2.8}$$

Where,

M_{Xo} is called the pitching moment=M_P.

M_{Yo} is called the rolling moment.

M_{Zo} is called the yawing moment=M_{Yaw}.

Practically, the flow of wind neither uniform, steady and unidirectional. Due to wind gradient and wind shift periodic

variations in torque, yawing moment and pitching moments can occur. These forces and moments will affect the overall system dynamics. The analysis has been applied to a downwind horizontal axis wind turbine, but can equally be applied to an upwind machine with suitable changes of sign.
The pitching moment coefficient is,

$$C_{mp}=\frac{M_P}{1/2\rho A V_\infty{}^2 R} \tag{4.7.2.9}$$

And, the yawing moment coefficient is,

$$C_{mz}=\frac{M_{Yaw}}{1/2\rho A V_\infty{}^2 R} \tag{4.7.2.10}$$

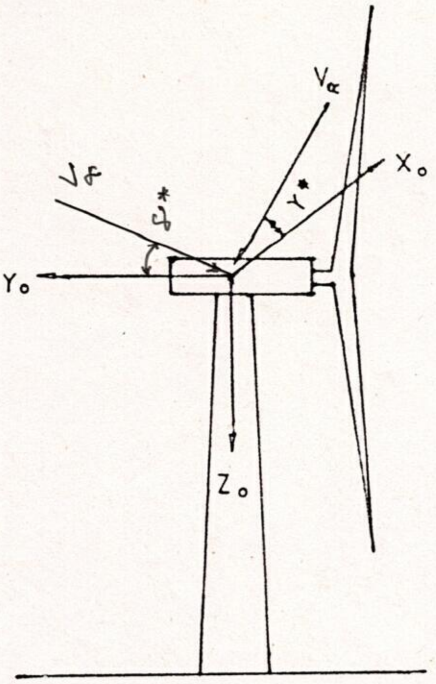

Fig.4·1·1: Reference Frame S_o.
(Coordinate System S_o)

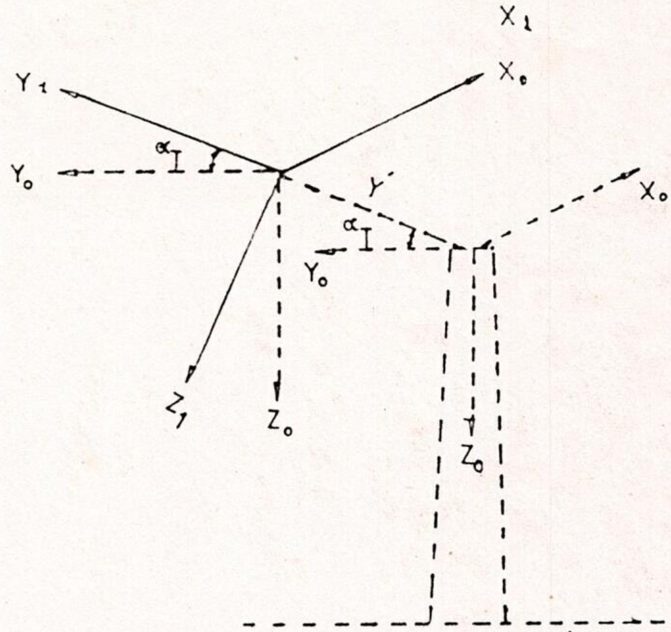

Fig.4·1·2: Translation Over Y' and Rotation About X_o by Angle α_T.
(Coordinate System S_1).

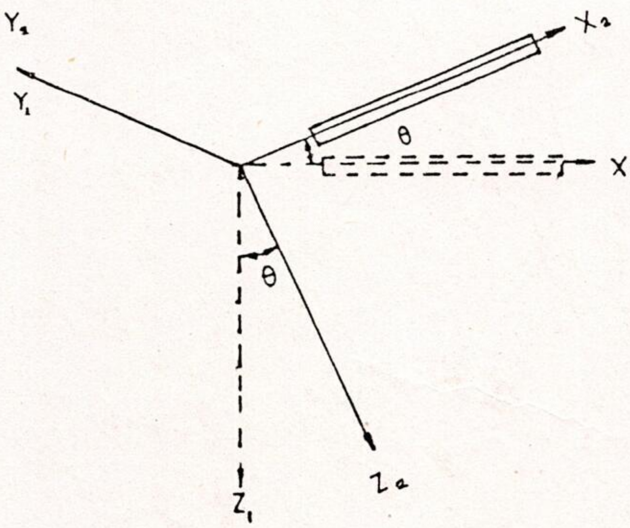

Fig. 4·1·3. : Rotation About Y_1 by Angle θ.
(Coordinate System S_2)

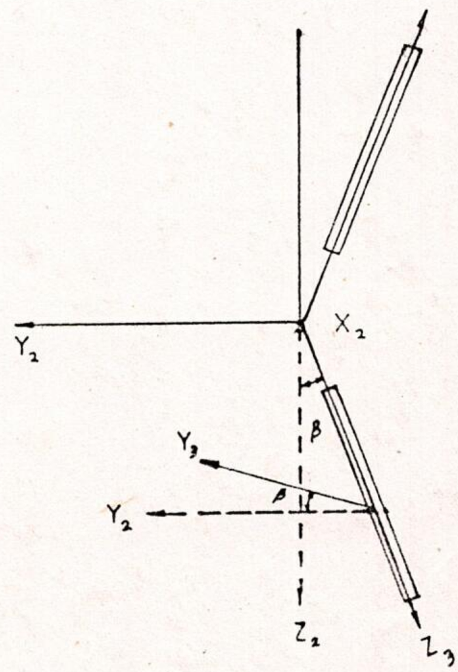

Fig. 4·1·4: Rotation About X_2 by Angle β.
(Coordinate System S_3.)

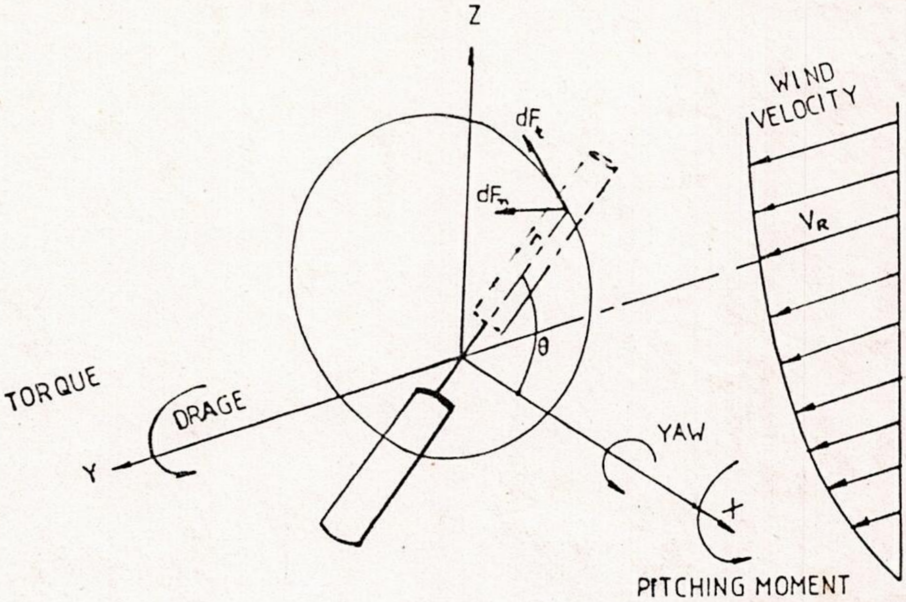

Fig.4.2.1 : Effect of wind gradient on rotor.

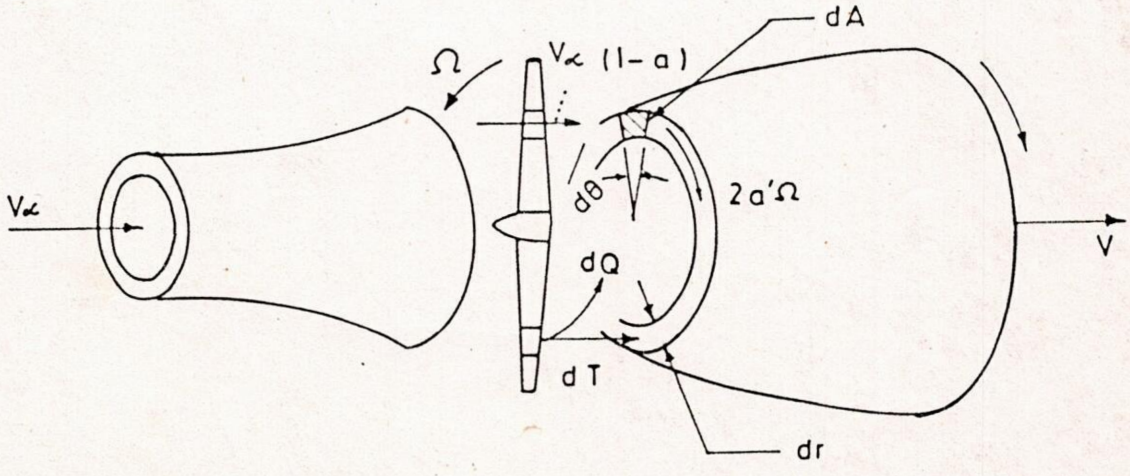

FIGURE 4.3.1. : ANALYTICAL MODEL FOR MOMENTUM THEORY.

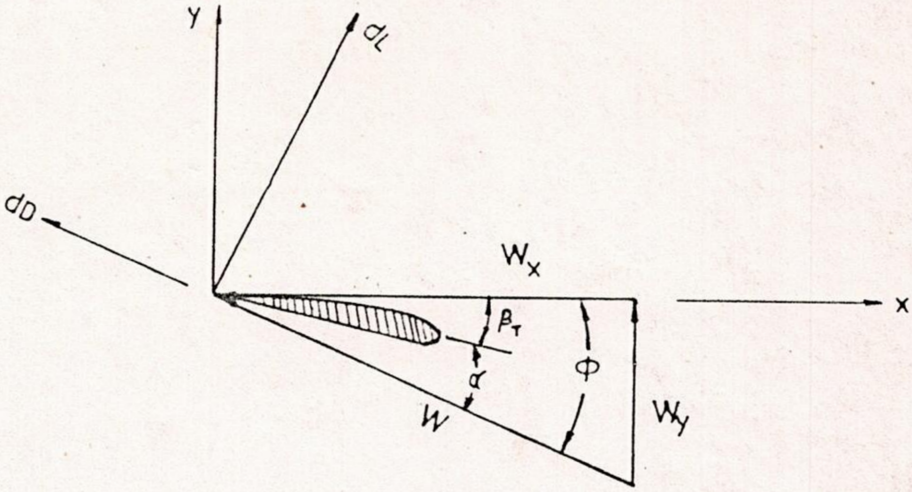

Fig.4.4.1 : Velocity Diagram for Rotor Blade Element.

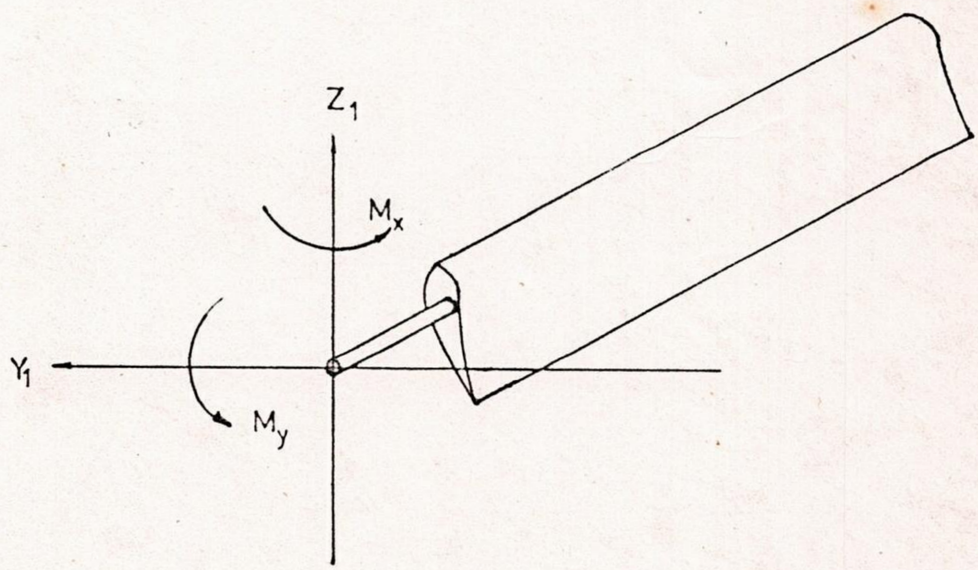

Fig.4.7.2.1: Flapwise and Edgewise Bending Moment.

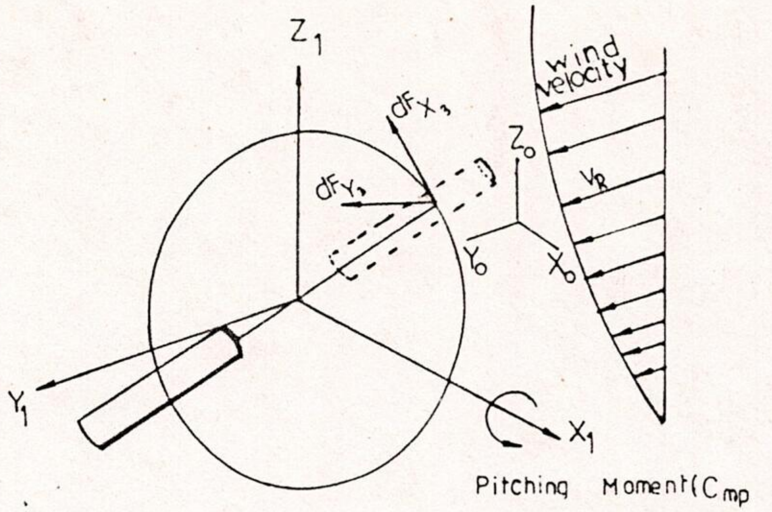

Fig. 4.7.2.2 : Pitching Moment due to Wind Gradient.

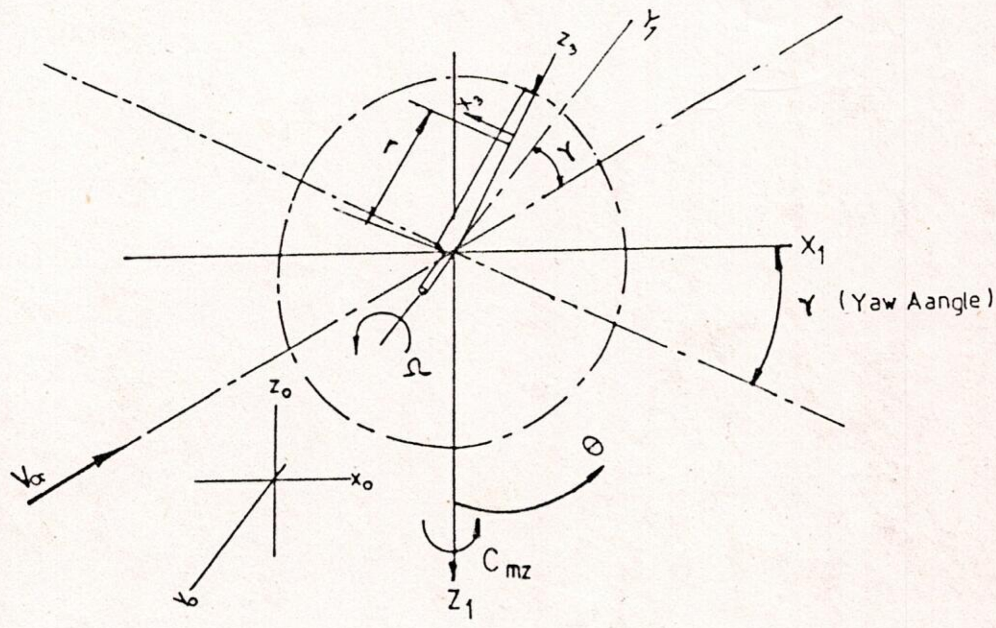

Fig. 4.7.2.3 : Yawing Moment Due to Wind Shift.

CHAPTER V

DESIGN OF WIND TURBINE

5.0 General

In this chapter, the important parameters for the design of a horizontal axis wind turbine have been discussed. This includes the design tip speed ratio, the number of blades and airfoil data. Calculation scheme for blade configuration and performance analysis are also discussed. The design of a wind rotor consists of two steps:

1. The choice of basic parameters such as the design tip speed ratio, amount of power to be extracted, the radius of the rotor, the type of airfoil section and the numbers of blades.

2. The calculations of the blade twist angle β_T and the chord C at a number of positions along the blade, in order to produce maximum power at a given tip speed ratio by every section of the blade.

The design procedure is discussed in the following sections,

5.1 Selection of Design Tip Speed Ratio and Number of Blades

In designing a wind turbine, the number of blades is an important parameter. If the number of blades increases, the cost of the wind turbine also increases, but we get improve performance and reduction of torque variations due to wind shear. Furthermore, power output increases with diminishing return [48]. The selection of λ_d and B is more or less related. The tip speed ratio is a dimensionless kinematic parameter and contains three most important variables of turbine design and analysis. These are, the wind speed, size of the rotor and the rotor rpm. Maximum power coefficient C_P versus λ for a rotor with a Rankine vortex wake [47] is shown in Figure 5.1.1, power coefficient approaches a larger value, for λ greater than 2. Also power output at larger values of λ depends upon the blades C_L/C_D ratio as shown in the Figure 5.1.2. Hence for maximum power output, the tip speed ratio which gives the maximum value of C_P in the C_L/C_D curve is to be selected. But this value of tip speed ratio must be greater than 2.

For the lower value of design tip speed ratios a higher number of blades is chosen. Because of the influence of B on C_P is larger at lower tip speed ratios. If the design tip speed ratio is higher, lower number of blades is selected. Because of the higher number of blades for high design tip speed ratio will lead to a vary thin and small blades which results in manufacturing problems. Type of load also influences upon the choice of the design tip speed ratio. If it is a slow running machine such as piston pump etc., will require a high starting torque, so the design speed of the rotor will usually be chosen low. Again, if it is a fast running machine such as generator or a centrifugal pump, it will require a low starting torque, so, a high design speed of the rotor should be selected. The following table can be considered as the guidelines for the choice of the design tip speed ratio and the number of blades [21].

λ_d	B
1	6-20
2	4-12
3	3-6
4	2-4
5-8	2-3
8-15	1-2

To obtain the optimum configuration, the blade is divided into a number of radial stations. Four formulas [20] will be used to describe the information about β_T and C :

Local tip speed ratio: $$\lambda_r = \lambda_d \frac{r}{R} \quad (3.2.8)$$

Relation for flow angle : $$\lambda_r = \frac{\text{Sin}\phi(2\text{Cos}\phi - 1)}{(1-\text{Cos}\phi)(2\text{Cos}\phi+1)} \quad (3.7.3)$$

or, $$\phi = \frac{2}{3}\tan^{-1}\frac{1}{\lambda_r} \quad (3.7.4)$$

Twist angle : $$\beta_T = \phi - \alpha \quad (3.7.4)$$

Chord : $$C = \frac{8\pi r(1-\text{Cos}\phi)}{BC_{Ld}} \quad (3.7.2)$$

The blade starting torque can be calculated by,

$$Q_{st} = \frac{1}{2} V_\infty^2 \rho B \int_r^R C(r) C_L[90-\beta_T(r)]\, rdr \quad (5.1.1)$$

The rotor configuration is determined using the assumption of zero drag and without any tip loss. Each radial element is optimised independently by continuously varying the chord and twist angle to obtain a maximum energy extraction.

5.2 Selection of Airfoil Data

Power coefficient of the wind turbine is affected by C_D and C_L values of the airfoil sections. For a fast running device, a high design tip speed ratio will be chosen and airfoil section with a low C_D/C_L ratio will be preferred. On the other hand, for a high starting torque device with low design tip speed ratio, a airfoil section having higher C_D/C_L ratio is preferred. For the design and performance calculations of the wind turbines two-dimensional airfoil data are to be used in terms of lift and drag coefficients which can be found in references [3], [4] and [9]. The available data are normally limited to a range of angles of attack upto maximum lift and the behaviour above this is not well known.

Once the airfoil section is selected, the design lift coefficient and the design angle of attack correspond to the minimum value of C_D/C_L ratio is found from the C_L-α characteristic curve, Figure 5.2.1 and C_L-C_D characteristics curve Figure 5.2.2, for that particular airfoil section. In C_L-C_D characteristics curve a tangent is drawn through the origin. The point where the tangent touches the curve, indicates the minimum C_D/C_L ratio. This ratio determines the maximum power coefficients that can be reached particularly at high tip speed ratio. With this value of C_L(for minimum value of C_D/C_L ratio), the design angle of attack is found from C_L-α characteristic curve.

5.3 Calculation Scheme for Blade Configuration

To determine the blade geometry the following data must be required beforehand:

Design tip speed ratio, λ_d.

Amount of power to be extracted, P_e.

Design wind velocity, V_d.

Type of airfoil section.

The following steps are to be carried out for getting the blade configurations:

1. Assume a definite number of radial stations for which the chord and blade twist angle are to be calculated.

2. Draw a tangent from the origin to C_L-C_D graph of airfoil section to locate the maximum value of C_L/C_D. Correspoding to this value find the design angle of attack α_d and design lift coefficient C_{Ld}. This is explained in Appendix B.

3. Select the number of blades B corresponding to the design tip speed ratio λ_d.

4. Assume a reasonable value of C_P. Estimation of the value of

C_P has been explained in Appendix B.

5. Calculate the blade radius from the equation,

$$C_P = \frac{P_e}{1/2\,\rho\,\pi R^2 V^3_\infty}$$

6. Select a fixed value of hub and tip radius ratio, $\frac{r_{hub}}{R}$

7. Calculate the value of λ_r for each radial station using the equation, $\lambda_r = \frac{r}{R}\,\lambda_d$

8. Determine the value of ϕ from the equation, $\phi = \frac{2}{3}\tan^{-1}\frac{1}{\lambda_r}$

9. Find the value of chord for each radial station from the equation,

$$C = \frac{8\pi r(1-\text{Cos}\phi)}{BC_{Ld}}$$

10. Calculate the value of blade twist angle using the equation, $\beta_T = \phi - \alpha_d$.

11. Consider a fixed value of coning or tilt angle.

12. After finding the blade geometry, the following iteration procedures are adopted to calculate the value of actual C_P.

a) Assume initial value of a and á.

b) Considering the wind shear and no yawing angle, calculate the components of relative velocity W from the equations,

$$W_X = V_{\infty o}\text{Sin}\theta_k\text{Sin}\alpha_T - \Omega r\text{Cos}\beta(1+\acute{a})$$

and, $W_Y = V_{\infty o}[\text{Cos}\alpha_T\text{Cos}\beta(1-a) - \text{Sin}\beta\text{Sin}\alpha_T\text{Cos}\theta_k]$

c) Determine ϕ from the equation, $\phi = \tan^{-1}\frac{W_Y}{W_X}$

d) Calculate the local angle of attack α by subtracting the local twist angle β_T from the relation $\alpha = \phi - \beta_T$.

e) Find the values of lift and drag coefficients from a given table or polynomial.

f) Determine the correction factor F for tip and hub losses.

g) Calculate a with, $a(1-aF)= \dfrac{\sigma W^2 Cos\phi C_L}{8Cos\beta Cos\alpha_T{}^2 V^2{}_{\infty o} F}$

h) Calculate á with, $á(1-aF)= \dfrac{\sigma W^2 Sin\phi C_L}{8rCos\beta^3 Cos\alpha_T \Omega V^2{}_{\infty o} F}$

i) Compare the values of a and á with the original assumed values and continue the iteration procedures until the new values are within the desired limit.

j) The values α, ϕ, C_L and C_D from the final iteration step are used to calculate local force components. The local power for the blade element is calculated from the equation,

$$dP= \frac{1}{2}\rho W^2 Sin\phi(C_L - \frac{C_D}{tan\phi})\ \sigma Cos\beta\ r^2\Omega dr d\theta$$

and the elemental power coefficient is determined from the equation,

$$dC_P= \frac{dP}{1/2\,\rho A V_\infty{}^3} = (\frac{W}{V_\infty})^2 \frac{r^2}{R^3} Sin\phi(C_L - \frac{C_D}{tan\phi})\,\lambda\sigma\ Cos\beta\ dr\ d\theta$$

h) Calculate the total power coefficient by integrating the elemental power coefficients using the Simpson's rule.

13) Compare the calculated power coefficient with the earlier assumed value. If it is not within certain desired accuracy, repeat all the procedures starting from step 4.

14) Find the starting torque from the equation,

$$Q_{st}= \frac{1}{2}\rho V_\infty{}^2\ B\int_r^R C(r)C_L[90-\beta_T(r)]rdr$$

15) If the starting torque is less than the desired load torque, increase number of blades and repeat all the procedure from the step 4.

5.4. Deviation from the Ideal Form

In the preceeding section it has been described how to calculate the ideal blade form. The chords as well as blade twist angle vary in a non-linear manner along the blade. Such blades are usually difficult to manufacture and may not have structural integrity. To minimize the above problems, the chords and the twist angles are linearized. Due to this linearization loss of power is small. If the linearization is done in a sensible way the loss of power will be a few percent. Due to this linearization, about 75% of the power is extracted by the rotor from the wind by the outer half of the blades. This is because, the blade swept area varies with the square of the radius and the efficiency of the blades is less at smaller radii, where the tip speed ratio λ_r is small. On the other hand, at the tip of the blade the efficiency is low due to the tip losses.

Due to the reasons mentioned previously it is better to linearize the chord and the blade angles β_T between r=0.5R and r=0.9R [25]. The equations for linearized chord and twist can be written in the following way,

$$C = rC_1+C_2$$
$$\beta_T= rC_3+C_4$$

Where C_1, C_2, C_3 and C_4 are constants. With the values of C and β_T at 0.5R and 0.9R from the ideal blade form the values of C_1, C_2, C_3 and C_4 can be calculated. The ultimate expressions for chord and twist of linearized blade can be written,

$$C=2.5(C_{90}-C_{50})\frac{r}{R} +2.25C_{50}-1.25C_{90} \tag{5.4.1}$$

$$\beta_T=2.5(\beta_{90}-\beta_{50})\frac{r}{R} +2.25\beta_{50}-1.25\beta_{90} \tag{5.4.2}$$

Where,

C_{50}= Chord of the ideal blade form at 0.5R
C_{90}= Chord of the ideal blade form at 0.9R
β_{50}= Twist angle of the ideal blade form at 0.5R
β_{90}= Twist angle of the ideal blade form at 0.9R

Further simplification of the blade shape consists of omitting the twist angle altogether. Rotor blade without twist angle results in a loss of power about 6% to 10% [44], which is acceptable for a single production unit. Since the main purpose is the design of a cheap wind turbine, an untwisted blade with a constant chord seems to be a good choice with only limited power losses.

5.5 Calculation Scheme for Performance Analysis

After the design of the rotor configurations has been completed according to the formulas in the preceeding sections, the characteristics of the rotor can now be determined.

The following data of the rotor are assumed to be available beforehand,

Rotor radius, R.
Chord(C) and twist angle(β_T) distribution along the radius.
Tip speed ratio, λ.
Number of blades, B.
Lift and drag coefficient of the blade airfoil section.

Now for a number of radial positions the values of axial and angular induction factors will be found out. As there is no analytical expressions for the induction factors, the following iteration procedures are to be performed for each radial station.

1) Assume reasonable value of a and á.

2) Find the values of W_X and W_Y from the following equations,

$$W_X = V_{\infty o}\cos\gamma\cos\theta_k + V_{\infty o}\sin\gamma\sin\theta_k\sin\alpha_T - \Omega r\cos\beta(1+á)$$

and, $$W_Y = V_{\infty o}[\sin\gamma\cos\alpha_T\cos\beta(1-a) - \sin\beta\sin\alpha_T\sin\gamma\cos\theta_k + \sin\beta\sin\theta_k\cos\gamma]$$

3) Calculate, ϕ by using the equation, $\tan\phi = \dfrac{W_Y}{W_X}$

4) Determine angle of attack α, from the equation, $\alpha = \phi - \beta_T$.

5) Calculate C_L with C_L-α graph or table.

6) Determine the total correction factor F for tip and hub losses.

7) Calculate a with, $$a(1-aF) = \frac{\sigma W^2\cos\phi C_L}{8\cos\beta\cos^2\alpha_T\sin^2\gamma V^2_{\infty o}F}$$

and, á with, $$á(1-aF) = \frac{\sigma W^2\sin\phi C_L}{8r\cos\beta^3\cos\alpha_T\sin\gamma\ \Omega V_{\infty o}F}$$

8) The new values of a and á are compared with those from step 1 and the iteration procedure is continued until the desired accuracy is reached.

9) The value of α, ϕ, C_L and C_D from the final iteration step are used to calculate local force components.

10) The local contributions to thrust, torque and power coefficients are calculated from the following equations,

$$dC_T = \frac{8}{\pi R^2}\left(\frac{V_{\infty o}}{V_\infty}\right)^2 aF(1-aF)\cos^2\alpha_T\sin^2\gamma\cos^2\beta\left(1+\frac{C_D}{C_L}\tan\phi\right)r\,dr\,d\theta$$

$$dC_Q = \frac{8}{\pi R^3}\left(\frac{V_{\infty o} r^2}{V_\infty^2}\right) \acute{a}F(1-aF)\mathrm{Cos}\,\alpha_T \mathrm{Sin}\,\gamma\ \mathrm{Cos}^4\beta\left(1 - \frac{C_D}{C_L \tan\phi}\right)\ \Omega\, dr\, d\theta$$

and,

$$dC_P = \frac{8\Omega^2}{\pi R^2}\ \frac{V_{\infty o}}{V_\infty^3}\ r^3 \acute{a}F(1-aF)\mathrm{Cos}\,\alpha_T \mathrm{Sin}\,\gamma\ \mathrm{Cos}^4\beta\left(1 - \frac{C_D}{C_L \tan\phi}\right) dr\, d\theta$$

When the iteration procedure is completed for all blade elements, the local contributions to torque, thrust and power are integrated by using Simson's rule to determine the performance of the rotor.

5.6 Design of a 350 kW Wind Turbine

For the present work, a two-bladed horizontal axis downwind turbine is selected. The design tip speed ratio is 8 and adjustable pitch angle changing mechanism is selected. Power to be extracted is 350 kW. The value of $r_{hub}/R=0.1$ is chosen arbitrarily. Throughout the theoretical studies NACA 4418 airfoil section and Prandtl's tip loss correction method are used to develop the curves. Average wind velocity 9 m/sec and no coning or tilting angles are considered to find out the optimum blade configuration. The blade radius has been divided into 12 number of radial stations. Each station has a distance r from the rotor centre having a local tip speed ratio λ_r. The relative wind velocity angle ϕ was found out by the equation (3.7.4). The chord and twist angle for each station were found out by equations (3.7.2) and (3.3.9). To find out the blade geometry, each radial station is optimised independently by continuously varying the chord and twist angle to obtain a maximum energy extraction. This is done when every element of the blade is operating at the maximum lift and drag ratio of the profile. The dependance of the torque at $\Omega=0$, on pitch angle is shown in the Figure 5.6.1. The starting torque coefficient increases with the increase of pitch angle. Where there is large internal resistance, high overall pitch angle is preferable to start the rotor from rest. Figures 5.6.2 and 5.6.3 show the distribution of twist angle and chord of the optimum designed wind turbine for the given conditions. Velocity distribution with tip speed ratio is shown in the Figure 5.6.4. At low tip speed ratio the velocity is high. The velocity decreases with the increase of tip speed ratio. The above mentioned configuration is studied because of, various test results concerning performance analysis are readily available [21] for comparison.

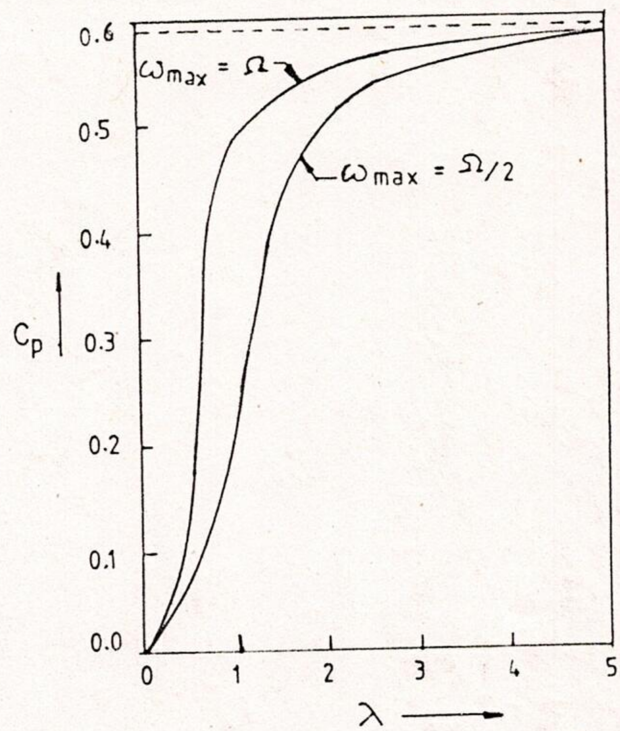

Fig.5.1.1 : Maximum Power Coefficient Versus λ for a Rotor with a Rankine Vortex wake

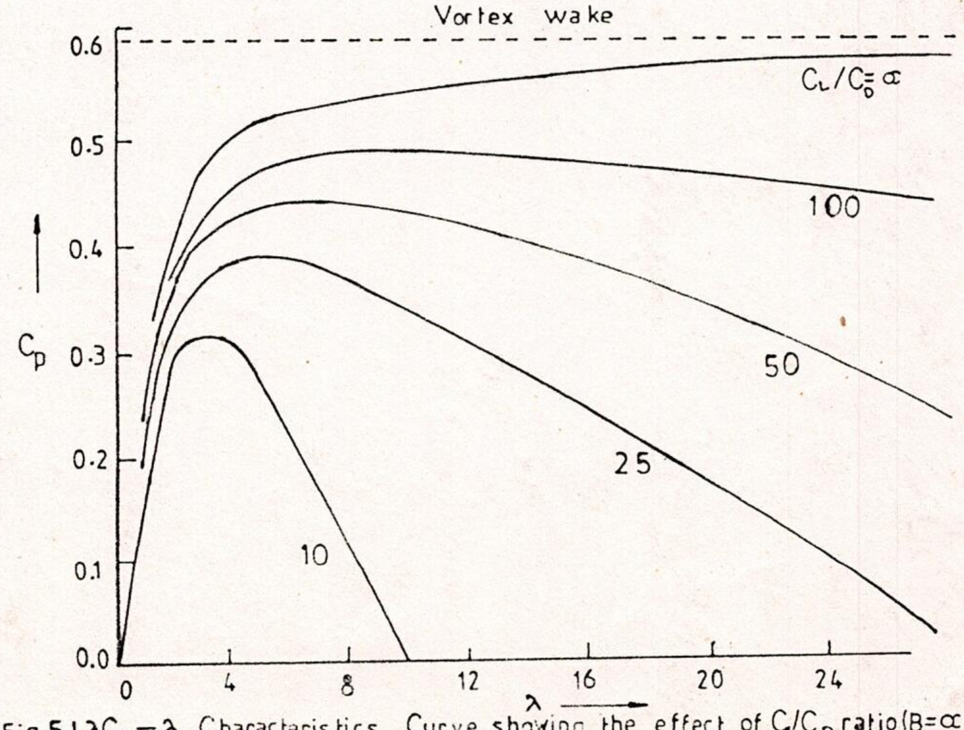

Fig.5.1.2 $C_p - \lambda$, Characteristics Curve showing the effect of C_L/C_D ratio $(B=\alpha)$.

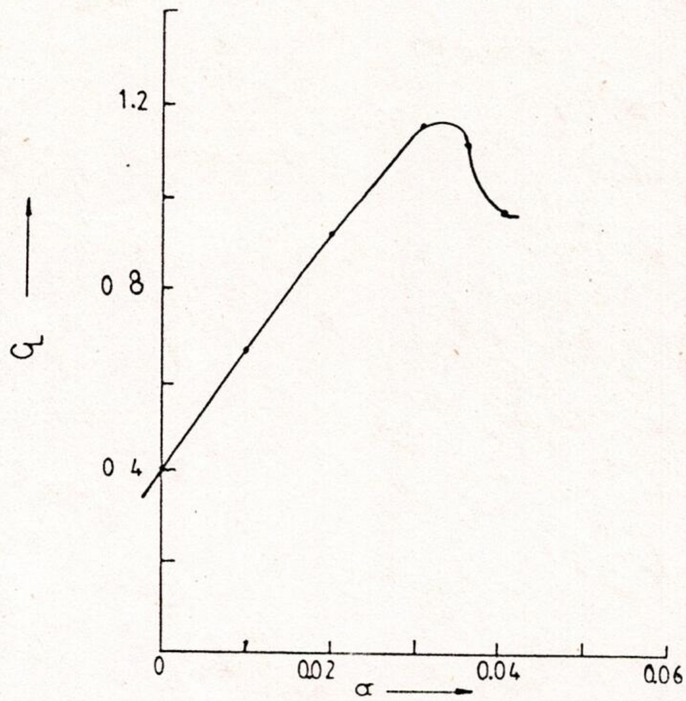

Fig. 5.2.1: $C_L - \alpha$, Characteristics Curve for a given Airfoil Section [21].

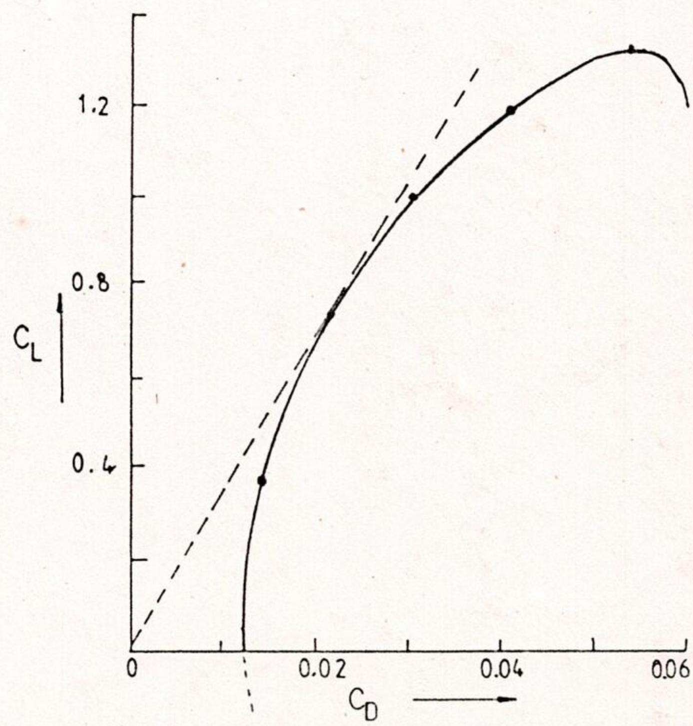

Fig. 5.2.2: $C_L - C_D$, Characteristics Curve for a given Airfoil Section [21].

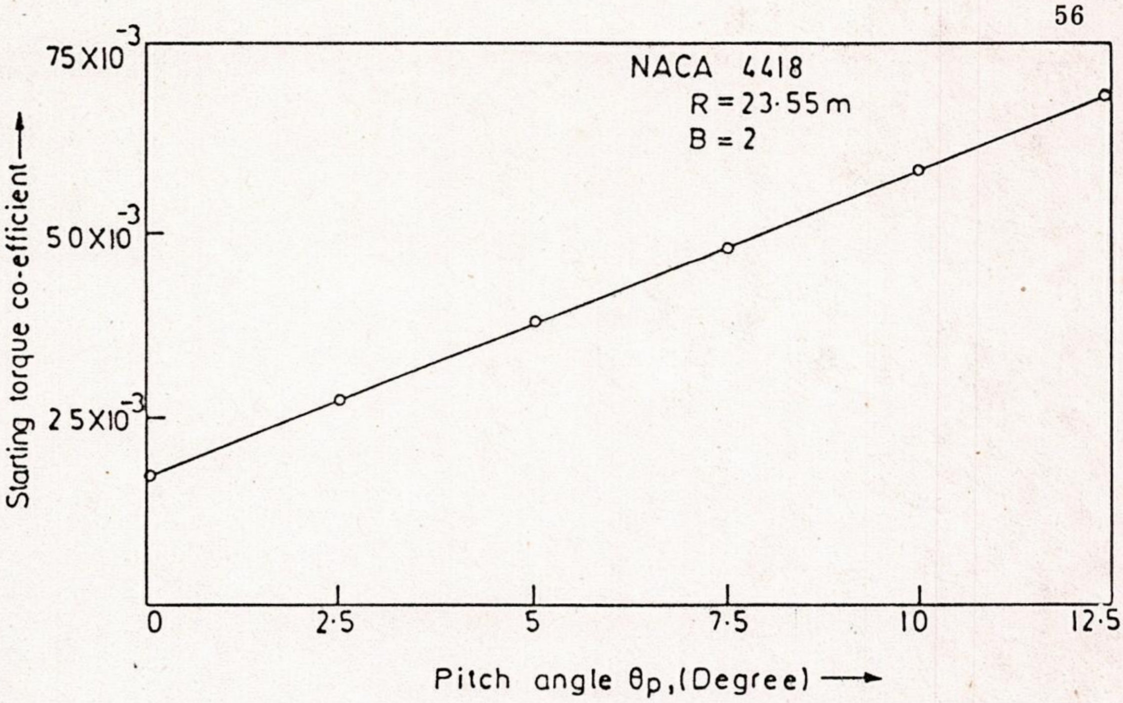

FIGURE 5·6·1: STARTING TORQUE CO-EFFICIENT AS A FUNCTION OF OVERALL PITCHING ANGLE.

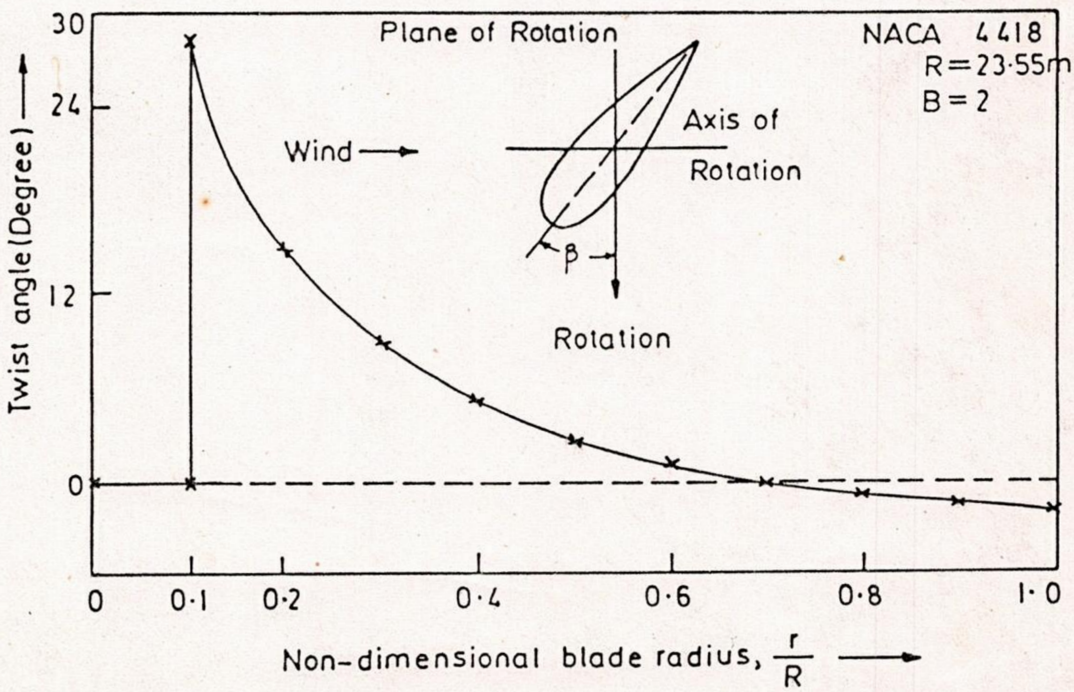

FIGURE 5·6·2: BLADE TWIST DISTRIBUTION FOR OPTIMUM DISTRIBUTION.

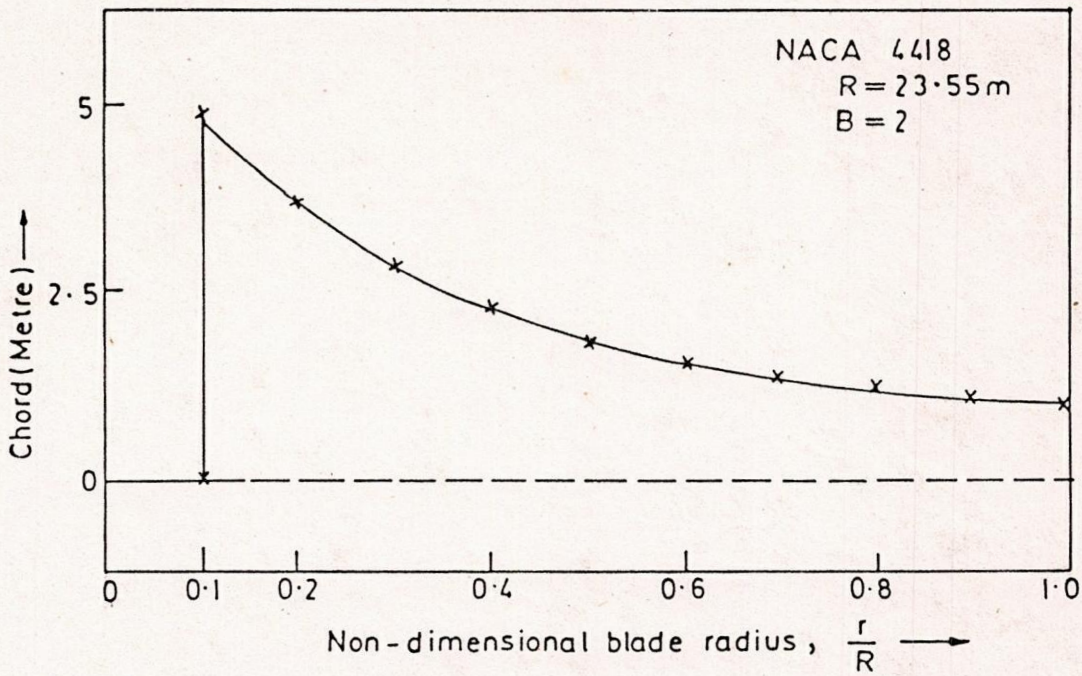

FIGURE 5.6.3: BLADE CHORD DISTRIBUTION FOR OPTIMUM PERFORMANCE.

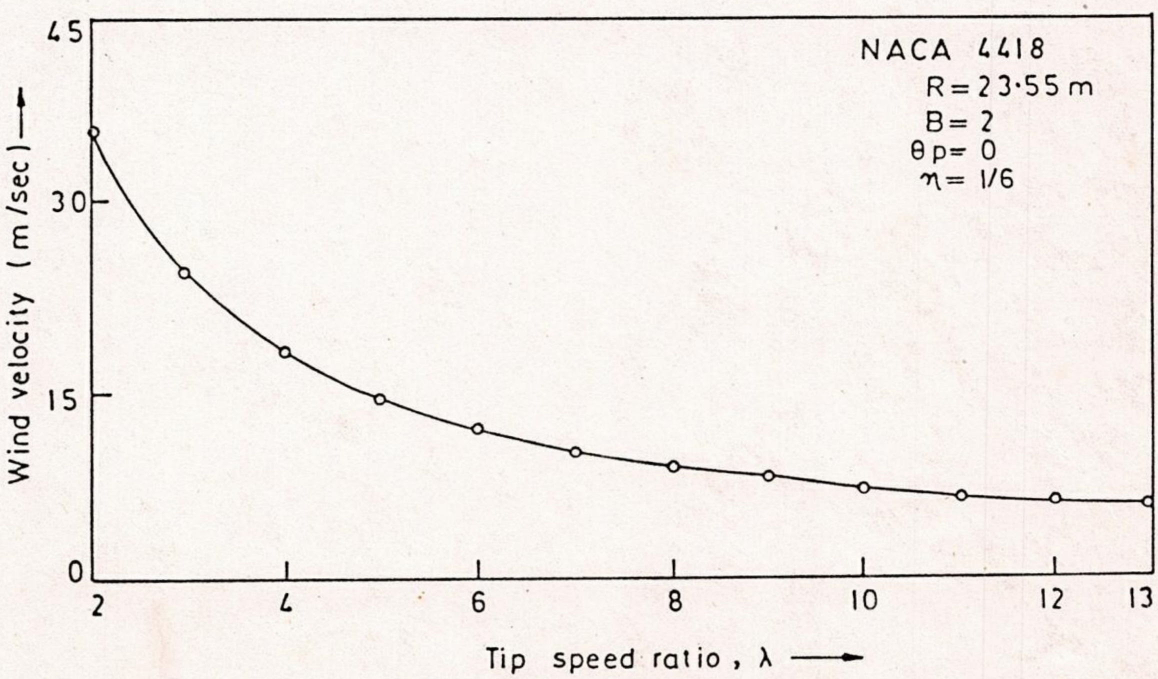

FIGURE 5.6.4: WIND VELOCITY VARIATION WITH TIP SPEED RATIO.

CHAPTER VI

YAW STABILITY ANALYSIS

6.0 General

In the present world of energy crisis, wind is an important source of energy. Which is one of the cheapest and renewable source of energy. In the region, where relatively strong wind exist, wind turbine may supply considerable energy for the generation of electrical power and for other use. Large horizontal axis wind turbine presently designed and studied for the generation of electrical power. The economic feasibility of this pollution free renewable source of energy must be effectively determined. An economically viable and proper designed wind turbine may be able to operate for long period without requiring excessive maintenance and replacement of parts. Therefore, for smooth running and to avoid fatigue problem of a wind turbine, it is necessary to minimize the vibratory loads and stresses in the rotor, tower and control system considerably.

A survey covering some aspects of the dynamic problems of large wind turbines is presented in reference [33], which contains a qualitative discussion of the effects of size, number of blades, hub configuration and type of control system on the turbine dynamic characteristics. Ormiston [34] considers the flapping response of a wind turbine blade using elementary analytic techniques for a simple rigid, centrally hinged and spring restrained blade model. Spera [39] has performed an approximate structural analysis for the NSF/NASA MOD-0 wind turbine rotors. Kaza and Hammond [27] have considered the flap-lag stability of wind turbine rotors in the presence of velocity gradients. Dugunji [10] has reviewed the whirl stability of a windmill on a flexible tower.

6.1 Present Approach

In the present work, the results obtained from a theoretical calculation model including the influence of yaw and tower shadow on the performance of horizontal axis wind turbines. Analytically a simple blade is chosen as the basic configuration for analysis to construct the rotor-tower model. A modified strip theory approach has been used to determine the effects of yaw and tower shadow. The tower shadow has a considerable effect on the flap response but leaves blade stability unchanged [28]. The model takes into account the drag of the blade and also the tip loss correction factor, modelled by Prandtl. Investigations are carried out for no coning and tilting angle. The analysis is applied to both downwind and upwind rotors.

6.2 Blade Forces Under the Influence of Yaw

If the rotor axis is not parallel to the direction of air flow, that is, where a yawing angle exists the aerodynsmic forces on the blades will vary during one revolution, although the windmill is situated in a steady flow of air. The reason behind this is, the change of both the magnitude and direction of the local resulting wind velocity, which varies with the cyclic movement of the blade and opposed to the direction of the wind alternately, which is shown in the Figure (6.2.1).

The effect of tower shadow on velocity at different azimuthal position greatly influence the forces and moments. Two blade configuration is chosen. when the blade is at 165^0 to 195^0, there is a considerable decrease of velocity, which results decrease of tower top forces and moments and therefore vibratory load produce on the tower as a result, reduced the fatigue life of the wind turbine and also increase the noise of the turbine.

6.3 Stability Analysis

In this section, the analysis of yaw stability is explained for different types of horizontal axis wind turbines. Generally, horizontal axis wind turbines can be classified according to the following six factors [40];

i) Rotor location with respect to the tower upwind or downwind.
ii) Rotor axis inclination levelled or tilted.
iii) Blade axis inclination radial or coned.
iv) Number fo blades two, three or more.
v) Blade root attachment cantelevered or hinged.
vi) Blade pitch angle fixed or variable.

Fixing up the last three factors listed above as follows: two blades, cantelevered root attachments and fixed pitch angle, the following sub-sections evaluates the analysis of yaw stability. According to equations (4.7.1.5) and (4.7.2.8), the tower top forces and moments can be written as,

$$\vec{F}_{so} = \begin{vmatrix} F_{x3}Cos\theta \quad +F_{Y3}Sin\theta Sin\beta + F_{z3}Sin\theta Cos\beta \\ F_{x3}Sin\alpha_T Sin\theta + F_{Y3}(Cos\beta Cos\alpha_T - Sin\alpha_T Sin\beta Cos\theta) \\ \qquad -F_{z3}(Sin\beta Cos\alpha_T + Sin\alpha_T Cos\beta Cos\theta) \\ -F_{x3}Cos\alpha_T Sin\theta + F_{Y3}(Cos\beta Sin\alpha_T + Cos\alpha_T Sin\beta Cos\theta) \\ \qquad +F_{z3}(Cos\beta Cos\alpha_T Cos\theta - Sin\beta Sin\alpha_T) \end{vmatrix} \tag{4.7.1.5}$$

and,

$$\begin{vmatrix} M_{xo} \\ M_{Yo} \\ M_{zo} \end{vmatrix} = \begin{vmatrix} Z_oF_{Yo} - Y_oF_{zo} \\ X_oF_{zo} - Z_oF_{xo} \\ Y_oF_{xo} - X_oF_{Yo} \end{vmatrix} \tag{4.7.2.8}$$

A. Downwind Rotor Without Coning and Tilting Angle

A downwind rotor without coning and tilting angle is shown in the Figure (6.3.1). Now putting $\alpha_T=0$ and $\beta=0$ in the above two equations then, tower top forces are

$$\vec{F}_{so}=\begin{vmatrix} F_{x3}Cos\theta & & +F_{z3}Sin\theta \\ & F_{Y3} & \\ -F_{x3}Sin\theta & & +F_{z3}Cos\theta \end{vmatrix} \qquad (6.3.1)$$

and the equation of yawing moment can be written as,

$$M_{zo} = Y_o(F_{x3}Cos\theta+F_{z3}Sin\theta) - X_oF_{Y3} \qquad (6.3.2)$$

B. Downwind Rotor With Coning Angle

For a downwind rotor with coned blade as shown in the Figure 6.3.2, putting $\alpha_T=0$ in equation (4.7.1.5) gives, tower top forces,

$$\vec{F}_{so} =\begin{vmatrix} F_{x3}Cos\theta & +F_{Y3}Sin\theta Sin\beta+F_{z3}Sin\theta Cos\beta \\ & F_{Y3}Cos\beta \quad -F_{z3}Sin\beta \\ -F_{x3}Sin\theta & +F_{Y3}Sin\beta Cos\theta+F_{z3}Cos\beta Cos\theta \end{vmatrix} \qquad (6.3.3)$$

and the equation of yawing moment is,

$$M_{zo} = Y_o(F_{x3}Cos\theta+F_{Y3}Sin\theta Sin\beta+F_{z3}Sin\theta Cos\beta) \\ -X_o(F_{Y3}Cos\beta-F_{z3}Sin\beta) \qquad (6.3.4)$$

C. Upwind Rotor Without Coning or Tilting

For a upwind rotor without coning and tilting angle is shown in the Figure (6.3.3). The equation of tower top forces and moments are same as the equations (6.3.1) and (6.3.2), but the moment arms considered with appropriate sign.

D. Upwind Rotor With Coning and Tilting

For a upwind rotor with coning and tilting angle is shown in the Figure (6.3.4) and the equation of yawing moment can be expressed as,

$$M_{zo}= Y_o(F_{x3}Cos\theta \quad +F_{Y3}Sin\theta Sin\beta+F_{z3}Sin\theta Cos\beta) \\ -X_o[(F_{x3}Sin\alpha_T Sin\theta \quad +F_{Y3}(Cos\beta Cos\alpha_T-Sin\alpha_T Sin\beta Cos\theta) \\ -F_{z3}(Cos\alpha_T Sin\beta+Sin\alpha_T Cos\beta Cos\theta)] \qquad (6.3.5)$$

The stability of a downwind rotor depends on a large number of parameters such as coning, tilting, azimuthal angles, wind shift, wind shear, tower shadow factor, pitching angle and the tip speed ratio. A wide investigation of these parameters are necessary.

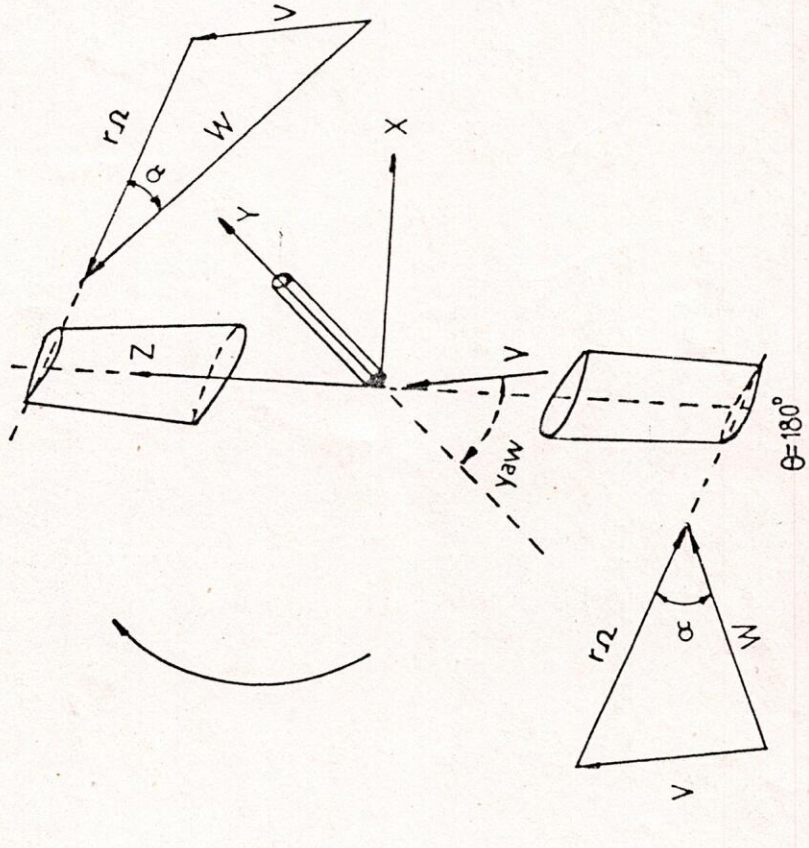

Fig.6·2·1: The Relative Wind Velocities at Upper and Lower Position of The Blade in yaw.

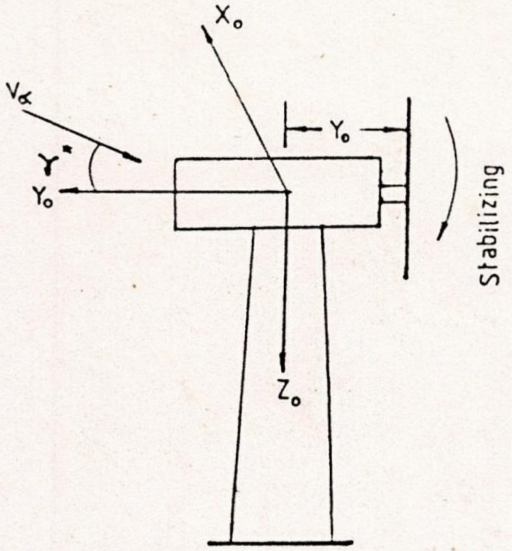

Fig. 6.3.1: Downwind Rotor Without Coning Angle.

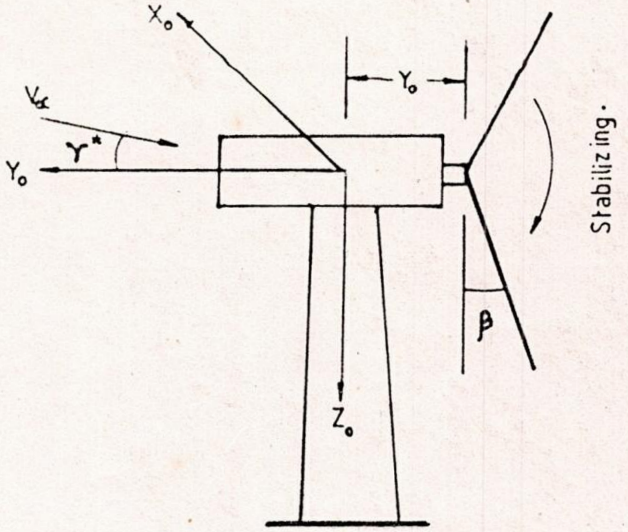

Fig. 6.3.2: Downwind Rotor With Coning Angle.

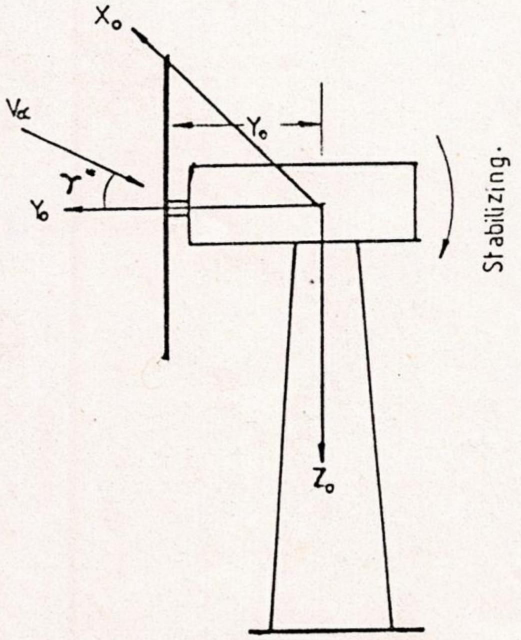

Fig. 6.3.3: Upwind Rotor Without Coning or Tilting Angle.

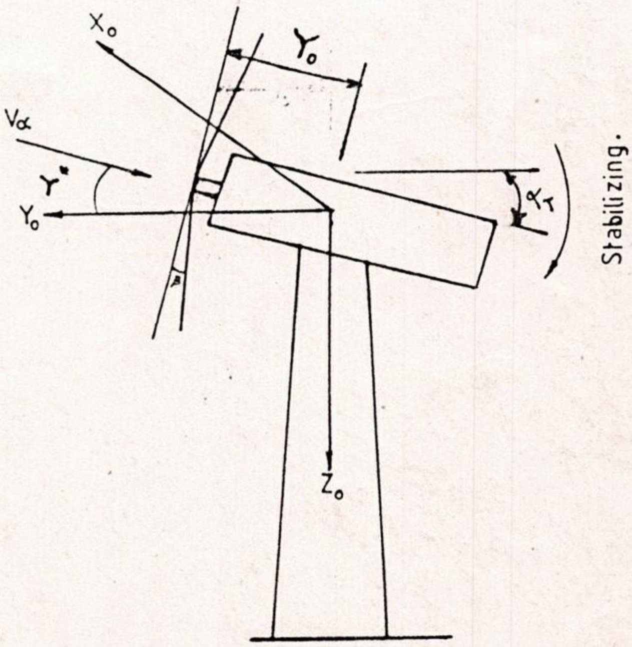

Fig. 6.3.4: Upwind Rotor With Coning and Tilting Angle.

CHAPTER VII

RESULTS AND DISCUSSION

7.0 General

In the following chapter, the results obtained are discussed. The effect of pitch angle variation, blade shapes, wind shift and tower blockage are discussed elaborately. Also a brief discussion about the effect of wind shear, number of blades, coning angle and tilting angle are presented in the following chapter for general information.

7.1 Results of a 350 KW Wind Turbine

The power, thrust and torque coefficients variation with respect to tip speed ratio for a 350 KW horizontal axis wind turbine are shown in Figures 7.1.1, 7.1.2 and 7.1.3, considering fixed pitch angle i.e. at zero pitch angle. The turbine is designed for a tip speed ratio 8 with a rated wind velocity 9 m/sec. The maximum power coefficient is attained at a tip speed ratio 8, as shown in the Figure 7.1.1. The thrust coefficient is found to increase continuously and the value greater than unity is reached, which is shown in the Figure 7.1.2. This is because at high tip speed ratios the velocities through the turbine actuator disc are less than half the free stream velocity. Which means that the velocity in the fully developed wake has reversed in direction and flows back towards the turbine actuator disc.

The flow reversal induces a vortex flow in the wake which violates a basic assumption for the blade element theory. The theoretical predictions in this condition by the present method are not accurate [21]. The variation of torque coefficient with respect to tip speed ratio is shown in the Figure 7.1.3. This Figure shows that the maximum torque coefficient moves towards a lower value of tip speed ratio than the value of the designed tip speed ratio.

7.2 Effect of Pitch Angle Variation

The calculated power, thrust and torque coefficient with respect to tip speed ratio at different pitch angle are shown in the Figures from 7.2.1 to 7.2.3. Increased pitch angle reduces the maximum power but increase the power available at low tip speed ratios. Figure 7.2.1 shows some general features of horizontal axis wind turbine. At low tip speed ratios, the power coefficient is strongly influenced by the maximum lift coefficient. The relative flow velocity angle ϕ is large at low tip speed ratios and much of the rotor, particularly the inboard stations, can be stalled when operating below the designed tip speed ratio. At tip speed ratios above the peak power coefficient, the effect of drag becomes dominant. Rapid power decrease will occur due to high drag coefficient and at some large tip speed ratio the net power output will become zero. The power curve is sensitive mainly due to blade pitch angle in the stalling region.

To avoid too much drop off of the power after the stalling point a blade pitch angle of 2.5^0 and 5^0 are seem to be more convenient than the performance at 0^0 pitch angle. Increase of pitch angle shows the shifting of maximum power coefficients to the lower values of tip speed ratios. Figure 7.2.2, shows that the thrust coefficient is increasing continuously with tip speed ratio and value becomes more than unity. With the increase of blade pitch angle the values of induced velocity factor a, close to or lower than 0.5, Which means a more relative solution. Therefore, the thrust coefficient more than unity is possible. But with the increase of pitch angle, peak value of thrust coefficients decreases and they occur at a particular tip speed ratio beyond which the coefficients gradually decreases. The variation of torque coefficient with tip speed ratio is shown in the Figure 7.2.3. With the increase of pitch angle the maximum value of torque coefficient moves towards the lower values of tip speed ratio. Change of the blade pitch angle means that the resulting angle of attack is reduced and the lift coefficient is shifted from the stalling region. Pitching of the blade provides higher torque coefficient at lower tip speed ratios. Therefore, a small change in pitch angle seems to give better results than other conditions.

When any part of the turbine blade operates with a resulting angle of attack that is higher than the value corresponding to maximum lift for the local blade section flow separation is encountered. This implies three dimensional effects which violates the basic assumption of the blade element theory and the solutions obtained are not fully reliable. Such conditions occur for high wind speeds. The flow angle ø will then increase with increasing wind speed keeping other parameters constant. Flow separation will first occur in the hub region, where ø has its largest values. Increasing the blade pitch angle means that the resulting angle of attack is lowered and more reliable solutions are obtained.

7.3 Effect of Blade Shapes

The normal procedure for determining the blade shape of a horizontal axis wind turbine is to optimize independently each radial element by cotinuously varying the chord and twist angle to obtain maximum energy extraction. This method results in complex blade shapes which is expensive to manufacture and may not have structural integrity. In order to reduce these problems it is possible to linearize the chord and twist angles. This results in a small loss of power. But, if the linearization is done in a sensible way the loss is only a few percent.

In the present analysis three types of blade shapes have been considered; optimum-chord optimum-twist, linear-chord linear-twist and linear-chord zero-twist. In the preceding section, the power,thrust and torque coefficient with respect to tip speed ratio at different pitch angles for optimum-chord optimum-twist are discussed. Figures 7.3.1 to 7.3.3 show the power, torque and thrust coefficient with tip speed ratio at different pitch angle for linear-chord linear-twist blade. Figures 7.3.4 to 7.3.6 show the power, torque and thrust coefficient with tip speed ratio at different pitch angles for linear-chord zero-twist blade. Figures 7.3.7 to 7.3.9 show the comparison among the power,torque and thrust coefficient at zero pitch angle. Figure 7.3.7 shows that the power coefficient for other two blade shapes is less than the optimum-chord optimum-twist blade at lower value of tip speed ratio but approximately same at high tip speed ratio. The Figure 7.3.8 shows the torque coefficient for three blade shapes. Optimum-chord optimum-twist blade has the greater value of torque coefficient than the other two blade shapes at lower tip speed ratio but same at higher tip speed ratio. Figure 7.3.9 shows the thrust coefficient for three type of blade shapes. Which also gives the same result as for power and torque coefficient.

The linearization of the chords and twist angles have been done by taking the values from the optimum blade cofiguration at r=0.5R and r=0.9R. Radial distribution of power coefficient, thrust coefficient and torque coefficient, power, thrust, torque, axial force and axial bending moment are shown in the Figures from 7.3.10 to 7.3.18. It is to be noted that about 75% of the power, thrust and torque are produced by the outer 50% radius. The reason behind this, is that, the blade swept area varies with the square of the radius and also the efficiency of the blades is less at small radii, where the local tip speed ratio $_r$ is small. On the other hand due to the tip losses there is a decrease of power, moments and forces near the tip of the blade.

Figures 7.3.19 to 7.3.20 show the distribution of chord and blade twist angles for three types of blades. From these Figures it is found that the changes in chords and twist angles are very small at the outer half of the blade. Large variations with the linear chord and twist distributions are found only at the lower part of the blade i.e. inner half of the blade. It is realized that about 75% of the power that is extracted by the rotor from the wind is extracted by the outer half of the rotor. So this will not lead to any significant loss of power but the starting torque will be less. In cases, where the starting torque is an important factor, this effect must be considered. Variation of starting torque for different blade configuration at various pitch angle is shown in the Figure 7.3.21, where approximately 30% differences may occur between the optimum blade and zero-twist blade.

7.4 Effect of Wind Shift

The aerodynamic forces on a blade will vary during a revolution in the case, where the rotor axis is not parallel to the wind direction, even though the wind speed is constant. This results from changes in both magnitude and direction of the resulting local wind speed for the profile, which alters with the varying moment of the blade with and against the wind conditions.

The power, thrust and torque coefficients with tip speed ratio produced by the wind turbine at different yaw angles are presented in Figures 7.4.1 to 7.4.3. It can be concluded that the rotor can be yawed into and out of the wind either to maintain power level as wind speed varies or to unload the rotor for shutdown. The changes in both wind speed and direction give rise to change in blade performance with blade azimuth. Figures 7.4.4 to 7.4.7 show the variation of the power, thrust coefficients and edgewise and flapwise bending moments with azimuth for different yaw angle for single blade. At low wind speeds the effects of changes in the resulting wind speed and angle of attack almost cancel each other, so that the forces remain nearly constant. At high wind speeds under stalled conditions both the higher resulting wind speeds and the combined lower angle of attack will cause increasing lift on the blade when moving upwards relative to the wind. It appears that the variation achieves a maximum at 180^0. At 0^0 and 180^0 a change of wind directions means a large change of angle of attack and hence of the blade force. Figures 7.4.8 to 7.4. 11 show the distribution of yawing and pitching moment coefficient and tower top axial and tangential forces at different yawing angle. The variation achieves a maximum twice per revolution for each case. These Figures are plotted by considering a two blade configuration. So after passing 180^0 repeatation of the same curve will take place. The two extremes will originate from structural coupling between the blades. The uniform sinusoidal wave shape indicates that it results primarily from wind shear, the wind velocity gradient with altitude. In non-axial flow, even at small angles of yaw, the cyclic variation in the aerodynamic forces at the blade root lead to resonance in either the blade, or the supporting structures and possibly reduce the lifetime of the turbine. The effects of non-axial flow, therefore, need to be considered in the design of horizontal axis wind turbine.

7.5 Effect of Tower Shadow

In case of horizontal axis wind turbine, the blades may be placed upwind or downwind of the tower support. Present analysis is for the downwind rotor arrangement. With a downwind rotor system, the blades can be mounted closer to the tower, because blade deflections will be away from the tower. Also the configuration tends to be self-orienting with changes in wind direction.

For a downwind rotor, the supporting tower of a wind turbine creates an aerodynamic shadow through which the rotor blades must pass. Tower shadow is typically a region of reduced wind speed and high turbulence. An important source of periodic wind load is the aerodynamic interference created by the tower, which is upwind of the rotor and known as the tower wake effect. The tower wake is a potential excitation source for variation at any integer multiple of the rotational speed. The flow disturbance caused by the presence of the rotor will, in general, have an adverse effect on rotor fatigue life. It may also increase acoustic noise generation from the turbines. The tower wake has a fundamental frequency of once per revolution for each blade. The magnitude of the periodic load created by the tower shadow depends on the projected area of the tower structural elements, their average drag coefficients, and the sector of the rotor area affected by the wake.

For the present analysis, the blades are assumed to be in the tower wake at azimuths from 165^0 to 195^0. When a blade is initially located vertically up behind the tower, $\theta=0^0$, and rotates to the vertically down at $\theta=180^0$, the tower blockage is modelled by [32],

$$V(\theta,\lambda)=(V_\theta)_K[1-\ B_f(1+\mathrm{Cos}\,n\theta)]$$

where,

λ =Tip speed ratio.

B_f=Tower blockage factor.

≤ 0.5 for $\pi(1-\dfrac{\pi}{12}) \leq\theta\leq \pi(1+\dfrac{\pi}{12})$

=0, for other angles.

$(V_\theta)_K$=Instantaneous wind velocity corresponding to azimuthal point.

$n\theta$=Angle of tower shadow area.

In the present work, an evaluation has conducted for the effects of tower shadow on the forces, power, thrust and moments for a downwind mounted wind turbine blade. The large and abrupt changes that occur as the blade passes through the tower shadow will obviously cause significant changes in blade. The effect of tower shadow is to give a sharp kick to the blades in an upwind direction. The tower shadow for present analysis was selected to occupy a total arc of 30^0. All the graphs have been plotted at design tip speed ratio 8 and the value of the wind power law exponent is taken as $\eta=1/6$. The value of the tower shadow factor is chosen as $B_f=0.25$.

Figures 7.5.1 to 7.5.8, show the combined effect of wind shift and tower shadow on power and thrust coefficients, bending moments, yawing and pitching moment coefficients and on the tower top forces, with azimuthal angle. The maximum effect will occur when one blade will be at vertical position i.e. at 0^0 and the other blade is shielded from the wind by the tower i.e. at 180^0. The blades of a downwind horizontal axis wind turbine passing through this shadow will be subjected to periodically varying large wind forces and bending moments, hence fatiguing stresses in the material of the blades. Since the tower wake has a fundamental frequency of once per revolution, it is a pulse load which can contain hermonics at many frequencies. In practice, once in the tower shadow the blade flapping motion results in increase in the blade angle of attack and thrust. When the blade leaving the tower shadow, the abrupt increase in wind velocity combines with the already increasing angle fo attack due to flapping, give a thrust that is briefly higher than the steady state value. One way to reduce the effect of the tower shadow is to make the tower aerodynamically smooth, thereby reducing the flow disturbance. However, because of changes in wind direction, it is necessary to have an aerodynamic tower fairing which is free to move in yaw with the turbine nacelle.

The wake behind the fairing is both narrower and much less deep. With the tower alone, there is an extensive turbulent region with no forward velocity at all, therefore the wind speed takes some distance to recover original speed. But the tower with fairing, there is no such turbulent region, and only much smaller velocity is reduced, which extends over a very narrow width. The addition of the fairing will greatly reduce the loss of the wind speed, so reducing the vibrations of the blade. In turbulent conditions, the recovery of velocity behind tower or fairing is expected to be quicker due to more rapid mixing at the boundary of the wake. The use of fairing also extending the fatigue life of the blade and reduces the noise levels to environmentally acceptable range [51].

7.6 Effect of Wind Shear

A wind shear arises from the fact that the wind near the surface of the earth is not entirely uniform. There exists a large scale boundary layer which may cause significant load variations on a blade as it passes from the bottom to the top of a rotation. The rotor of a wind turbine is placed at some distance above the ground level to avoid the extreme low velocity part of the atmospheric boundary layer. The influence of wind shear on the power output and the blade loading of a horizontal axis wind turbine is complicated, because each blade element is subjected to a varying wind velocity during one revolution of the rotor.There is some arbitariness in the choice of the reference wind velocity to calculate C_p. Wind shear has been previously discussed in section 4.2.

7.7 Effect of Number of Blades

In designing a wind turbine, the question arises how many number of blades should be used. The increased number of blades increases the cost of blade manufacture. The advantages of increasing the number of blades are improved performance and lower torque variation due to wind shear. The maximum power coefficient is also affected by the number of blades, because of the tip losses that occur at the tips of the blades. These losses depend on the number of blades and tip speed ratios. For the lower design tip speed ratios, generally a high number of blades is chosen. This is done because the influence of number of blades on power coefficient is larger at lower tip speed ratios. For a high design tip speed ratio with a high number of blades will lead to very small and thin blades which results in manufacturing problems and a negative influence on the lift and drag properties of blades. Increase of number of blades shows that the region of higher power coefficients move to the region of smaller values of tip speed ratios. Increase of pitch angle results in a decrease of angle of attack, therefore lift coefficient is shifted from the stall region and it moves towards the higher values at low tip speed ratios. Effect fo number of blades has been previously discussed in section 5.1.

7.8 Effect of Blade Coning Angle

Since stresses induced by flapwise loads are greatest in magnitude, blades are frequently coned so that bending stresses are induced by centrifugal forces, cancelling those due to aerodynamic loads. Exact cancellation is only possible at one aerodynamic loading condition and equalisation of stresses along the entire blade requires a complex blade curvature. Blade coning introduces an additional cyclic gravitational bending moment in the flapwise direction. The amplitude of this load is normally small in comparison to other loads. The coning angle, as expected, is inversely proportional to the mass of the blade. Due to coning, there is change in both magnitude and direction of the resulting local wind speed for the profile which alters with azimuth.

For a downwind rotor coning angle tends to align the net drag force of the rotor with the wind direction and this cause a freely yawing wind turbine to track the wind. On the other hand, the effects of coning are statically destabilizing and the wind turbine must be mechanically aligned with the wind. Increase of the coning angle reduces the aerodynamic forces due to the reduction of the swept area of the rotor and a better balance between centrifugal forces and thrust, which reduces the flapping moment. At zero coning angle wind shear produces a periodic wind load with a frequency of twice per revolution for a two bladed wind turbine.

7.9 Effect of Rotor Tilt Angle

If rotor blades are coned it follows that the blade tip at its lowest point is moved nearer the supporting tower for an upwind rotor. To reduce the overhung required at the hub the axis of the drive shaft is frequently tilted. If the tilt angle equals the cone angle the blade becomes vertical when pointing downwards. Due to tilting of the rotor the blade angle is changed as it moves round the blade circle, the amplitude being equal to the tilt angle.Significant changes of the blade loads are induced by these changes which must be calculated with great accuracy by the simplified methods since the rotor tilt angle is not small relative to the design angle of attack. There will be change in both magnitude and direction of the local wind speed for the blade profile due to tilting of the rotor. This will also alter with the blade azimuth. For a blade in the lower half of the rotor disc the blade will move towards the wind and for the upper half of the disc it will be further away from the wind. Also due to wind shear the blade will receive higher wind velocity when it will be at the upper half of the circle.

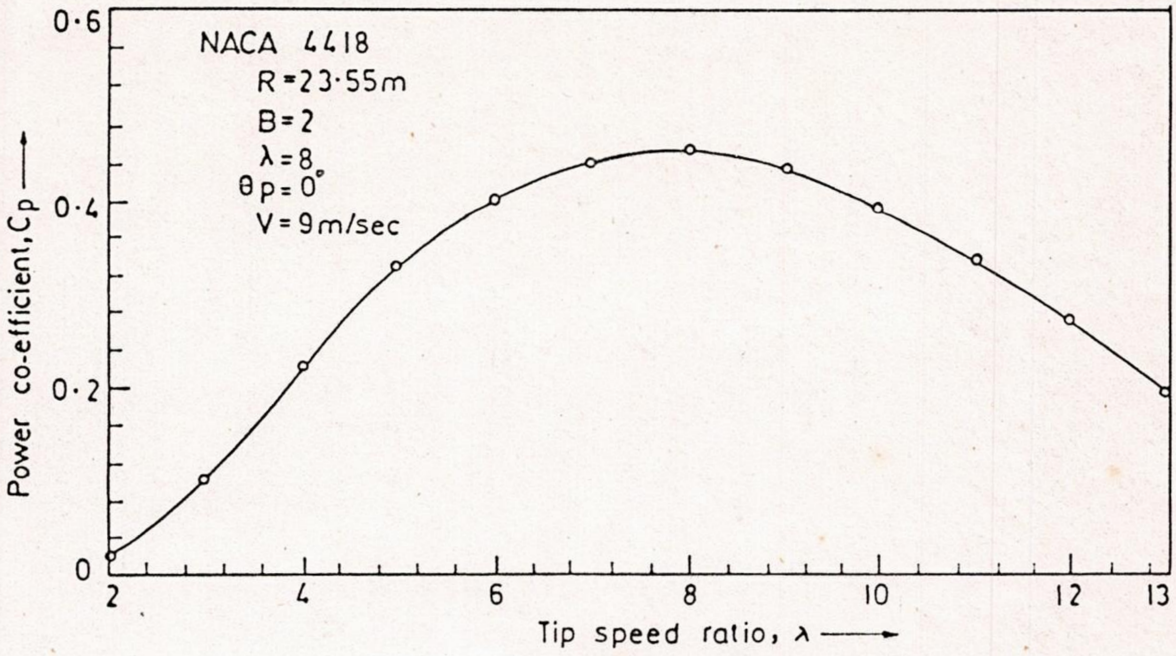

FIGURE 7·1·1: VARIATION OF POWER CO-EFFICIENT WITH TIP SPEED RATIO.

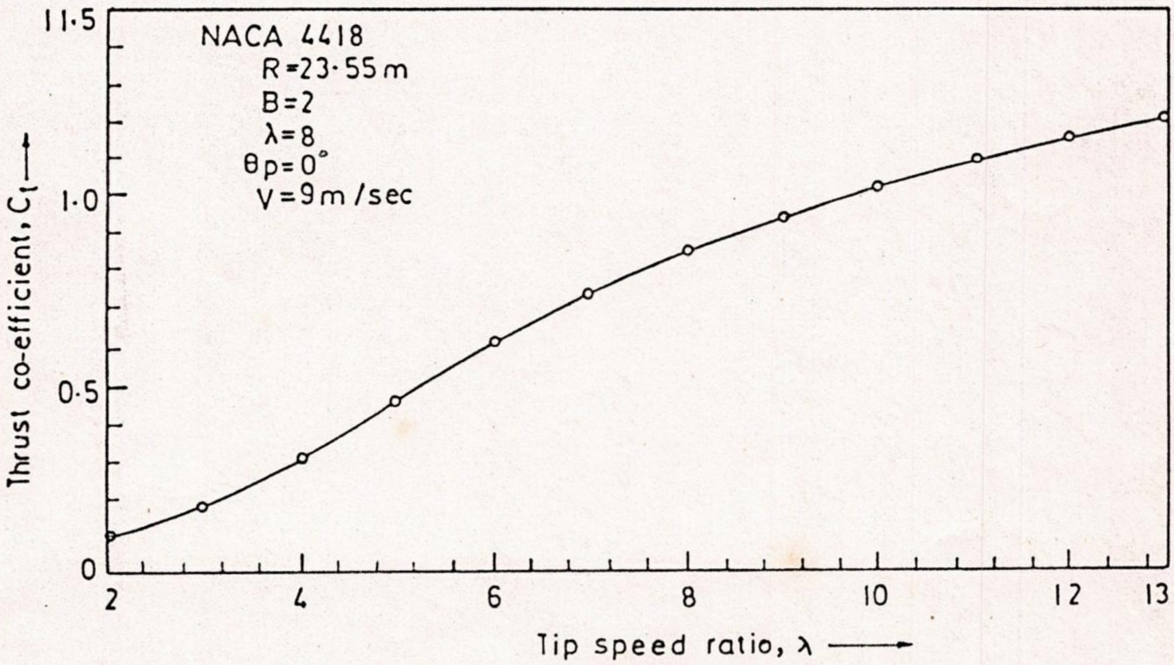

FIGURE 7·1·2: VARIATION OF THRUST CO-EFFICIENT WITH TIP SPEED RATIO.

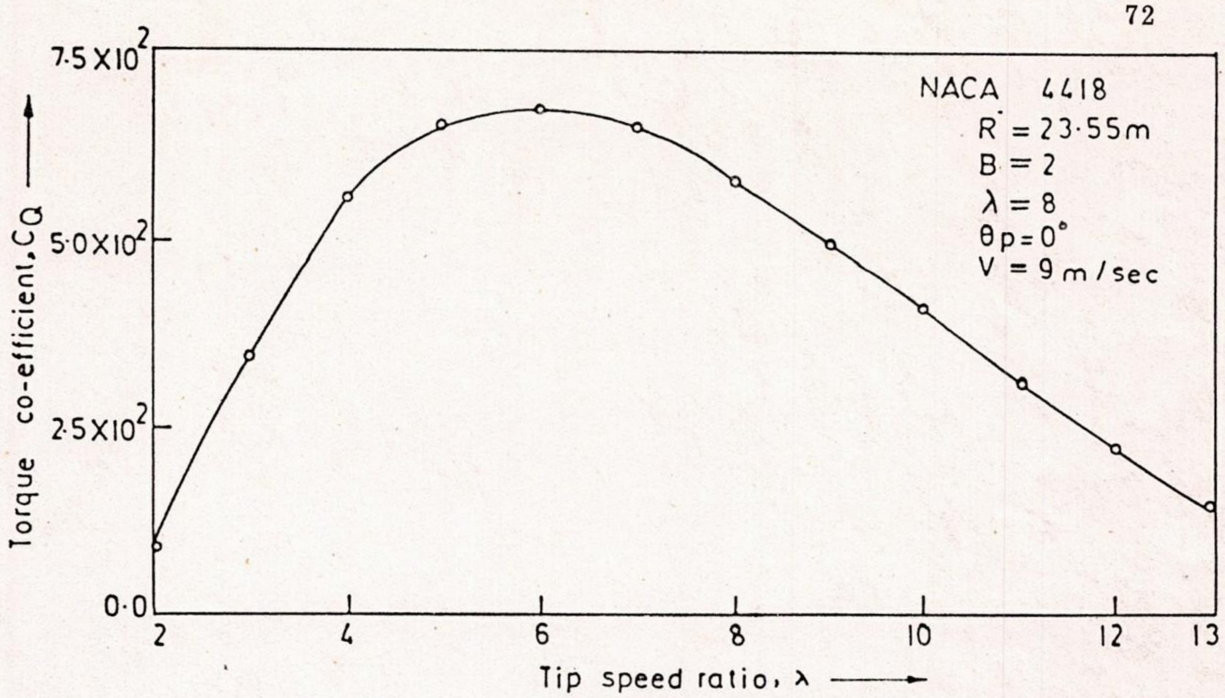

FIGURE 7.1.3:VARIATION OF TORQUE CO-EFFICIENT WITH TIP SPEED RATIO.

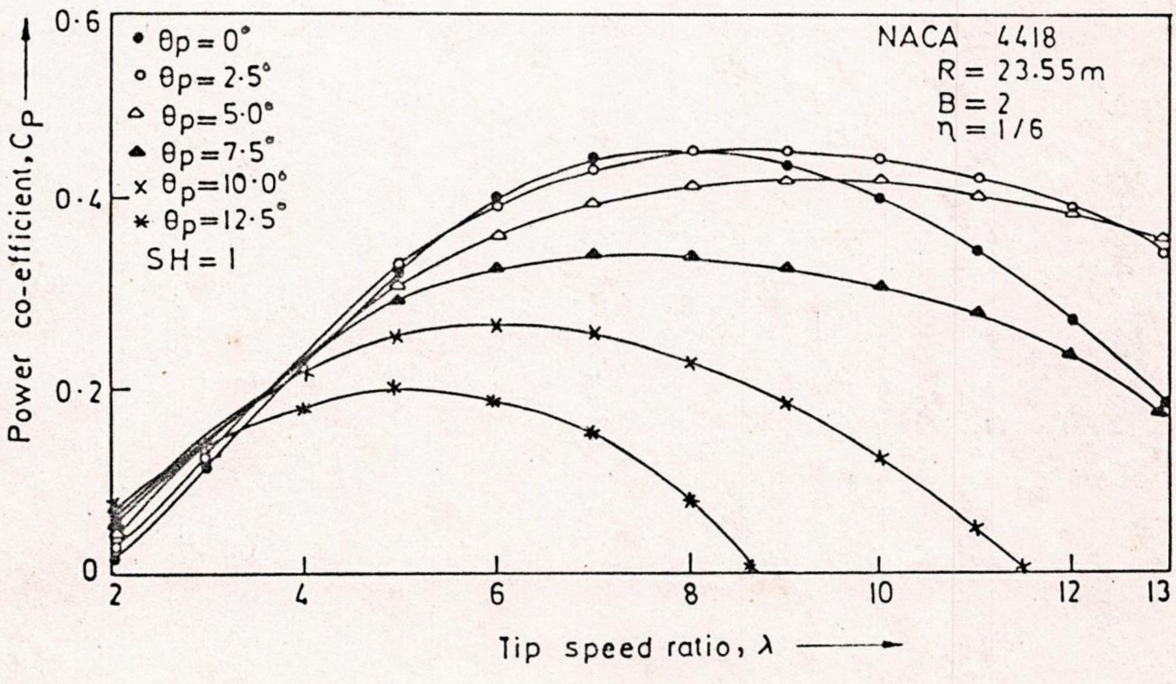

FIGURE 7.2.1:VARIATION OF POWER CO-EFFICIENT WITH TIP SPEED RATIO AT DIFFERENT PITCHING ANGLE.

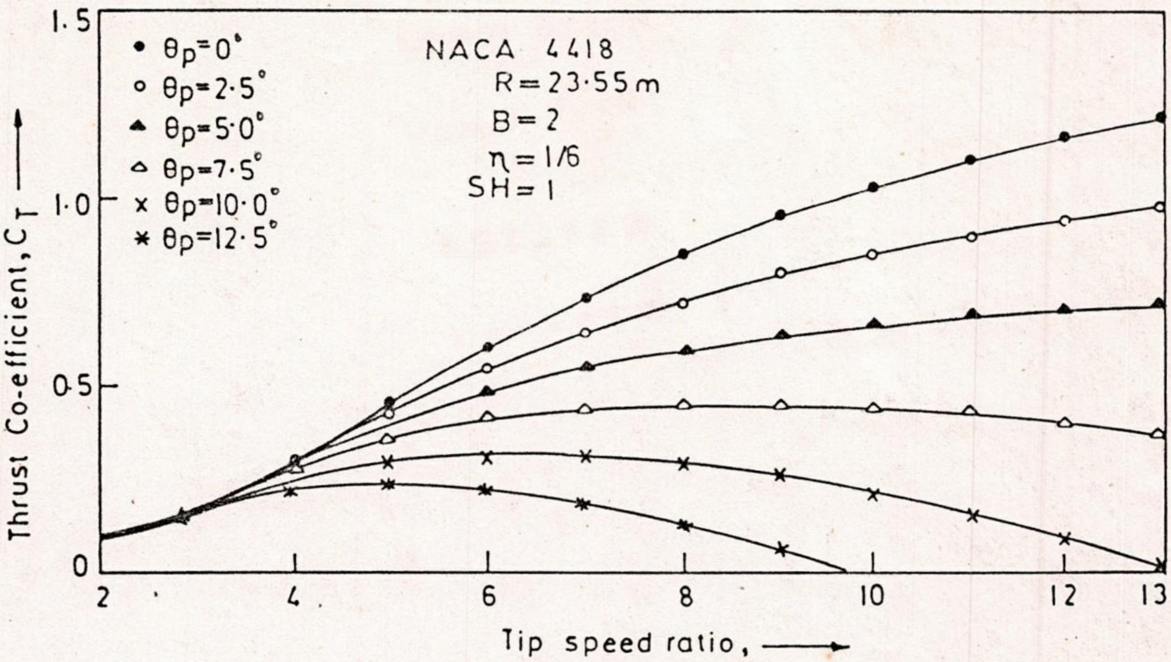

FIGURE 7.2.2: VARIATION OF THRUST CO-EFFICIENT WITH TIP SPEED RATIO AT DIFFERENT PITCHING ANGLES.

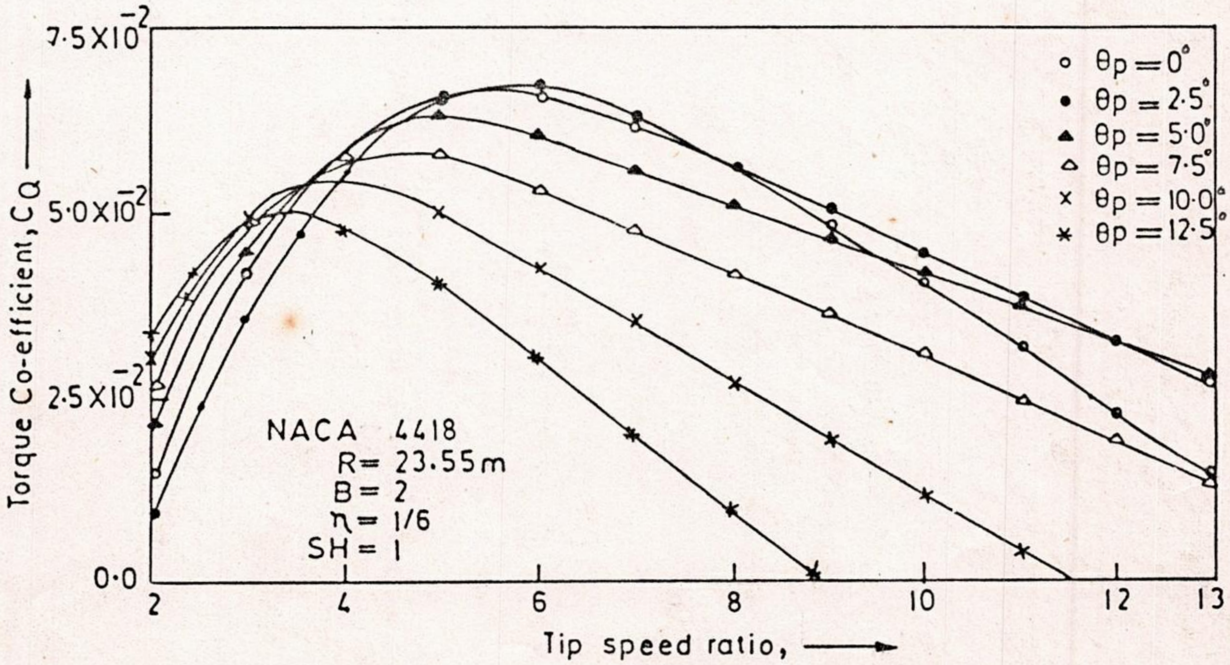

FIGURE 7.2.3: VARIATION OF TORQUE CO-EFFICIENT WITH TIP SPEED RATIO AT DIFFERENT PITCHINC ANGLES.

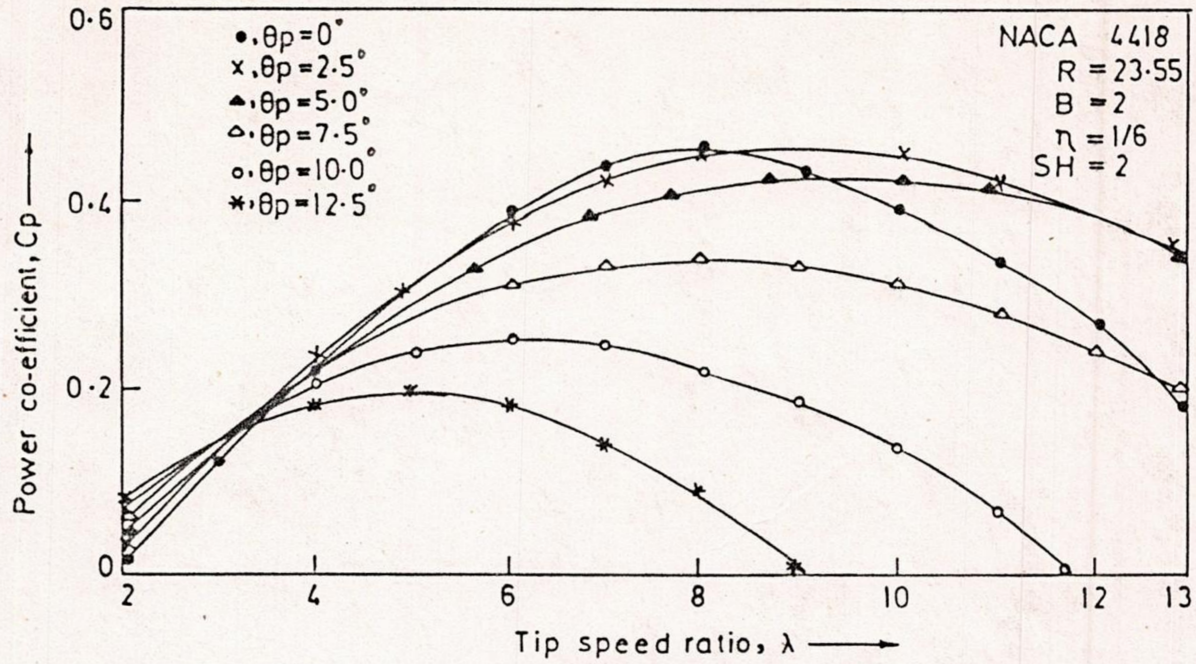

FIGURE 7·3·1: VARIATION OF POWER CO-EFFICIENT WITH TIP SPEED RATIO FOR LINEAR-CHORD-LINEAR-TWIST BLADE AT DIFFERENT PITCHING ANGLES.

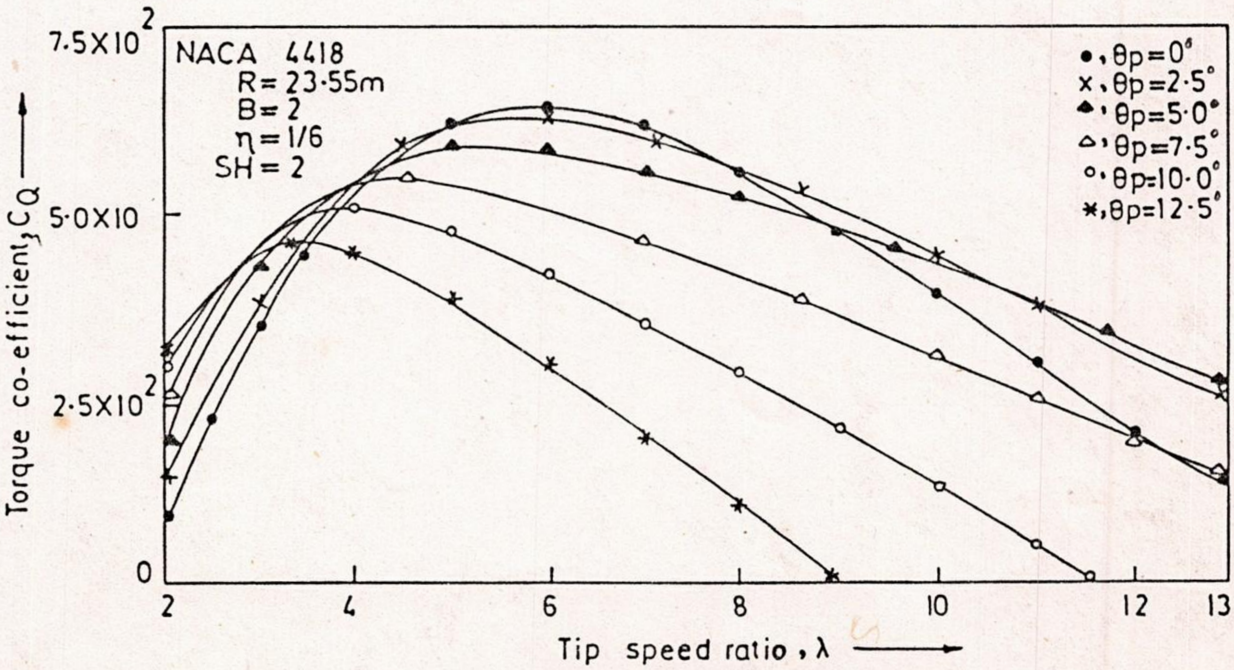

FIGURE 7·3·2: VARIATION OF TORQUE CO-EFFICIENT WITH TIP SPEED RATIO FOR LINEAR-CHORD-LINEAR-TWIST BLADE AT DIFFERENT PITCHING ANGLES.

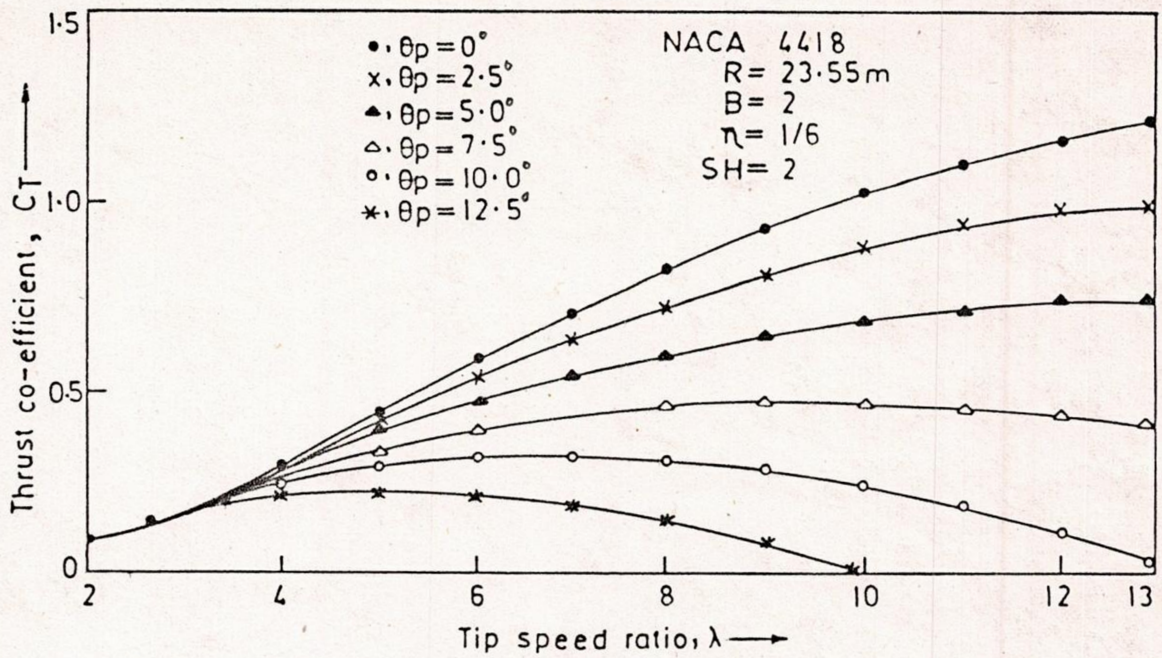

FIGURE 7·3·3: VARIATION OF THRUST CO-EFFICIENT WITH TIP SPEED RATIO FOR LINEAR-CHORD-LINEAR-TWIST BLADE AT DIFFERENT PITCHING ANGLES.

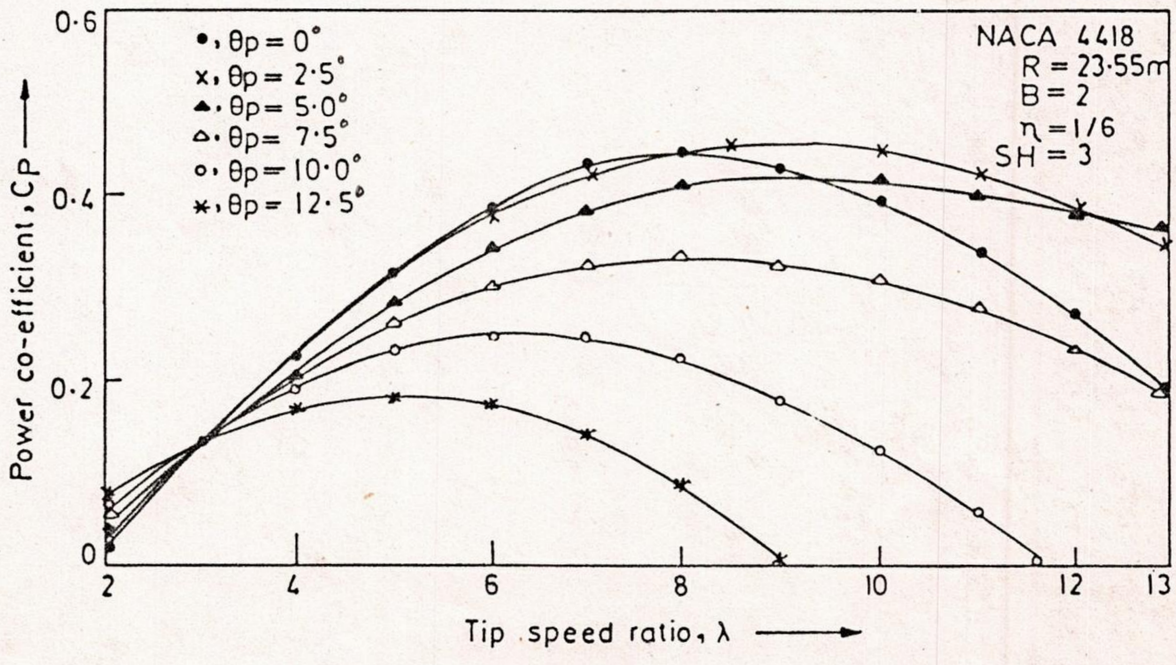

FIGURE 7·3·4: VARIATION OF POWER CO-EFFICIENT WITH TIP SPEED RATIO FOR LINEAR-CHORD-ZERO-TWIST BLADE AT DIFFERENT PITCHING ANGLES.

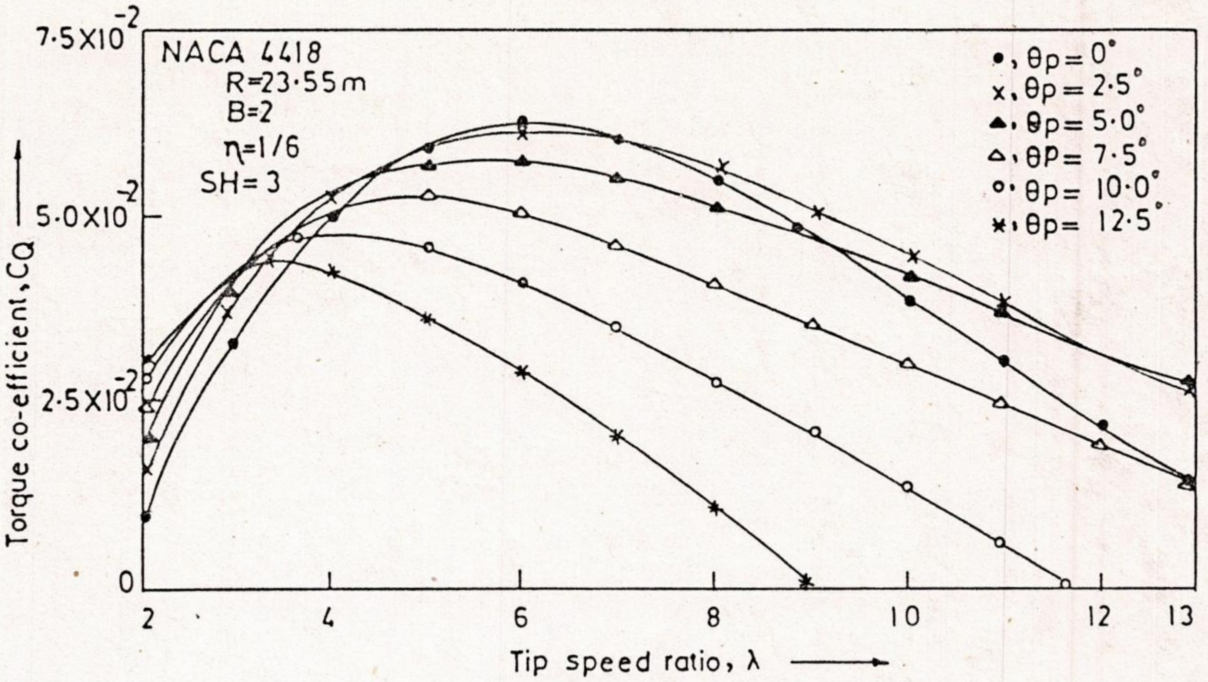

FIGURE 7·3·5: VARIATION OF TORQUE CO-EFFICIENT WITH TIP SPEED RATIO FOR LINEAR-CHORD-ZERO-TWIST BLADE AT DIFFERENT PITCHING ANGLES.

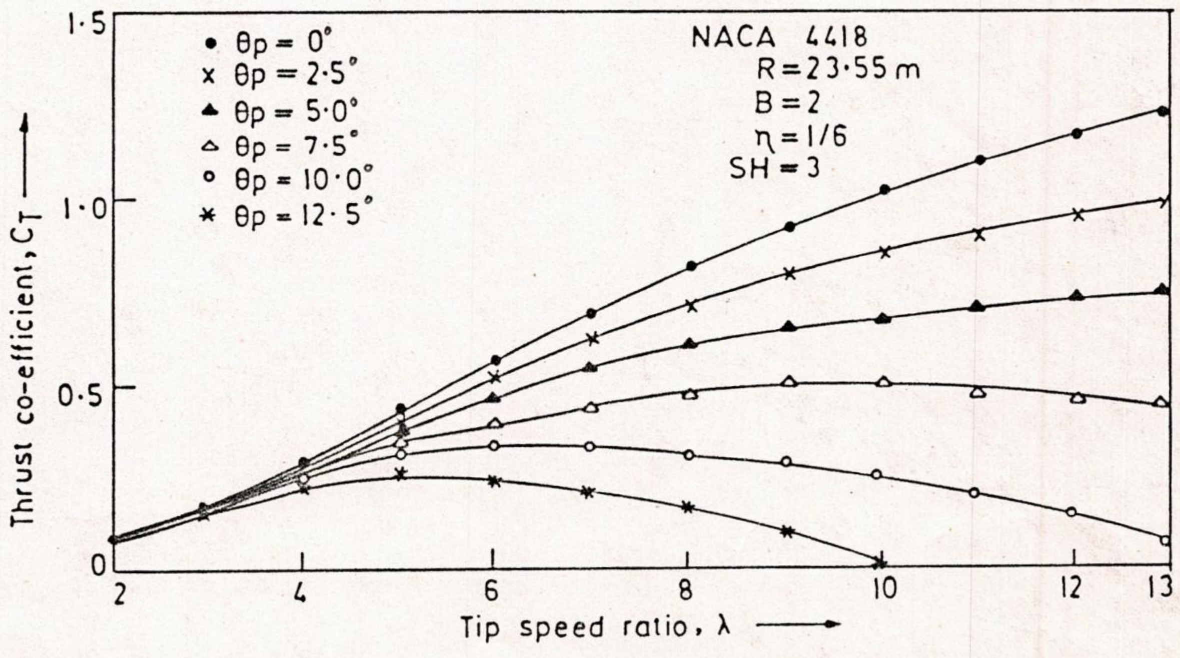

FIGURE 7·3·6: VARIATION OF THRUST CO-EFFICIENT WITH TIP SPEED RATIO FOR LINEAR-CHORD-ZERO-TWIST BLADE AT DIFFERENT PITCHING ANGLES.

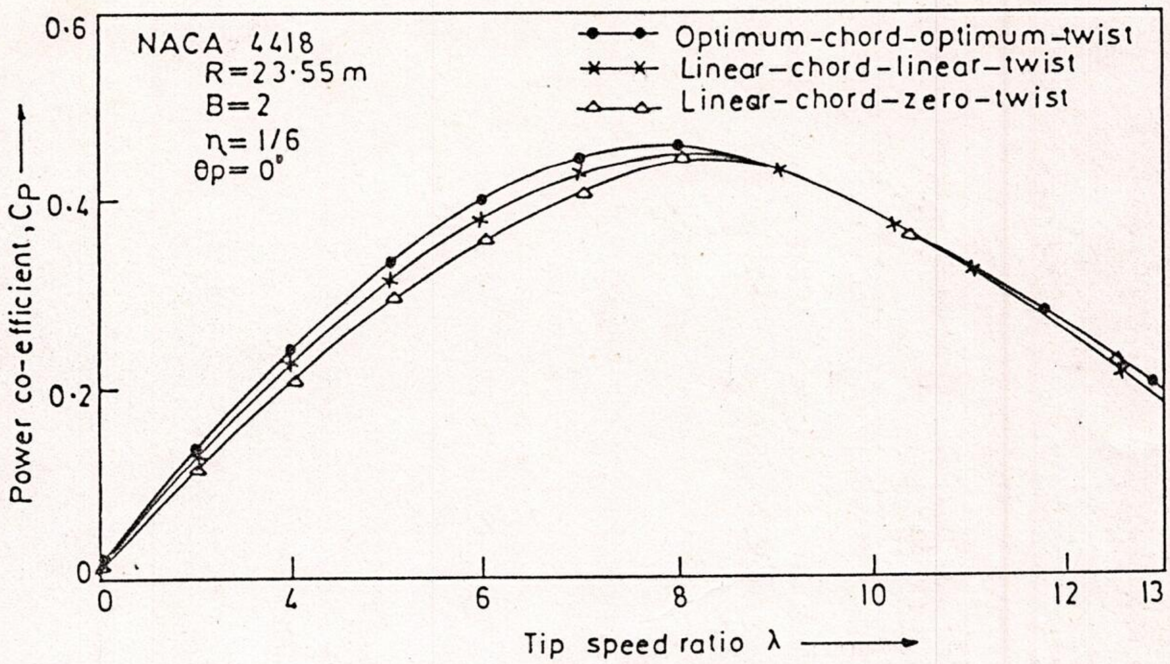

FIGURE 7·3·7 : VARIATION OF POWER CO-EFFICIENT WITH TIP SPEED RATIO FOR DIFFERENT BLADE SHAPES.

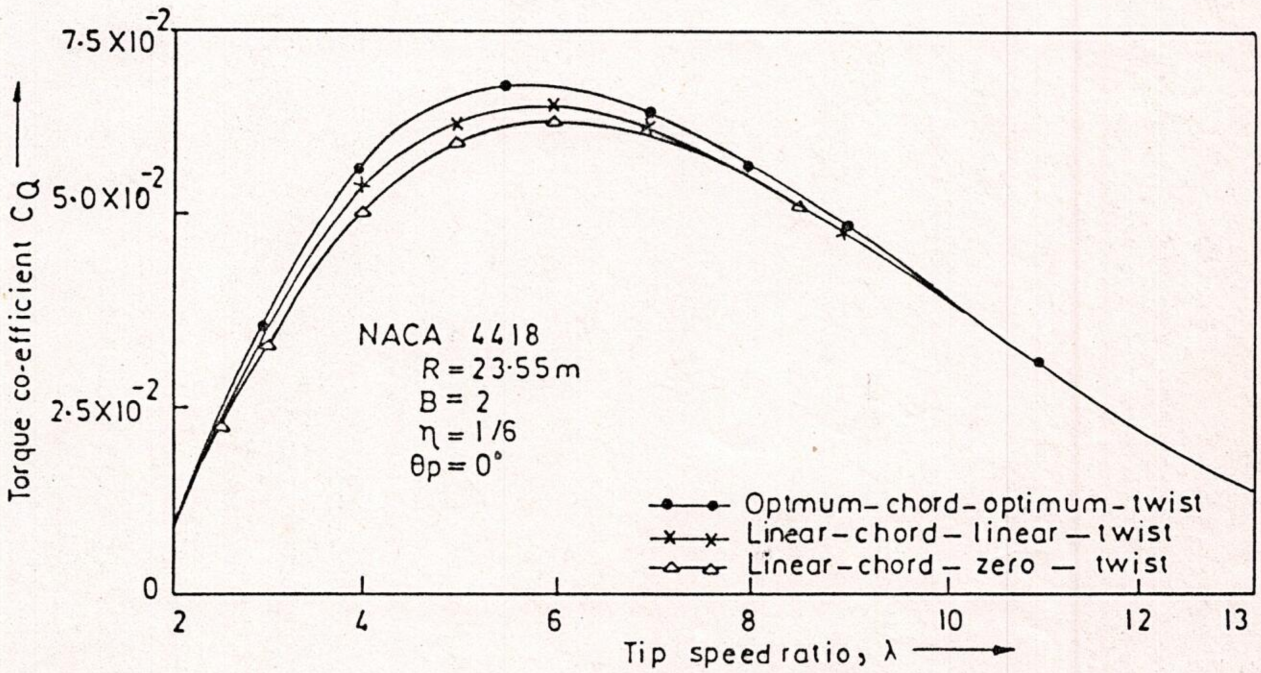

FIGURE 7·3·8 : VARIATION OF TORQUE CO-EFFICIENT WITH TIP SPEED RATIO FOR DIFFERENT BLADE SHAPES.

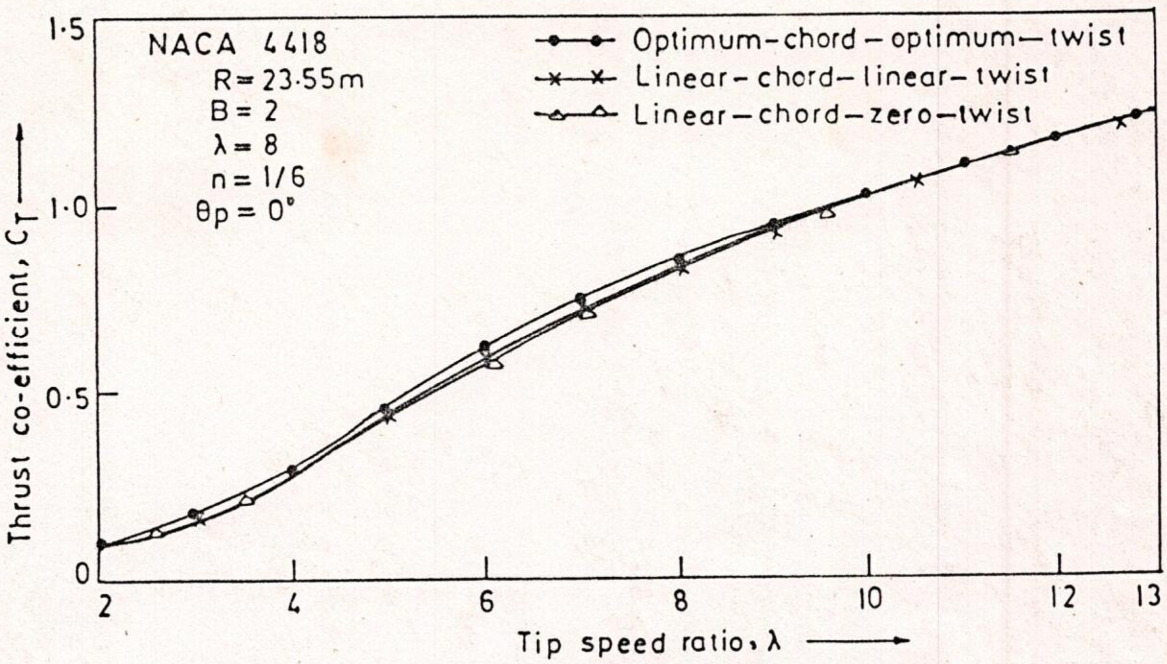

FIGURE 7·3·9: VARIATION OF THRUST CO-EFFICIENT WITH TIP SPEED RATIO FOR DIFFERENT BLADE SHAPES

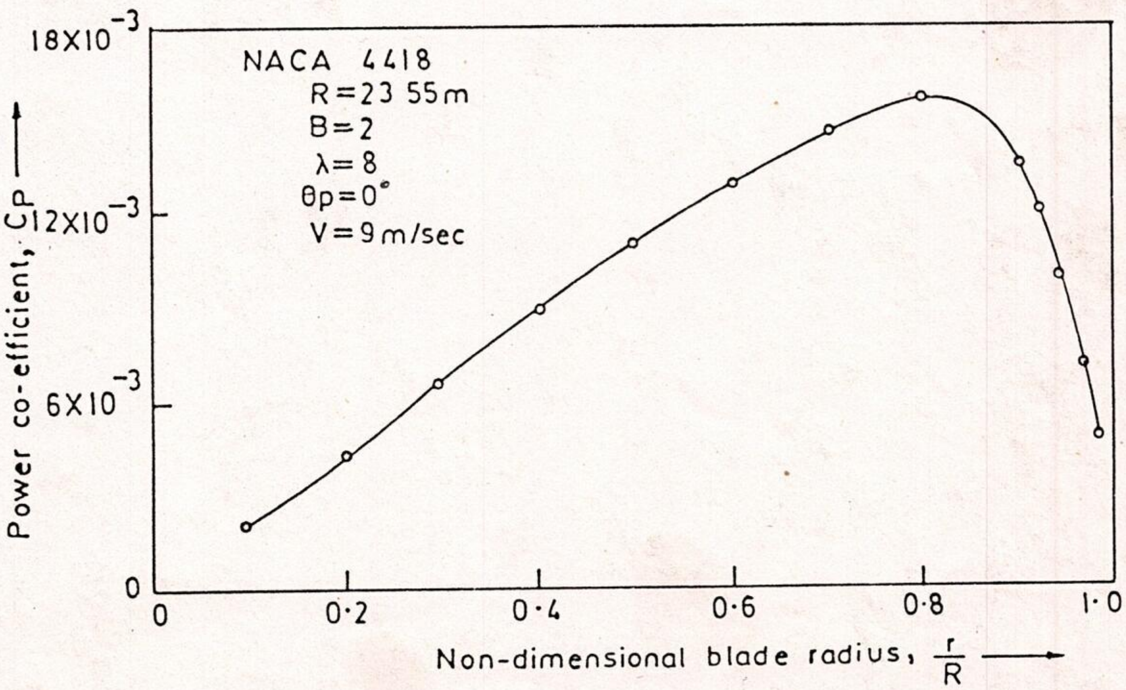

FIGURE 7·3·10: RADIAL VARIATION OF POWER CO-EFFICIENT

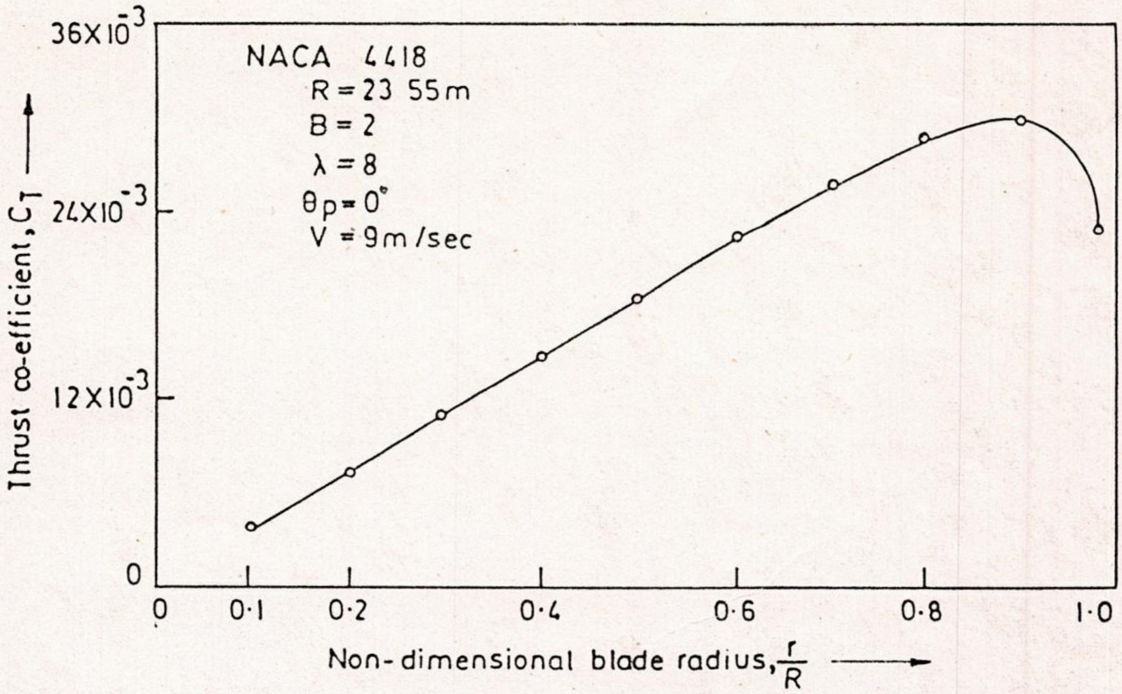

FIGURE 7·3·11: RADIAL VARIATION OF THRUST CO-EFFICIENT.

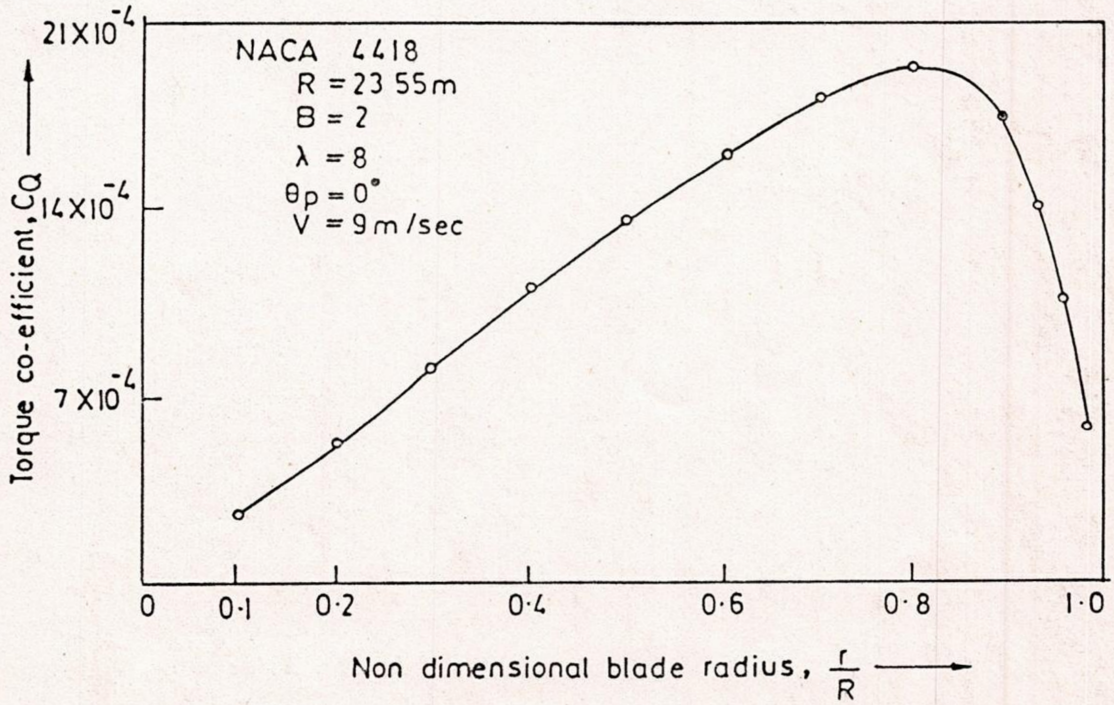

FIGURE 7·3·12: RADIAL VARIATION OF TORQUE CO-EFFICIENT.

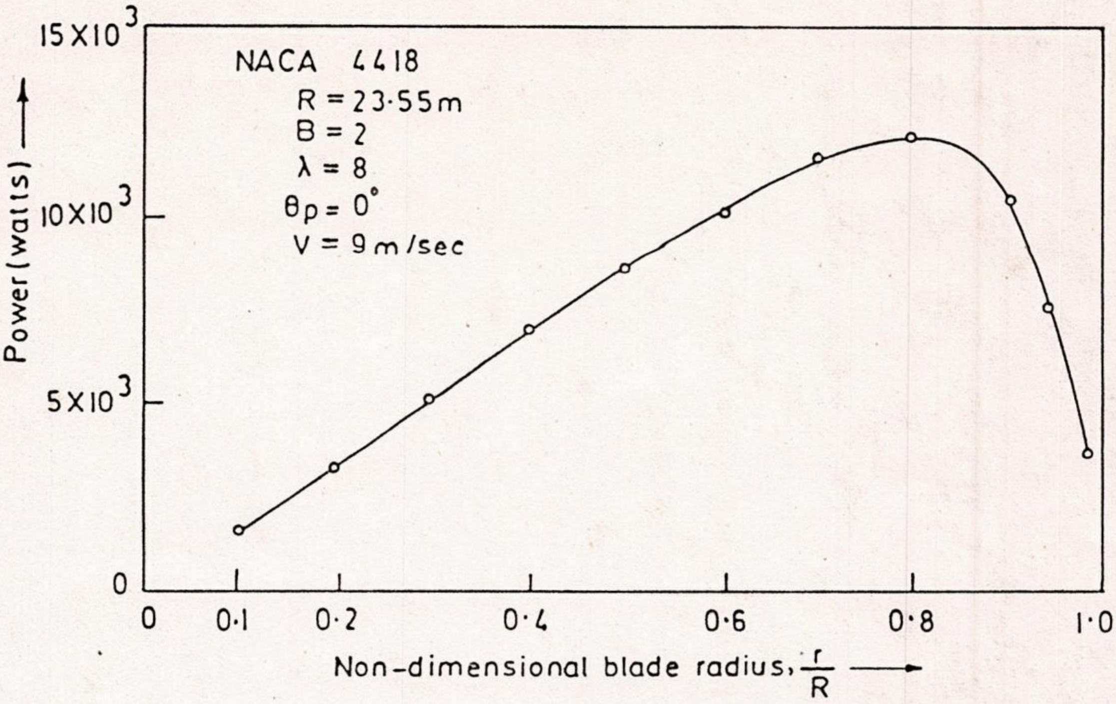

FIGURE 7·3·13: RADIAL VARIATION OF POWER.

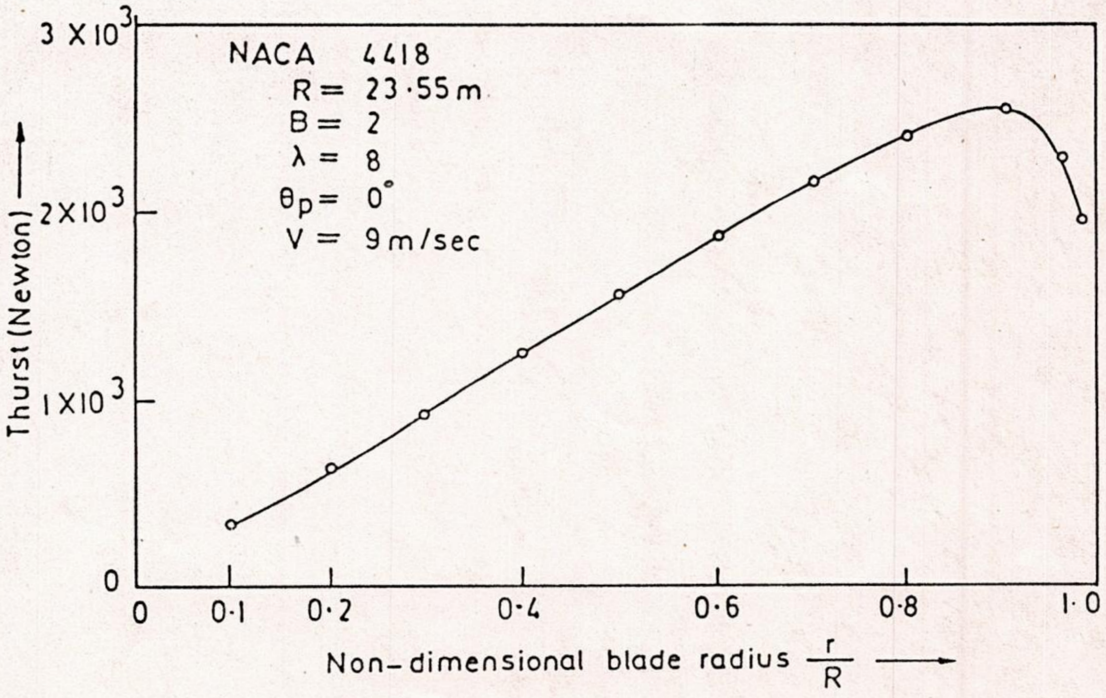

FIGURE 7·3·14: RADIAL VARIATION OF THRUST.

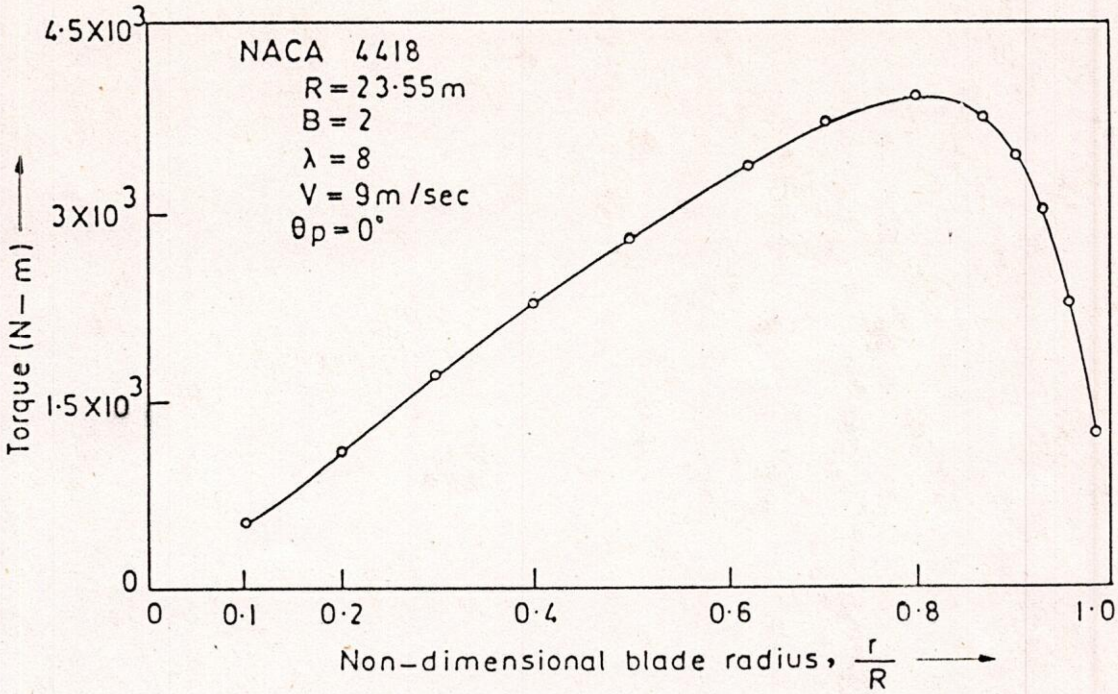

FIGURE 7·3·15: RADIAL VARIATION OF TORQUE.

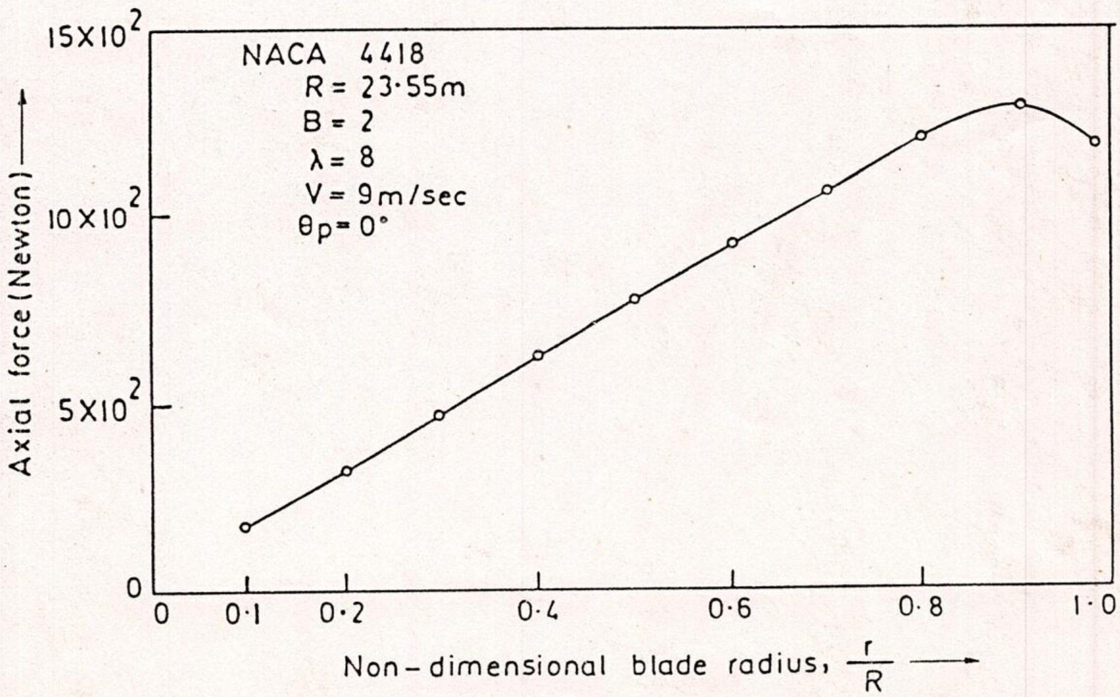

FIGURE 7·3·16: RADIAL VARIATION OF AXIAL FORCE.

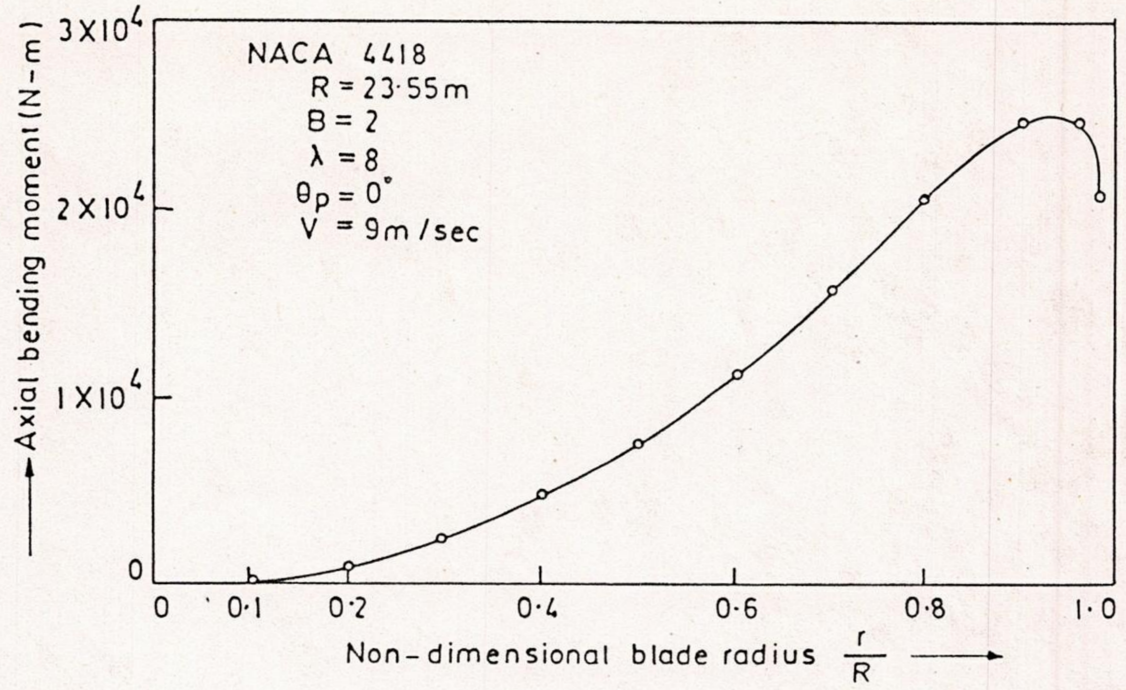

FIGURE 7·3·17: RADIAL VARIATION OF AXIAL BENDING MOMENT.

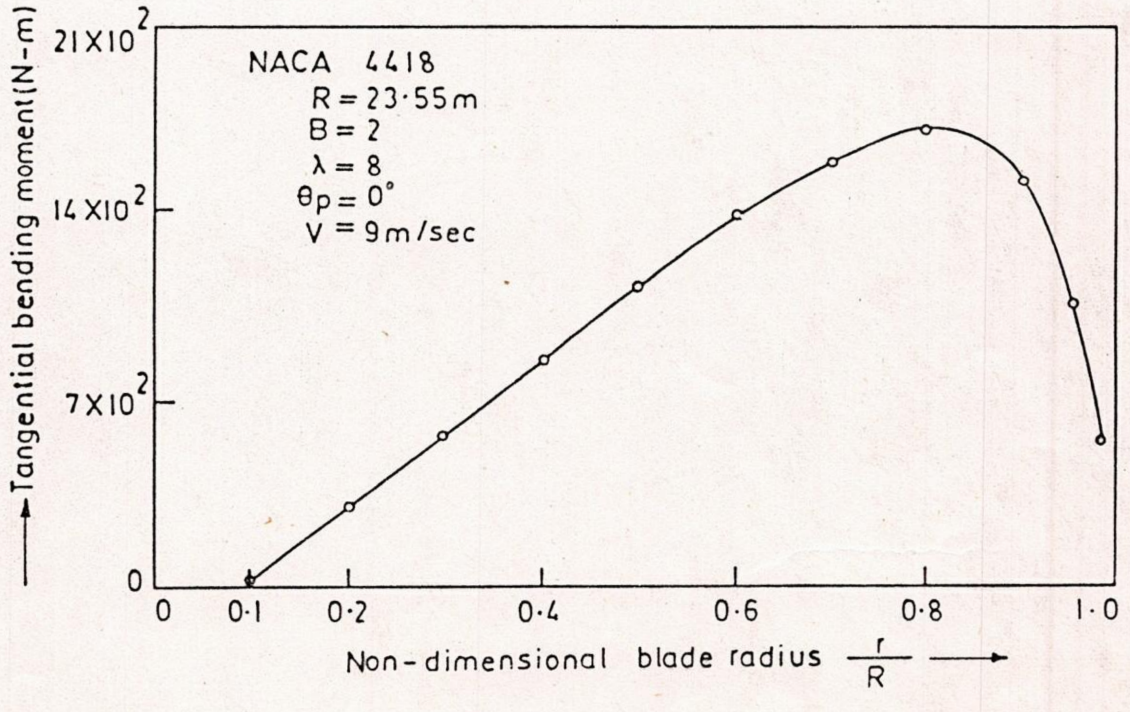

FIGURE 7·3·18: RADIAL VARIATION OF TANGENTIAL BENDING MOMENT.

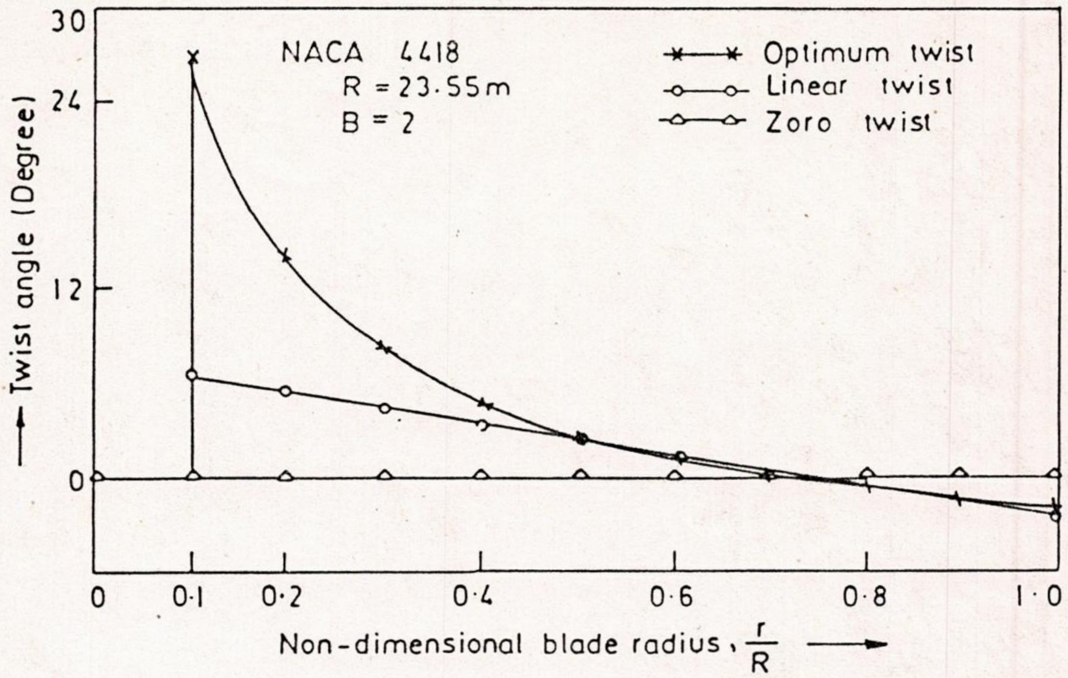

FIGURE 7·3·19: OPTIMUM AND LINEARIZED BLADE TWIST DISTRIBUTION.

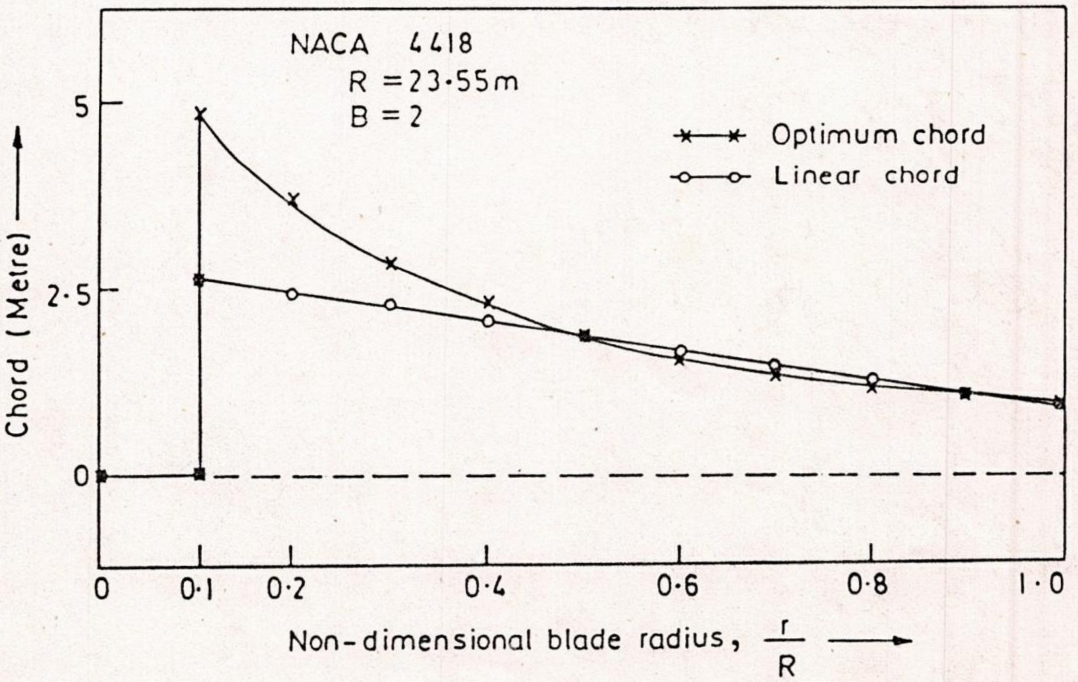

FIGURE 7·3·20: OPTIMUM AND LINEARIZED BLADE CHORD DISTRIBUTION.

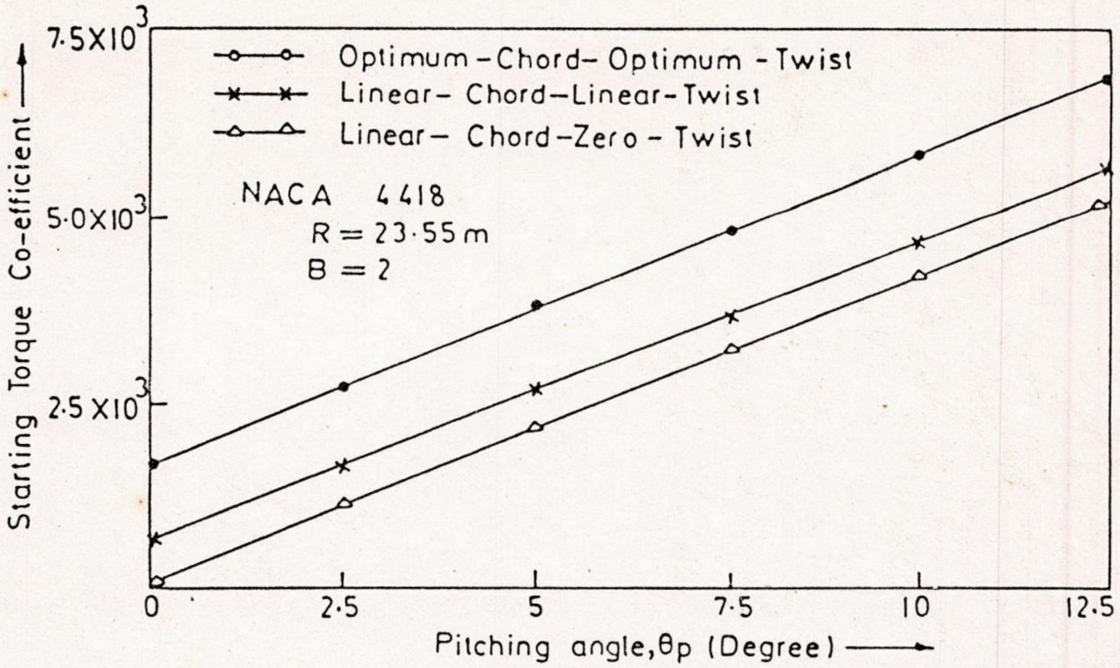

FIGURE 7.3.21: COMPARISON OF STARTING TORQUE CO-EFFICIENTS FOR DIFFERENT BLADE SHAPES WIND TURBINES.

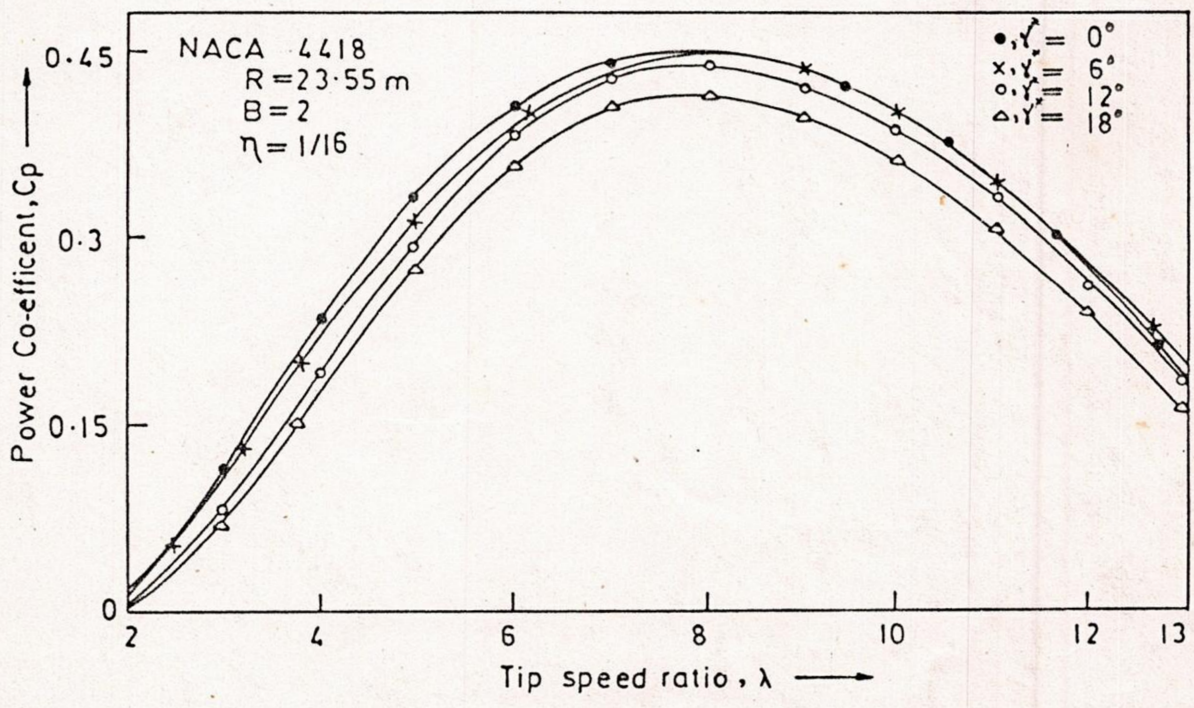

FIGURE 7.4.1: EFFECT OF YAW ANGLES ON POWER CO-EFFICIENT.

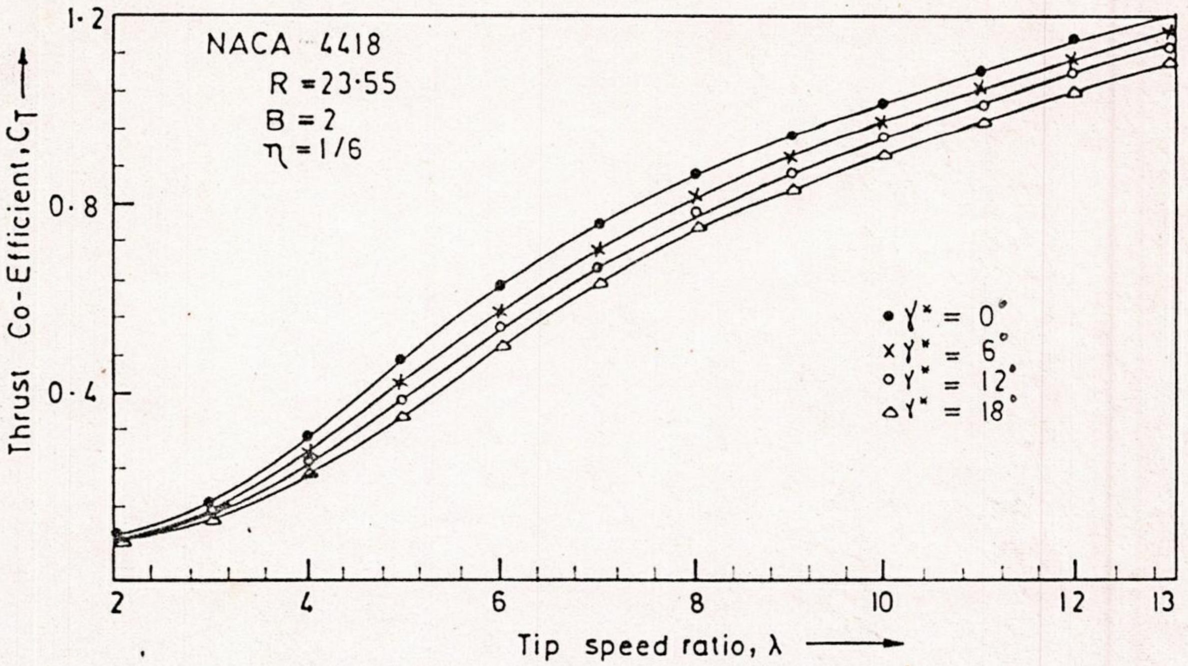

FIGURE 7.4.2: EFFECT OF YAW ANGLES ON THRUST CO-EFFICIENT.

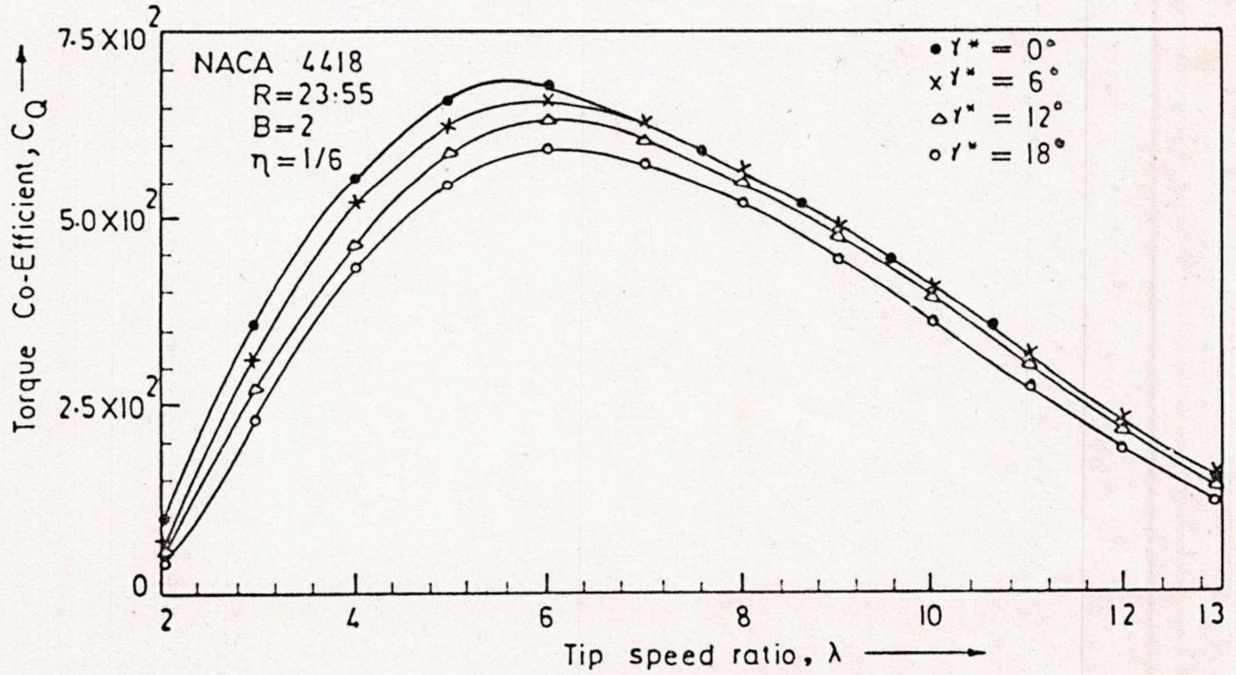

FIGURE 7·4·3: EFFECT OF YAW ANGLES ON TORQUE CO-EFFICIENT.

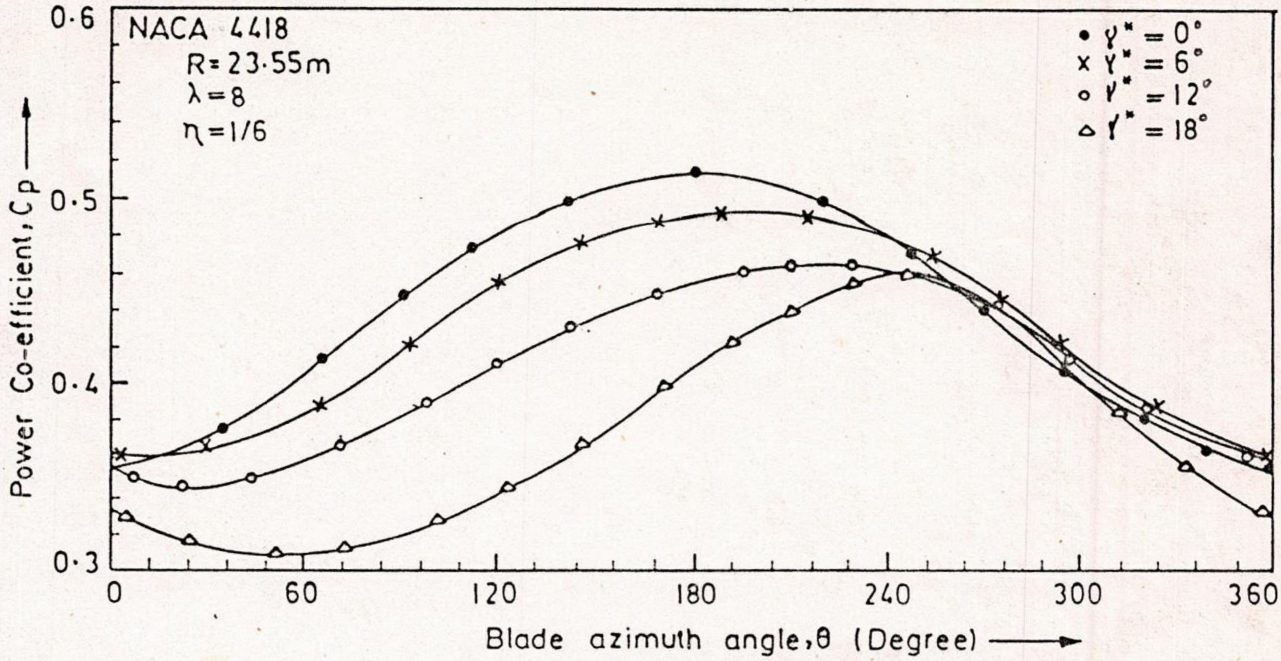

FIGURE 7.4.4: POWER CO-EFFICIENT AS A FUNCTION OF BLADE ANGULAR POSITION AT DIFFERENT YAWING ANGLES.

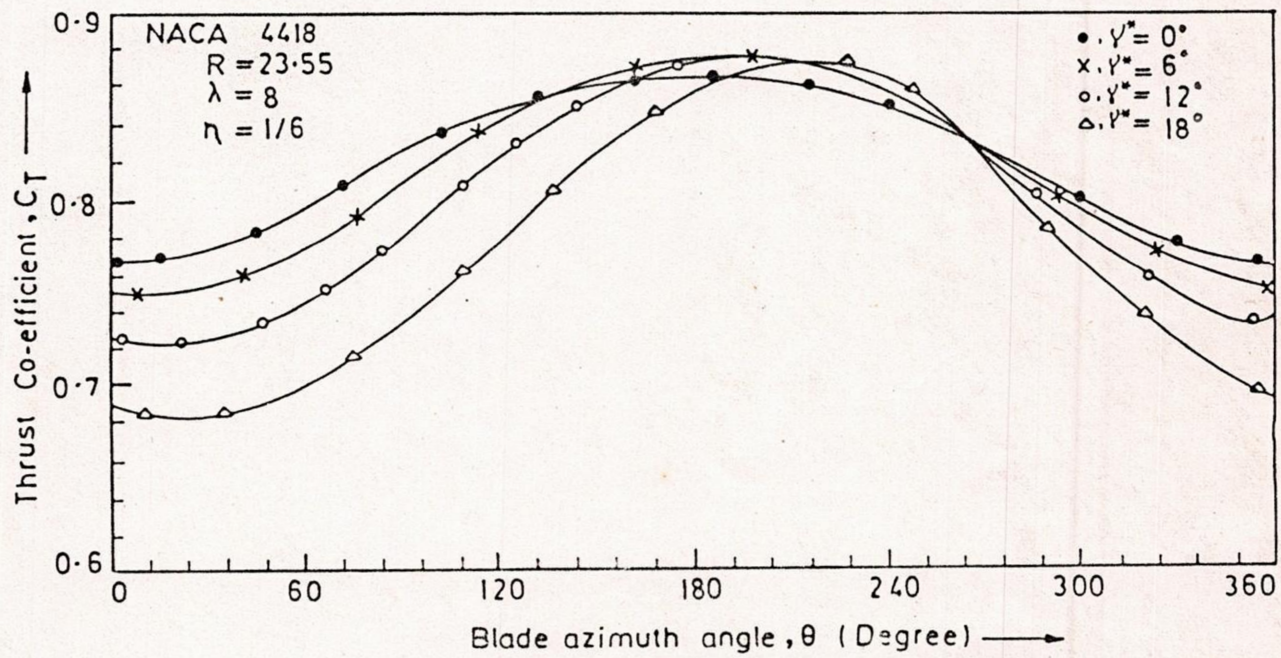

FIGURE 7.4.5: THRUST CO-EFFICIENT AS A FUNCTION OF BLADE ANGULAR POSITION AT DIFFERENT YAWING ANGLES.

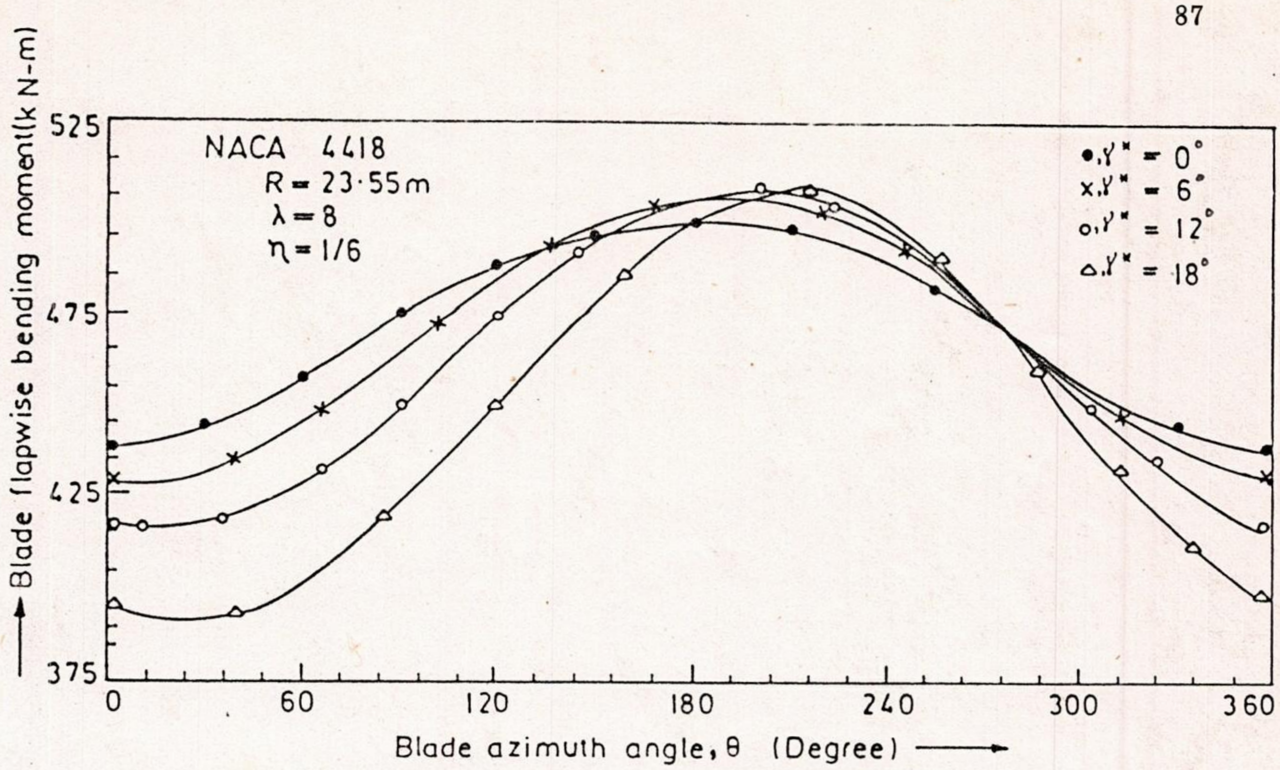

FIGURE 7·4·6: BLADE FLAPWISE BENDING MOMENT AS A FUNCTION OF BLADE ANGULAR POSITION AT DIFFERENT YAWING ANGLES.

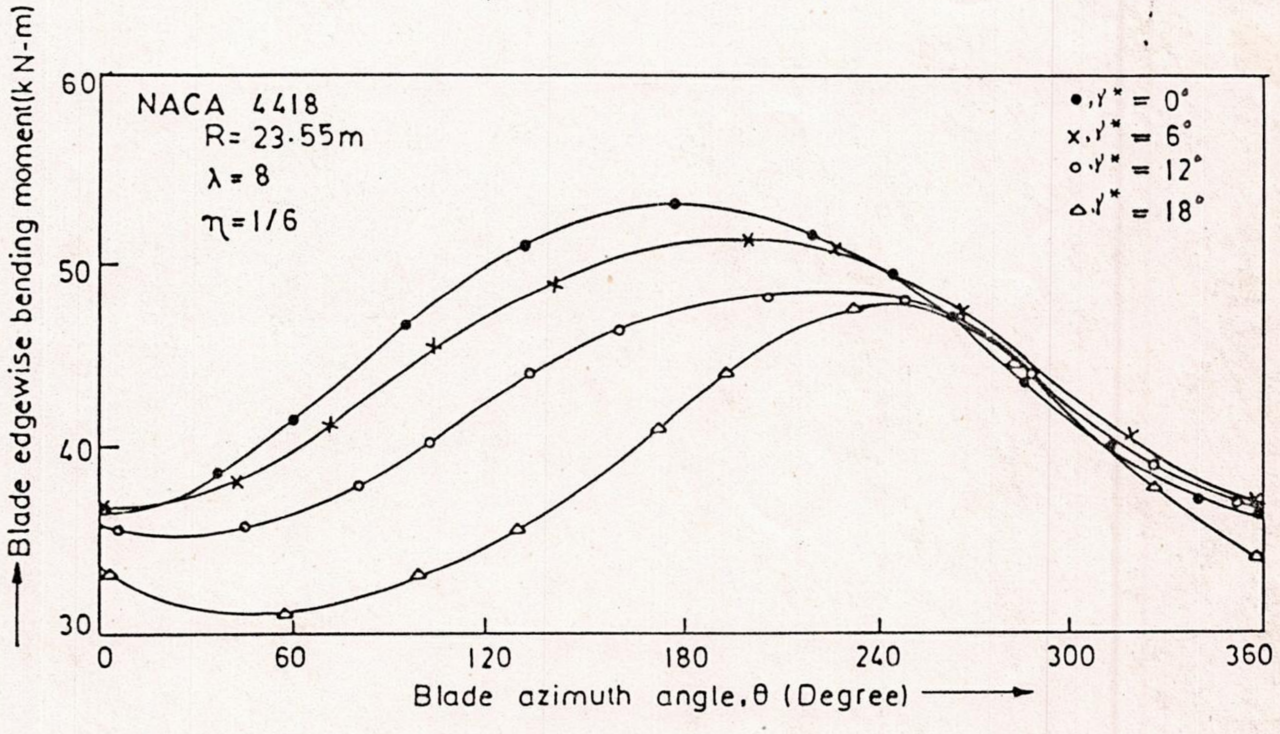

FIGURE 7·4·7: BLADE EDGEWISE BENDING MOMENT AS A FUNCTION OF BLADE ANGULAR POSITION AT DIFFERENT YAWING ANGLES.

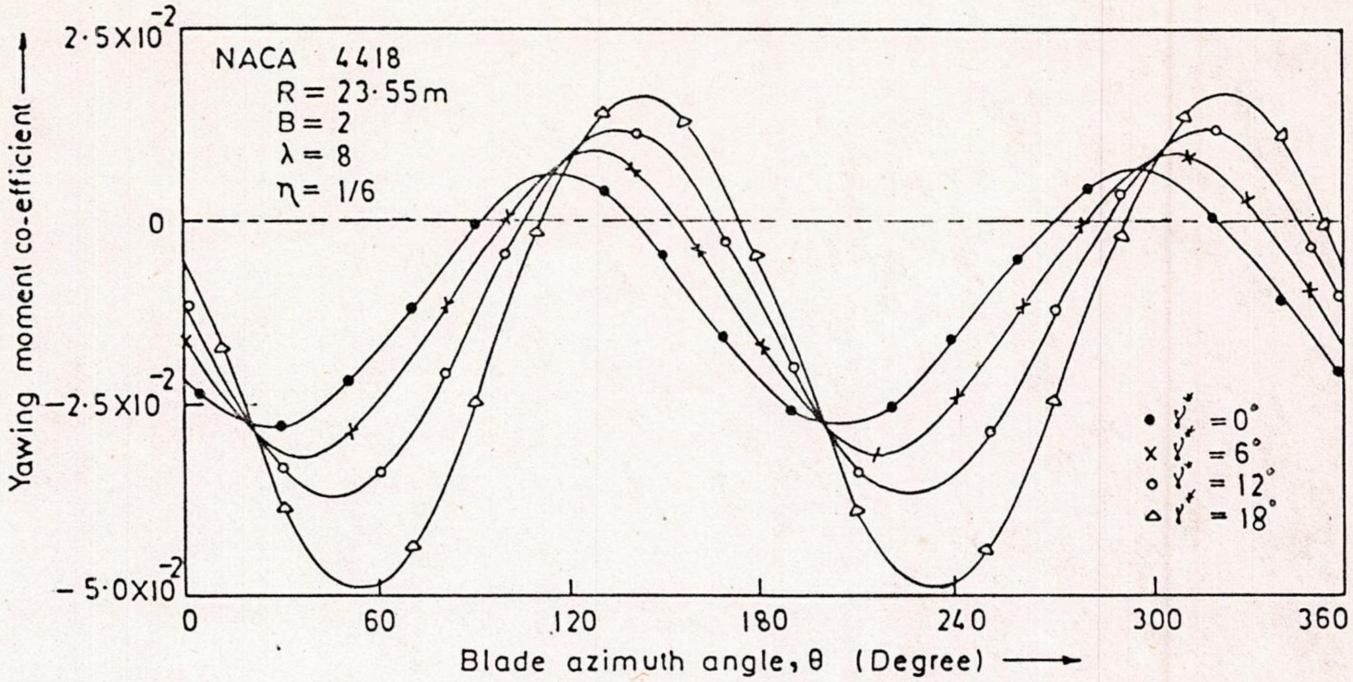

FIGURE 7·4·8: EFFECT OF YAWING ANGLES ON YAWING MOMENT CO-EFFICIENT DURING ONE REVOLUTION.

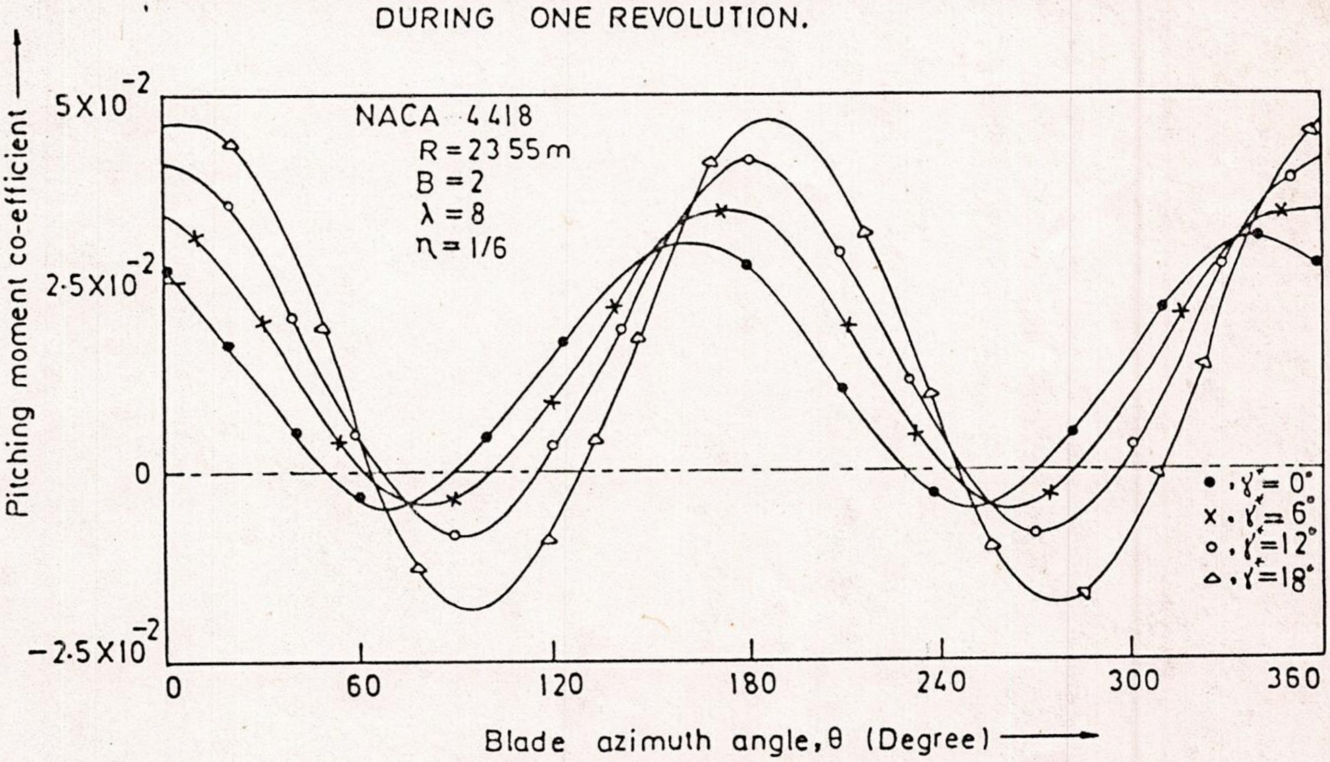

FIGURE 7·4·9: EFFECT OF YAWING ANGLES ON PITCHING MOMENT CO-EFFICIENT DURING ONE REVOLUTION.

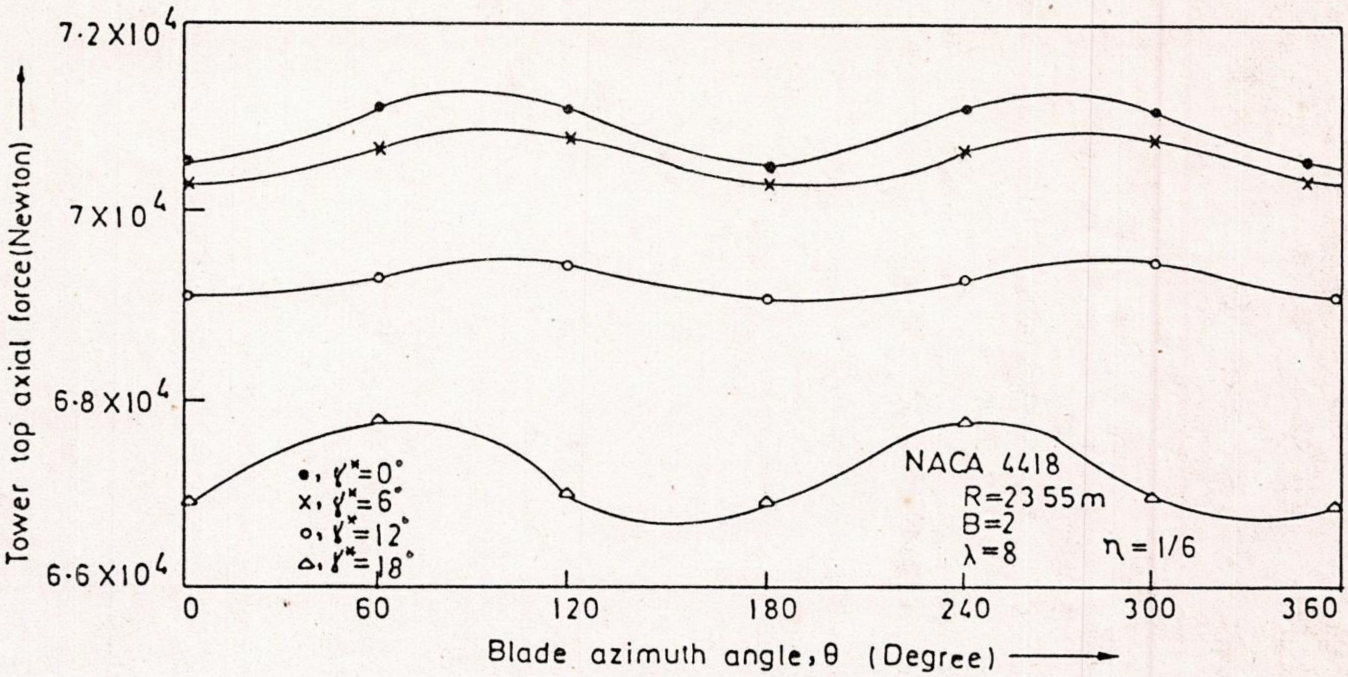

FIGURE 7·4·10: TOWER TOP AXIAL FORCE DURING ONE REVOLUTION AT DIFFERENT YAWING ANGLES.

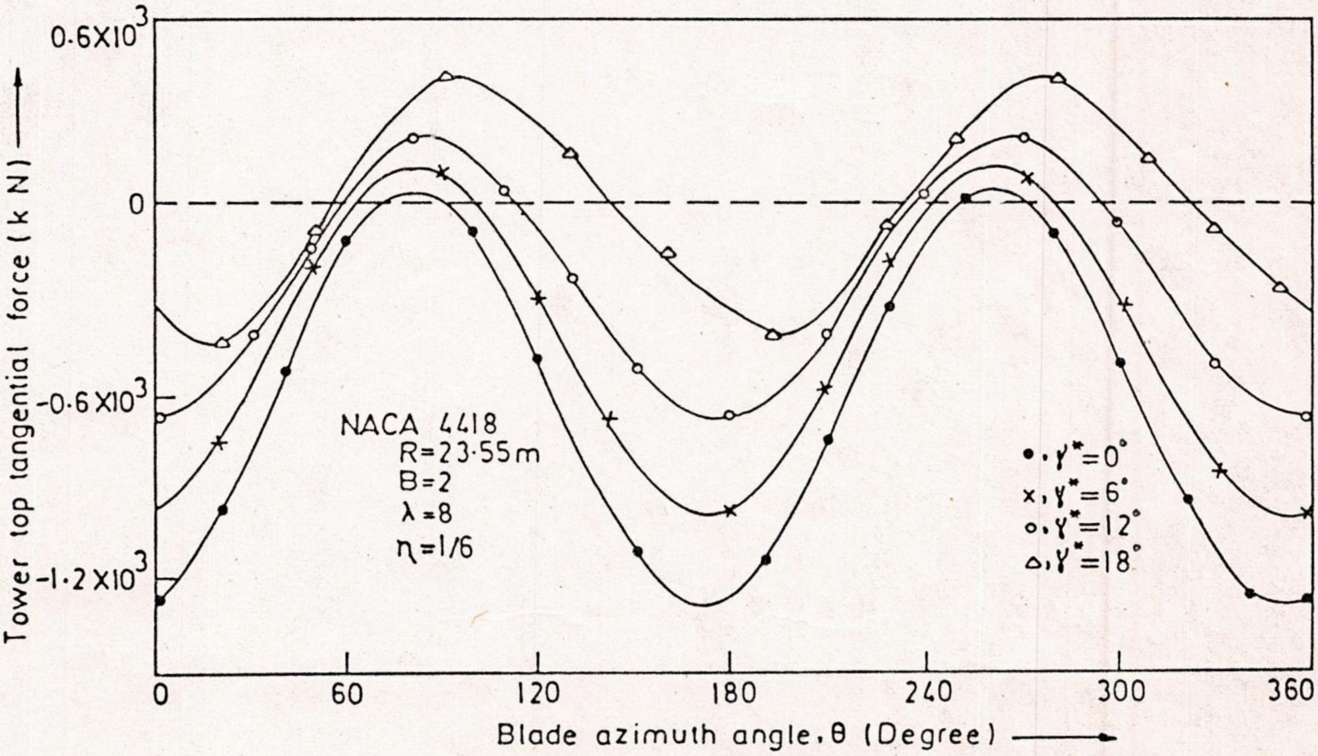

FIGURE 7·4·11: EFFECT OF YAWING ANGLE ON TOWER TOP TANGENTIAL FORCE DURING ONE REVOLUTION.

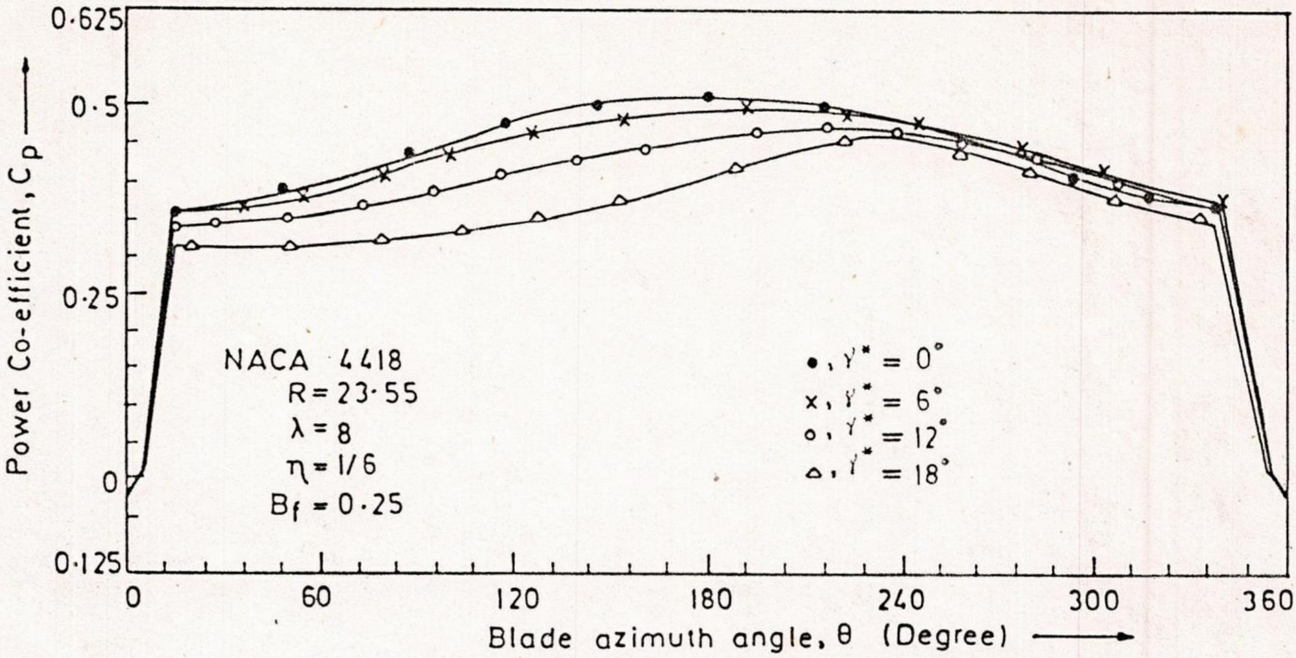

FIGURE 7·5·1: EFFECT OF TOWER SHADOW ON POWER CO-EFFICIENT AT DIFFERENT YAW ANGLES DURING ONE REVOLUTION.

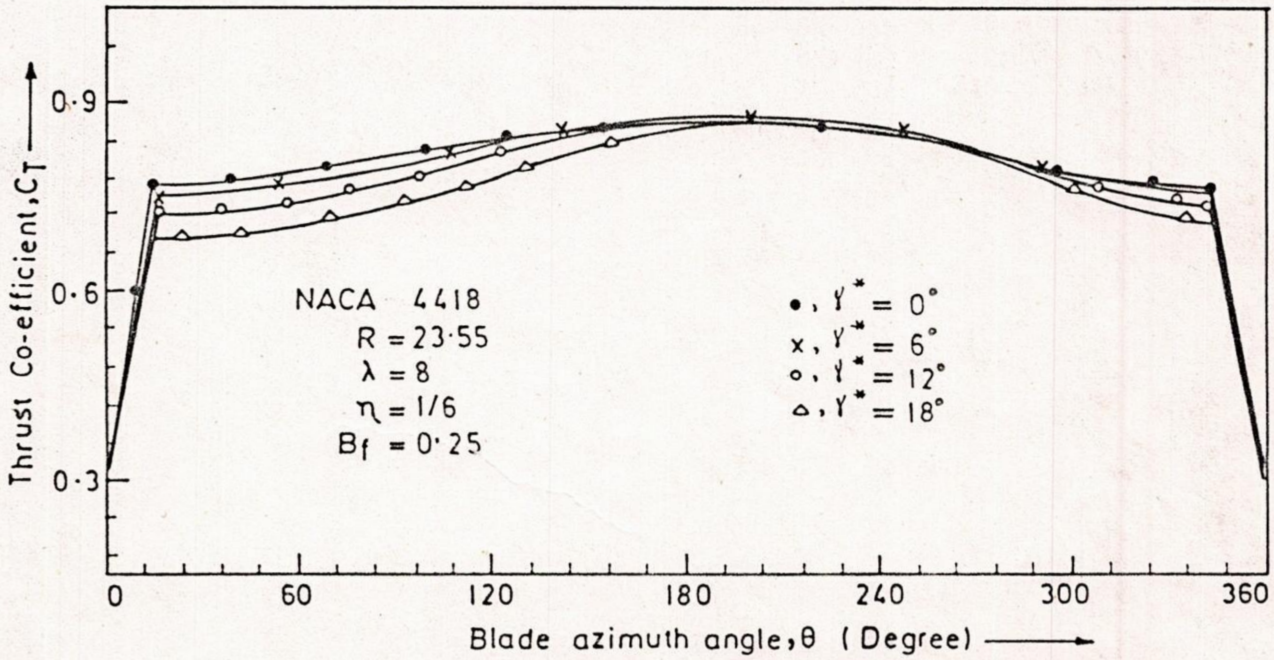

FIGURE 7·5·2: VARIATION OF THRUST CO-EFFICIENT DUE TO TOWER SHADOW AND YAWING ANGLES DURING ONE REVOLUTION.

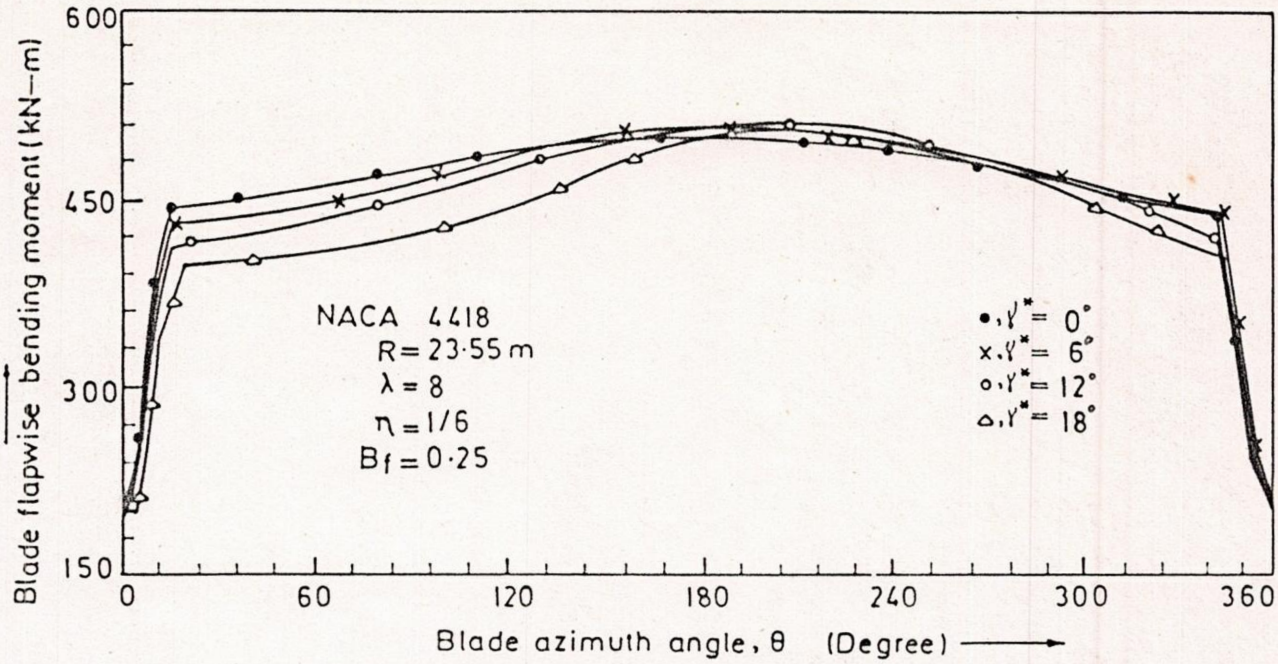

FIGURE 7.5.3: EFFECT OF TOWER SHADOW ON BLADE FLAPWISE ROOT BENDING MOMENT AT DIFFERENT YAW ANGLES DURING ONE REVOLUTION.

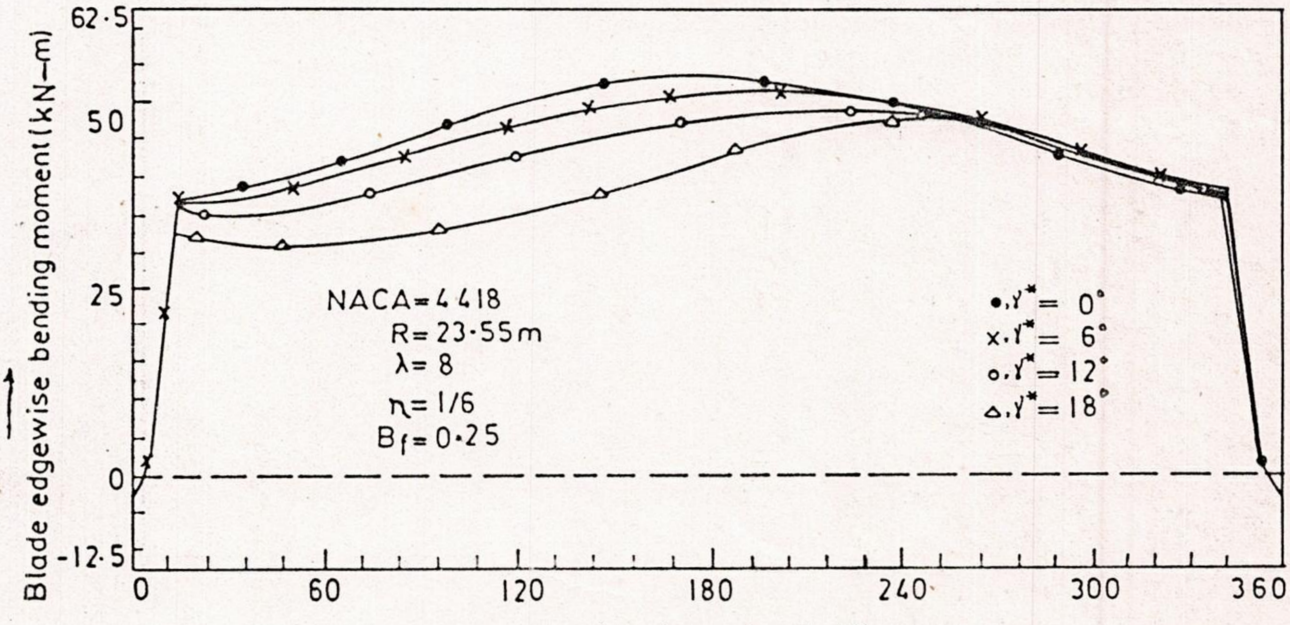

FIGURE 7.5.4: EFFECT OF TOWER SHADOW ON BLADE EDGEWISE BENDING MOMENT AT DIFFERENT YAW ANGLES DURING ONE REVOLUTION.

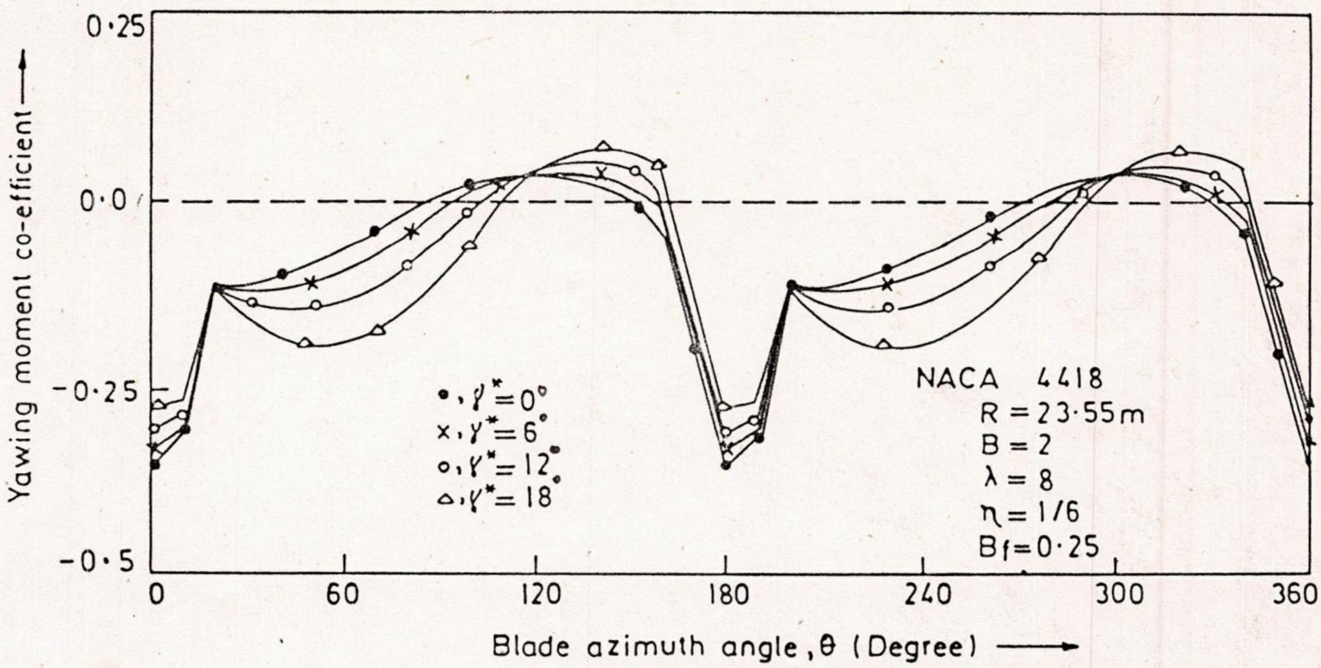

FIGURE 7·5·5: EFFECT OF TOWER SHADOW ON YAWING MOMENT CO-EFFICIENT AT DIFFERENT YAW ANGLES DURING ONE REVOLUTION.

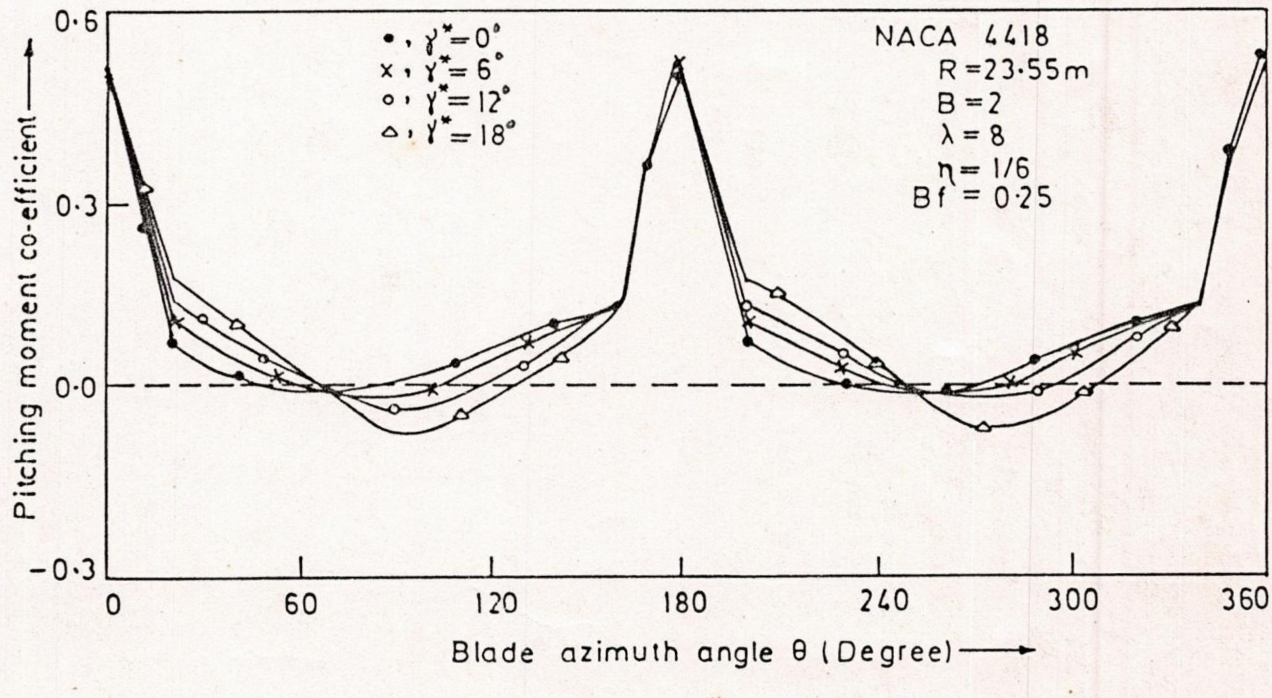

FIGURE 7.5 ·6: VARIATION OF PITCHING MOMENT CO-EFFICIENT DUE TO TOWER SHADOW AND YAW ANGLES DURING ONE REVOLUTION.

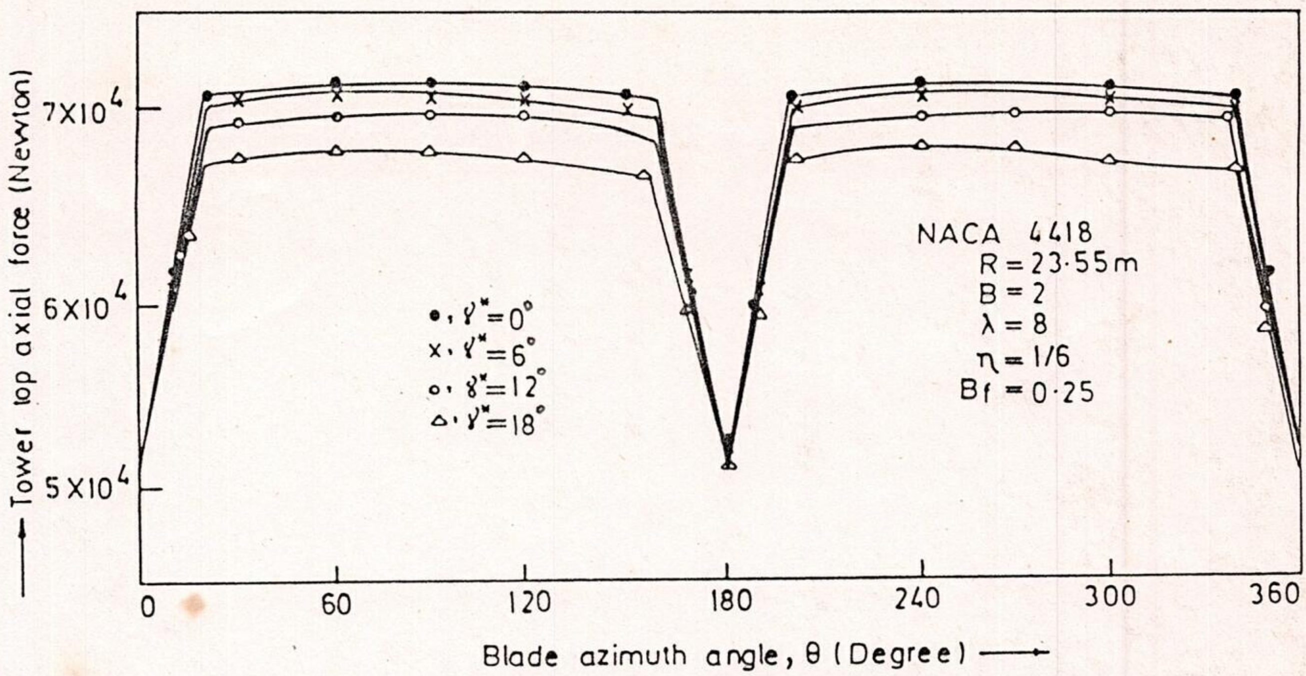

FIGURE 7·5·7: EFFECT OF TOWER SHADOW ON TOWER TOP AXIAL FORCE AT DIFFERENT YAW ANGLES DURING ONE REVOLUTION.

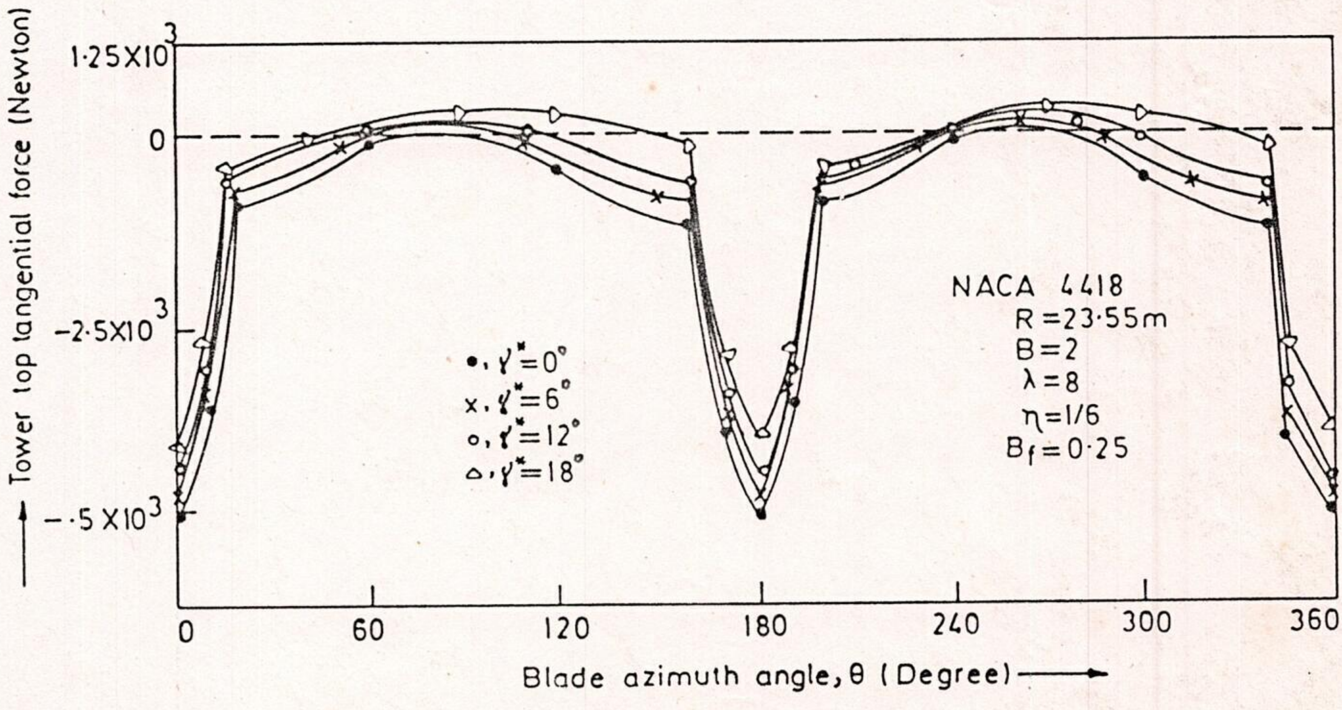

FIGURE 7·5·8: EFFECT OF TOWER SHADOW ON TOWER TOP TANGENTIAL FORCE AT DIFFERENT YAW ANGLES DURING ONE REVOLUTION.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study is to find out a suitable method to calculate the overall design and performance analysis for a horizontal axis wind turbine utilizing a simplified method. Most of existing design programs are considered separately the important effects of wind shear, wind shift, coning angle, tilting angle, azimuth angle, and tower shadow. But the present analysis considers all the effects of these parameters combinedly. The theoretical results obtained by the present method are studied thoroughly. From the study the following conclusions may be drawn,

1. In the designing of a horizontal axis wind turbine the combined influence of coning, tilting, azimuthal, and wind shear, wind shift and tower shadow should be considered.

2. By the existing blade element theory, a rotor blade can be designed that produces an optimum C_P at a fixed value of the tip speed ratio. A first condition to get optimum C_P is to choose the maximum value of lift and drag ratio $(C_L/C_D)_{max.}$ of the given profile and to keep it constant along the entire blade span. That is, the corresponding C_L and α, is to be taken constant along the entire span. A second condition is that, the continuously varing blade chord and twist angle along the entire length of the blade are accepted. This gives very complicated rotor blade that is expensive to manufacture and may not have structural integrity. It is possible to achieve very close value of the optimum C_P by considering linear-chord linear-twist blade. The power difference between a ideal blade and zero twist blade only about 6% to 10%. This might be acceptable for a single production unit, but looses its attraction in case of mass production. Choosing an untwisted blade design, the constant chord blade seems rather attractive, because the blade area in the hub region is reduced, where the blade stall starts at relatively high values of λ.

3. For two-bladed horizontal axis wind turbine operating with uniform velocity and without any disturbances the loads will be steady. The introduction of wind shear and a certain yawing angle will cause each blade to experience a periodic force and moment.

4. With the variation of blade pitch angle θ_P, the maximum power reduces but increases the power available at relatively low tip speed ratios. The power coefficient Versus tip speed ratio curves are sensitive to blade pitch angle in the stalling region. To avoid too much drop off, of power after stalling region, blade pitch angle between 2.5° to 5° are seem to be more convenient than the performance at

0° pitch angle.

5. The minimum clearence between blade and tower depends on the coning angle and the dynamic response of the blade on sudden gusts. This clearence creates a larger problem in case of upwind turbines and reduced by tilting the rotor. The downwind rotor has the disadvantages of the fatigue load due to tower blockage. Again, the tilted rotor shows the disadvantages of fluctuating loads due to misalignment with the wind direction.

6. The present study shows that large thrust, power, torque, moments and tower top forces variation can occur as the blade passes through the tower shadow. The magnitude of this variations is a function of the amount of flow blockage occuring and duration of the turbine blade remains in the tower shadow. The results show that studies of future wind turbines should include a careful evaluation of the tower shadow effect to ensure that the rotor blades and support system can withstand the stresses set up as the blades transit through the varying wind environment of tower shadow. Works need to be done on wake modelling of specific tower designs.

7. The effect of tower shadow on turbine noise is another area that needs additional study. The vibration of the tower must be considered in relation to blade vibrations because of the possibility of its natural period of oscillation coinciding with that of some of the alternating forces on the blades.

8. In non-axial flow, even at small angles of yaw, the cyclic variation in the forces and moments at the blade root could lead to resonance in either the blade, or the supporting structure and possibly reduce the lifetime of the turbine. The effects of non-axial flow therefore need to be considered in the design of horizontal axis wind turbines.

9. Power losses due to aerodynamic profile drag can be reduced by increasing the rotor solidity and reducing tip speed ratio, but at the expense of increased blade weight and cost. Improvements in airfoil lift and drag ratio will permit reduced solidity and higher tip speed ratios.

10. To start a low speed rotor that has high internal resistance primarily requires a high pitch angle. Usually the internal resistance decreases as the device started and accelerated. So after attainment to a certain angular speed, the pitch angle is to be reduced to the required value.

11. Location of rotor, whether, it will be upwind or downwind, has a significant effects on yaw stability of the horizontal axis wind turbines.

12. Yaw has a significant effect upon the power developed by the

rotor. This effect can be approximated for yaw angles less than 30° by defining rotor power as a function of the wind velocity normal to the rotor plane cubed or equal to f[Cosγ]^3.

Present world of energy crisis, demands large scale wind power generating systems with sufficient aerodynamic design, lightweight, mechanically simple design, requiring a minimum maintanance, low initial and operating costs but maximum power output per unit cost.

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APPENDICES

APPENDIX-A

Local Frame of Reference

To calculate aerodynamic forces acting on the rotor,several coordinate systems are introduced in the present analysis. These frames include a reference frame S_o, fixed at the top of the tower of the wind turbine with Z_o being the vertical axis and (X_o,Y_o) forming horizontal plane. A second non-rotating frame S_1 is fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance Y and a rotation of tilting angle α_T around the X_o axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally, a local reference frame S_3 is attached to a particular point of the blade at a distance r from the hub and is rotated over a coning angle ß. These are shown in Figures (A.1) to (A.4). The relationships between the reference frames can be expressed as,

$$S_1=[K_T]S_o$$

$$S_2=[K_\theta]S_1$$

$$S_3=[K_\beta]S_2$$

and inversely,

$$S_0=[K_T]^TS_1$$

$$S_1=[K_\theta]^TS_2$$

$$S_2=[K_\beta]^TS_3$$

the superscript T indicates the transposed matrix. The transformation matrices are,

$$\text{for tilting,}\quad K_T=\begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_T & -\sin\alpha_T \\ 0 & \sin\alpha_T & \cos\alpha_T \end{vmatrix} \tag{A.1}$$

$$\text{for azimuth,}\quad K_\theta=\begin{vmatrix} \cos\theta_k & 0 & \sin\theta_k \\ 0 & 1 & 0 \\ -\sin\theta_k & 0 & \cos\theta_k \end{vmatrix} \tag{A.2}$$

$$\text{for coning,} \quad K_\beta = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{vmatrix} \tag{A.3}$$

Thus a point on the blade can be expressed in the local S_3 coordinate system as,

$$\overleftarrow{r}_{s3} = \begin{vmatrix} X_3 \\ Y_3 \\ Z_3 \end{vmatrix} \tag{A.4}$$

Considering the non-rotating system S_1 attached to the hub, above equation becomes,

$$\overleftarrow{r}_{s1} = [K_\theta][K_\beta] r_{s3} \tag{A.5}$$

This can be written as,

$$\begin{vmatrix} X_1 \\ Y_1 \\ Z_1 \end{vmatrix} = \begin{vmatrix} r\cos\beta\sin\theta \\ -r\sin\beta \\ r\cos\beta\cos\theta \end{vmatrix} \tag{A.6}$$

In S_o coordinate system,

$$\overleftarrow{r}_{so} = [K_T][K_\theta][K_\beta]\overleftarrow{r}_{s3} \tag{A.7}$$

The following equation can be obtained from the above equation,

$$\begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} = \begin{vmatrix} r\cos\theta\sin\beta \\ -r\sin\beta\cos\alpha_T - r\cos\theta\cos\beta\sin\alpha_T \\ -r\sin\beta\sin\alpha_T + r\cos\theta\cos\beta\cos\alpha_T \end{vmatrix} \tag{A.8}$$

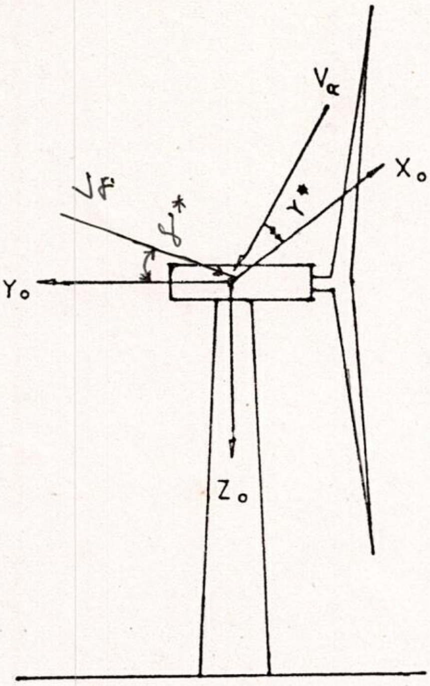

Fig. A·1 : Reference Frame S_0.
(Coordinate System S_0)

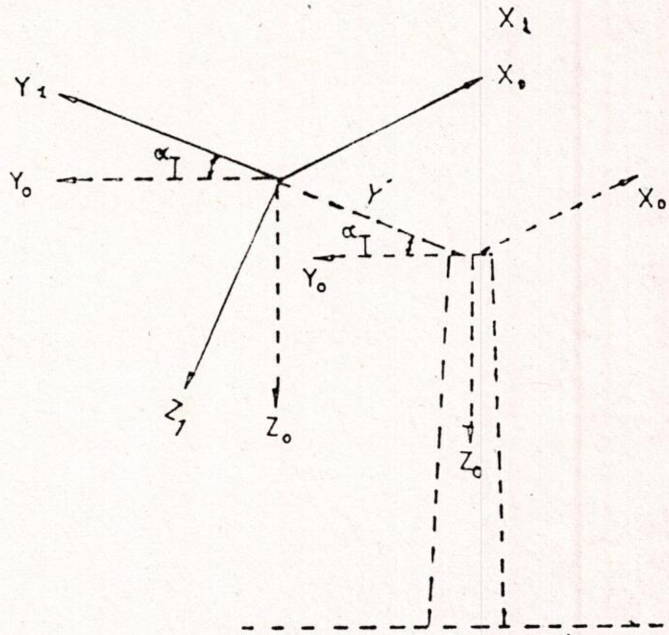

Fig. A·2 : Translation Over Y' and Rotation About X_0 by Angle α_τ.
(Coordinate System S_1).

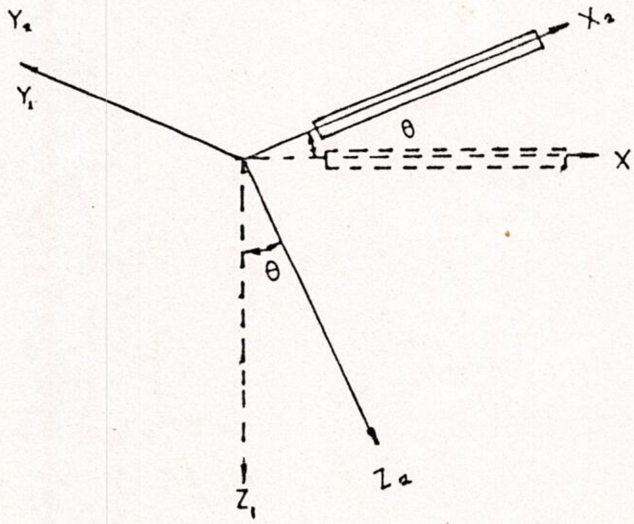

Fig. A·3 : Rotation About Y_1 by Angle θ.
(Coordinate System S_2)

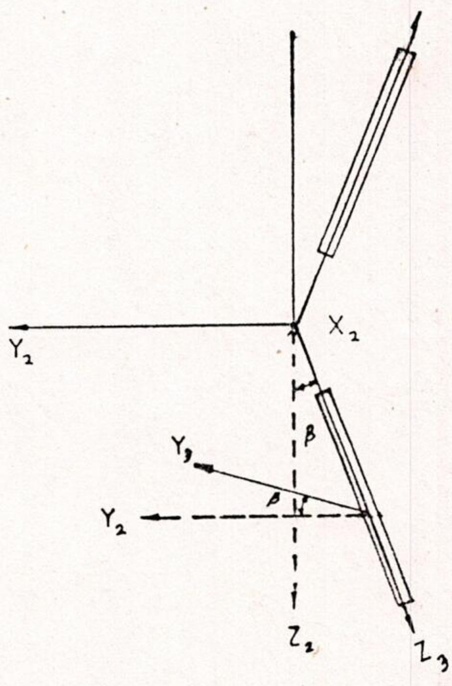

Fig. A·4 : Rotation About X_2 by Angle β.
(Coordinate System S_3.)

APPENDIX-B

Selection of Design Parameters

In the Desigining of a horizontal axis wind turbine it is important to find the value of design lift coefficient and design angle of attack from graphs that correspond to a minimum value of C_D/C_L ratio. To carry out the iteration procedure for the blade configuration, estimation of design power coefficient is also important. Determination of these parameters are discussed in the following sections.

B.1 Determination of Minimum C_D/C_L Ratio

The lift and drag coefficients of a given airfoil for a given Reynolds number are shown in Figure B.1.1.

In the C_D/C_L graph a tangent is drawn through $C_D=C_L=0$. From the point where the tangent touches the curve indicates the minimum C_D/C_L ratio. This ratio detrmines the maximum power coefficients that can be reached, particularly at high tip speed ratios. From the C_L-α curve corresponding to minimum C_D/C_L ratio, the values of lift coefficient and angle of attack are found. The C_L and α values found in this way are known as design lift coefficient, C_{Ld} and design angle of attack,α_d and these are very important parameters in the design process.

B.2 Determination of the Design C_P

The power coefficient is affected by the profile drag via the C_D/C_L ratio. The reduction of the maximum power coefficient is proportional to the tip speed ratio and to C_D/C_L ratio. The result is shown in the Figure B.2.1. The curve shows for each the maximum attainable power coefficient with the number of blade and C_D/C_L ratio as a parameter. In this collection of maximum power coefficient it is seen that for a range of design tip speed ratio from 1 to 10 the maximum theoretically attainable power coefficients lie between 0.3 to 0.5.

Now we can design a windmill rotor for a given wind speed V_∞ and a power demand. First the minimum C_D/C_L ratio of an airfoil is to be determined. The procedure for selection of number of blades, B and the design tip speed ratio λ_d has been explained in chapter-5. From Figure B.2.1 the maximum expected power coefficient can be found out. For conservative design, the design C_P is calculated as [25]

$$C_P=0.8 x C_{Pmax}.$$

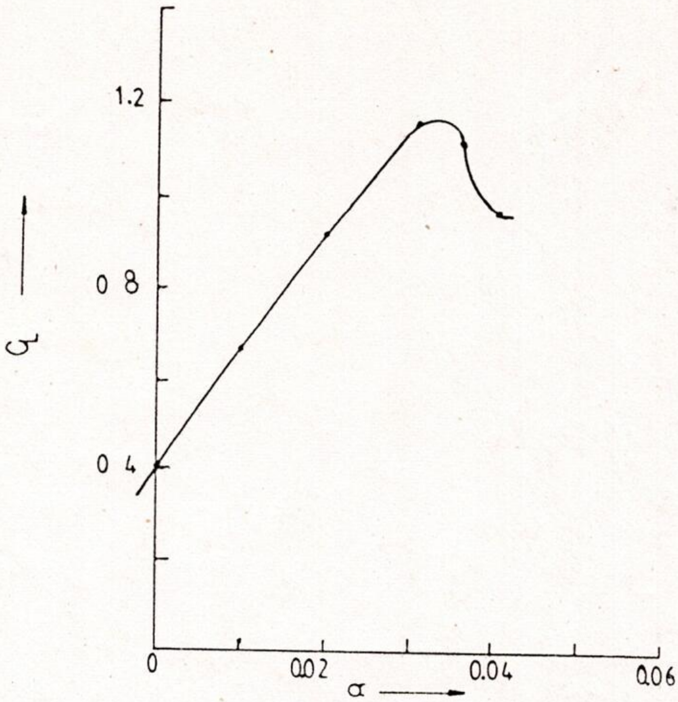

Fig. B.1.1: $C_L - \alpha$, Characteristics Curve for a given Airfoil Section [21].

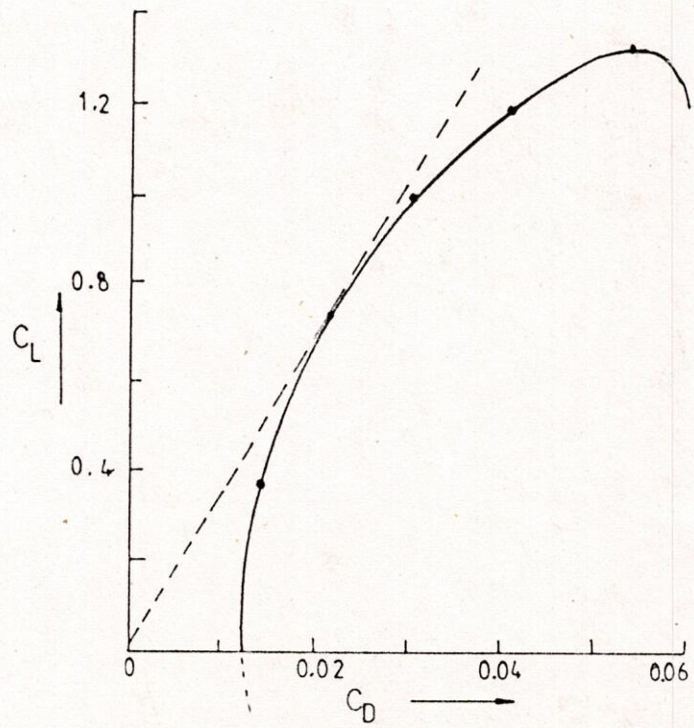

Fig. B.2.1: $C_L - C_D$, Characteristics Curve for a given Airfoil Section [21].

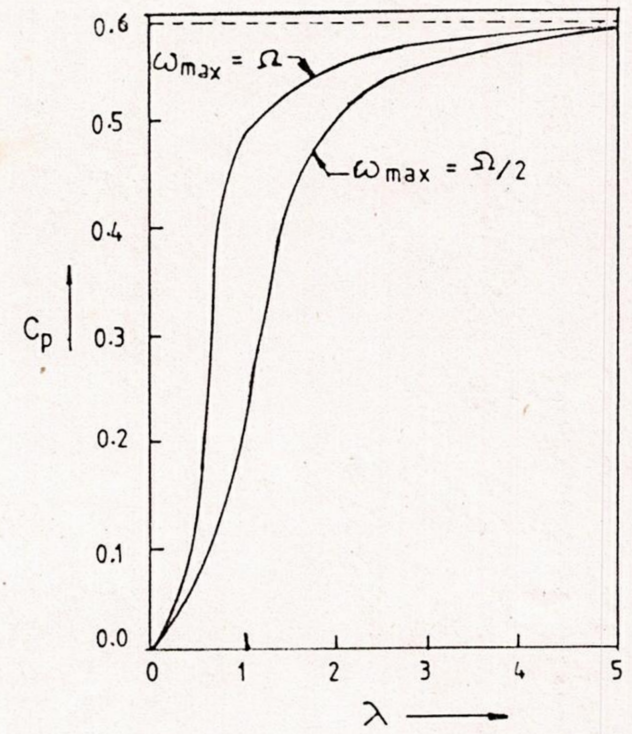

Fig.B.2.2 : Maximum Power Coefficient Versus λ for a Rotor with a Rankine Vortex wake

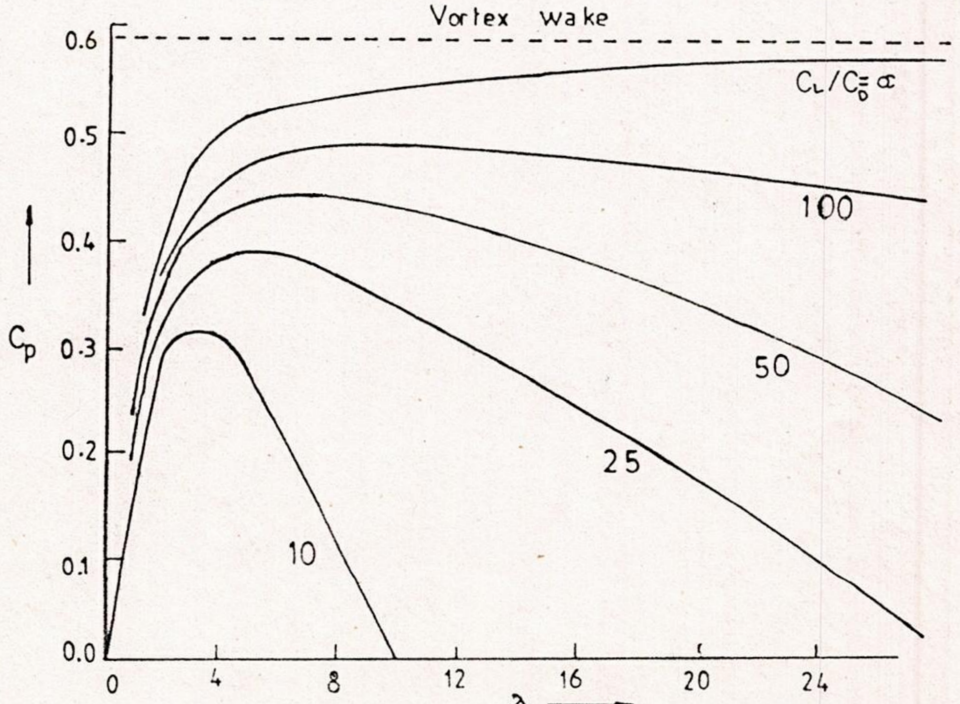

Fig.B.2.3 $C_p - \lambda$, Characteristics Curve showing the effect of C_L/C_D ratio ($B = \infty$).

APPENDIX-C

C.1 Maximum Power Coefficient

The plot C_P versus a, shows that the maximum value of C_P occurs when a=1/3 and the value is 16/27 or 0.593.
Analytically, by equation (3.1.14):

$$C_P=4a(1-a)^2=4(a-2a^2+a^3)$$

Now for maximum power coefficient,

$$\frac{dC_P}{da}=0, \quad \text{gives} \quad 4(1-4a+3a^2)$$

or, a=1/3, and a=1
For a=1, C_P has the minimum value zero.
For a=1/3, C_P has the mmaximum value of 16/27.

C.2 Influence of Wake Axial Induction Factor.

The addition of a vortex componenet reduce the torque and power given to the actuator disk, because it produces rotational kinetic energy to the fluid passing through the disk. It is shown in reference [49] that, for constant angular momentum, in the wake flow(irrotational vortex, r^2=constant).

$$a=\frac{1}{2}b[1-\{\frac{b^2(1-a)}{4\lambda^2(b-a)}\}] \qquad (C.2.1)$$

108

Where, $b=1-(V/V_\infty)$, is called the wake axial induction factor.

Figure C.2.1 (reference [49]) shows a plot of the relationship of equation (C.2.1) and it is seen that for the ideal actuator disk, with $\lambda=\infty$, b is exactly equal to 2a. For $\lambda \geq 2$, b is little different from 2a, but rapidly differs from this value for $\lambda<1$. The second terms within brackets in equation (C.2.1) is usually small and if b is replaced therein by its approximate value 2a, the equation reduces to,

$$a \approx \frac{1}{2}b[1-\{\frac{a(1-a)}{\lambda^2}\}] \qquad (C.2.2)$$

The power coefficient with a rotational component is then found to be,

$$C_P=\frac{b^2(1-a)^2}{(b-a)} \qquad (C.2.3)$$

with b=2a, this reduces to,

$$C_P=4a(1-a)^2,$$

the same as found from the axial momentum theory.

C.3 Relation between ω and U.

With respect to the disk, the relative angular velocity Ω changes to ($\Omega + \omega$) and the other components of the velocity remain unchanged. Applying the Bernoulli's equation at upstream and downstream of the actuator disk,

$$P^{+}+1/2\,\rho(U^2+\omega^2+\Omega^2r^2)=P^{-}+1/2\,\rho[U^2+\omega^2+(\Omega+\omega)^2r^2]$$

or, $P^{+}-P_{-}=1/2\,\rho(2\,\Omega\omega+\omega^2)r^2$ (C.3.1)

Thrust force on the annulas is given by,

$$dT=(P^{+}-P^{-})dA$$

Using the equation (C.3.1) which becomes,

$$dT=1/2\,\rho(2\Omega\omega+\omega^2)r^2dA \qquad \text{(C.3.2)}$$

But thrust force of the wind is,

$$dT=\rho UdA(V_\infty-U_1)=2\rho UdA(V_\infty-U) \qquad \text{(C.3.3)}$$

Equating equations (C.2.3) and (C.3.3) we ahve,

$$2\rho UdA(V_\infty-U)=1/2\,\rho(2\Omega\omega+\omega^2)r^2dA$$

or, $U(V_\infty-U)=1/2(\Omega\omega+\omega^2/2)r^2$

or, $U(V_\infty-U)=1/2\,\omega r(\Omega+\omega/2)r$

Rearranging in the form,

$$\frac{U}{V_\infty}\left(1-\frac{U}{V_\infty}\right)V_\infty^2=\frac{1}{2}\omega r\left(\Omega r+\frac{1}{2}\omega r\right) \qquad \text{(C.3.5)}$$

Since $á=\frac{\omega}{2\Omega}$ i.e. $1/2\,\omega r=á\,\Omega r$

and $\lambda_r=r\Omega/V_\infty$ i.e. $\Omega r=\lambda_r V_\infty$

$1-U/V_\infty=a$, one may get from equation (C.3.5),

$$\begin{aligned}(1-a)aV_\infty^2 &= a\,\Omega r(\lambda_r V_\infty+á\,\Omega r)\\ &= á\,\lambda_r V_\infty(\lambda_r V_\infty+a\lambda_r V_\infty)\\ &= a\,\lambda_r^2V_\infty^2(1+á)\end{aligned}$$

That is, $a(1-a)=á(1+á)\lambda_r$ (C.3.6)

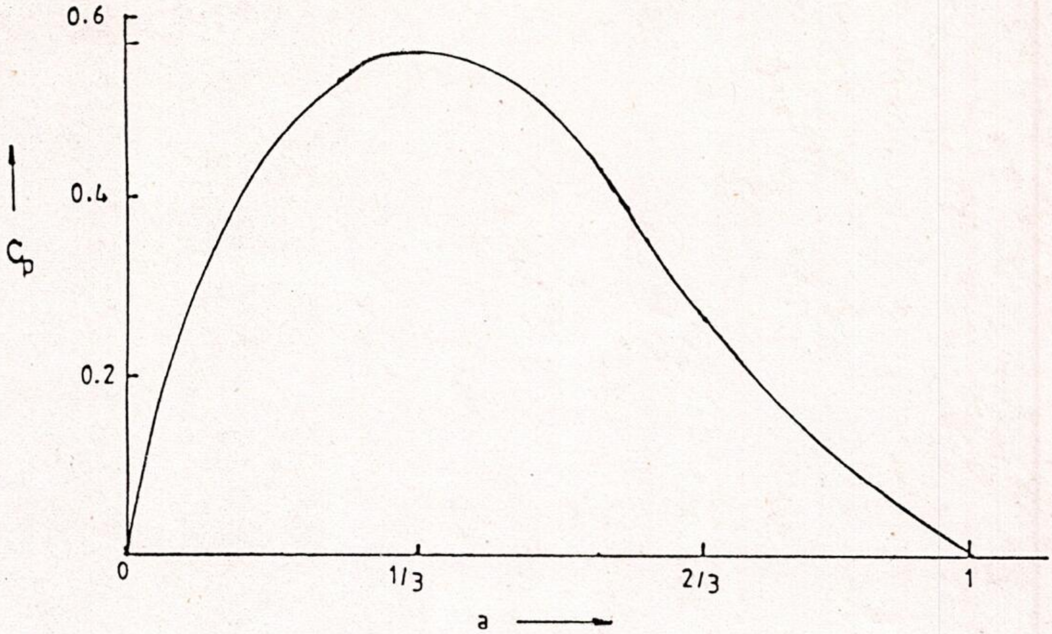

Fig. C.1.1: C_p Vs. a curve

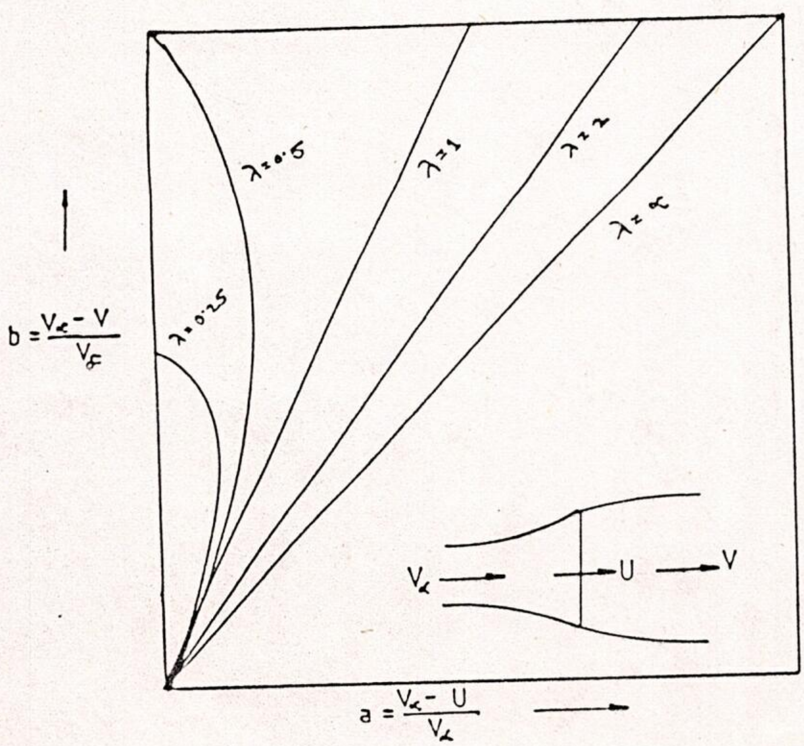

Fig. C.2.1: Effect of Tip Speed Ratio on Induction Factors.

APPENDIX-D

D.1 C_P in Terms of λ

we have, $$C_P = \frac{P}{1/2\,\rho A V_\infty^3}$$

$$= \frac{P}{1/2\,\rho \pi R^2 V_\infty^3}$$

Using the equation (3.2.9) & (3.2.10) this becomes,

$$C_P = \frac{2\rho\pi\Omega \int_0^R \omega r^3 dr}{1/2\,\rho\pi R^2 V_\infty^3}$$

Using the equation (3.1.10) this becomes,

$$C_P = \frac{4\ \Omega \int_0^R (1-a)V_\infty\ r^3 dr\,\omega}{R^2 V_\infty^3} \tag{D.1.1}$$

Since, $\lambda_r = \Omega r / V_\infty$ i.e. $r = \lambda_r V_\infty / \Omega$ (D.1.2)

$dr = (V_\infty / \Omega\ ) d\lambda_r$ and when $r \longrightarrow R$, $\lambda_r \longrightarrow \lambda$ (D.1.3)

Substituting all these values into equation (D.1.1),

$$C_P = \frac{4\ \Omega \int_0^\lambda (1-a)V_\infty\ (\ \lambda_r^3 V_\infty^3 /\ \Omega^3)(V_\infty\ /\Omega\ ) d\lambda_r}{R^2 V_\infty^3}$$

$$= 4 \int_0^\lambda (1-a)\frac{V_\infty^2}{R^2\ \Omega^2}\left(\frac{\omega}{\Omega}\right) \lambda_r^3 d\lambda_r$$

$$= \frac{8}{\lambda^2} \int_0^\lambda (1-a)\acute{a}\ \lambda_r^3 d\lambda_r \tag{D.1.4}$$

D.2 Relation Between a and á

Differentiating equation (D.1.4) with respect to a, and equating to zero (Since, at a particular radius λ_r=constant), one have,

$$\frac{dC_P}{da}=á(1-a)=0$$

or, $$(1-a)\frac{dá}{da}+á\frac{d}{da}(1-a)=0$$

or, $$(1-a)\frac{dá}{da}=á$$

or, $$\frac{dá}{da}=\frac{á}{(1-a)} \qquad (D.2.1)$$

Now differentiating equation (C.3.6), with λ_r=constant as before, with respect to a,

$$\frac{d}{da}[a(1-a)]=\frac{d}{da}[á(1-a)\ \lambda_r^2]$$

$$1-2a=(1+2á)\lambda^2{}_r\frac{dá}{da} \qquad (D.2.2)$$

Substituting the value of $\frac{dá}{da}$ from (D.2.1) it becomes,

$$1-2a=(1+2á)\lambda_r{}^2\frac{á}{(1-a)}$$

or, $$(1-a)(1-2a)=á(1+2á)\lambda_r{}^2$$

inserting the value of $\lambda_r{}^2$ from the equation (C.3.6) in it one may get,

$$(1-a)(1-2a)=á(1+2á)\left[\frac{(1-a)a}{á(1+á)}\right]$$

or, $$á=\frac{1-3a}{4a-1} \qquad (D.2.3)$$

D.3 Table for Average Wind Speed

Ten stations from all over the Bangladesh analysed the availability of wind velocity. The following stations are found to have good potential for the use of wind power [22].

Sl. No.	Stations	Potential mounth(s) for power generation by windmill.	Average wind speed(m/sec).
1.	Chittagong	March to September	4.0
2.	Dhaka	March to October	3.2
3.	Khepupara	February to September	3.5
4.	Comilla.	March to September	2.8
5.	Teknaf	June to September	2.3
6.	Jessore	April to September	2.1
7.	Cox's Bazer	May to Augest	2.0
8.	Hatiya	Ends of April to July	1.9
9.	Dinajpur	March to Augest	2.0
10.	Rangamati	April to May	1.9

* Source : Dhaka Meteorological Department.

COMPUTER PROGRAM

```
CM=1 --------------------------------------------------------------------------
C]      MM        MM     AAAAAA      II  NN         NN           ]  ]
C]      MM MM MM MM  AA       AA     II  NN NN      NN           ]  ]
C]      MM  MMM  MM  AA       AA     II  NN    NN   NN           ]  ]
C]      MM   M   MM  AAAAAAAAAA      II  NN      NN NN           ]  ]
C]      MM       MM  AA       AA     II  NN         NN           ]  ]
CM ____________________________________________________________________________
C                            ]                   ]                     ]
C KHANDKAR AFTAB HOSSAIN     ] ROLL NO.-871402F  ] REDG NO.85502       ]
C                            ]                   ]                     ]
C______________________________________________________________________]
C
C  THIS PROGRAM IS USED TO GET THE RESULTS OF YAW ANGLE VARIATIONS
C
C        _______________ MAIN PROGRAM _______________
C
C     IT IS USED TO DESIGN A NEW HORIZONTAL AXIS WIND TURBINE (H.A.W.T)
C     & CALCULATION OF ITS PERFORMANCE OR TO  CALCULATE THE PERFORMANCE
C     OF AN EXISTING H.A.W.T.  IT UTILIZES A SIMPSON'S-RULE METHOD /
C     THREE PASS TECHNIQUE OF NUMERICALINTEGRATION.
CM_____________________________________________________________________________
      INTEGER T,PRINT,SH,PITCH,GO,APP,VAL,Z1,CAT,CNTROL
      DIMENSION RR(45),CI(45),THETI(45),AAT(45),CLT(45),CDT(45),
     &CPCONT(45),PCCCR(45),CP(23,7),XX(23),CT(23,7),RR1(45),PY4(45),
     &TY4(45),QY2(45),CQY(23,7),TP1(23,7),V1(23),TP2(23,7),V2(23),
     &XMYY2(23,7),RA(23,7),XMYY3(45),V33(21,4),THETP1(22),V3(14),
     &T1P(13,21),WWR(4),DXMAS(45),SM(45),N1(22),XX1(23),XMYAW(37,4),
     &ANGL(8),FY8(45),POYA(12,4),TOYA(12,4),CQCONT(45),CTCONT(45),
     &SQC(8),RR8(45),SIA3(37),QOYA(12,4),PIM(37,4),TOFX(37,4),Y9(45,5),
     &C11(45),SIA2(29),FLAP1(29,3),EDGE1(29,3),SUP(12,2),T3P(29),
     &CPYS1(29,3),CTYS1(29,3),FX8(45),C12(45),XMYAH(37,4),PIM1(37,4),
     &TOFY(37,4),TOFZ(37,4),TOFX1(37,4),TOFY1(37,4),TOFZ1(37,4)
      EQUIVALENCE (Y9(1,1),PY4),(Y9(1,2),TY4),
     &(Y9(1,3),QY2),(Y9(1,4),XMYY3)
      COMMON/C1/R,DR,HB,B,V,X,THETP,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,W,
     &NPROF,APP,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,ALO,AC,TH
      COMMON/C2/DTSR,PRINT,T
      COMMON/C3/Z1,SH,CONST,ZOAN1
      COMMON/C4/QU,RR8,NFIT,RHOM,GAL
      COMMON/C5/OME,EIGV,EIGL,EXCI,GAM
      COMMON/C6/SIA,BF,COAN1,YA
      COMMON/C7/GG
      COMMON/C9/TILT
      COMMON/AA1/WWR,XXP
C.............................................................................
      OPEN(UNIT=60,FILE='INPUT',STATUS='OLD')
      OPEN(UNIT=61,FILE='OUTPUT',STATUS='NEW')
C.............................................................................
```

```
C
C --------------- PROGRAM OPERATION CONTROLLER -----------------------
C                ____________________________
C______________________________________________________________________
C.......T-TYPE
C..............1-NEW
C..............2-EXISTING
C
C.......PITCH-CONTROL
C..............1-FIXED
C..............2-VARIABLE
C
C.......R=RADIUS OF BLADE(METER)
C.......P=POWER TO BE EXTRACTED(KW)
C
C.......SH=SHAPE OF BLADE
C..............1-IDEAL
C..............2-LINEARIZING CHORD AND TWIST ANGLES BETEWEEN
C                0.5R & 0.9R.
C..............3-LINEARIZING CHORD BETEWEEN 0.5R & 0.9R BUT
C                TWIST ANGLE IS ZERO.
C
C.......PRINT-CODE
C..............1-RESULTS & GRAPHS
C..............2-GRAPHS ONLY
C..............3-RESULTS IN DETAILS
C..............4-RESULTS IN SHORT
C.......HB   =HUB RADIUS      (NEW-10% OF RADIUS)
C.......DTSR =DESIGN TIP SPEED RATIO
C.......DR   =INCREMENTAL %  (0.02 OR HIGHER)
C.......THETP=PITCH ANGLE  (DEGREE)
C.......B    =NO. OF BLADE
C.......V    =WIND VELOCITY (M/SEC)
C.......VM   =WIND VELOCITY (M/SEC)
C.......XETA =VELOCITY POWER LAW EXPONENT (1/6 FOR NEW D/N)
C.......RPM  =ANGULAR VELOCITY OF THE TURBINE (RPM)
C.......XMIN =STARTING TIP SPEED RATIO FOR EVALUATION
C.......DX   =TIP SPEED RATIO INTERVAL FOR EVALUATION
C.......XMAX =MAXIMUM TIP SPEED RATIO FOR EVALUATION
C.......H    =HUB ABOVE SEA LEVEL (METER ; FOR NEW D/N=2*R)
C.......HH   =HUB ABOVE GROUND LEVEL (METER ; FOR NEW D/N=2*R)
C.......SI   =CONING ANGLE  (DEGREE)
C.......NF   =NO. OF INPUTED STATIONS FOR BLADE GEOMETRY
C.......NPROF=NACA PROFILE ( SUBROUTINE)
C.......      4418
C.......      0012
C.......      0015
C.......      23024
C.......      8888    DATA IN TABULER FORM
C.......      9999    PROFILE CURVE FIT IN NACAXX
C
C.......GO=TIP LOSS MODEL CONTROLLER
C.............0-PRANDTL
C.............1-GOLDSTEIN
C.............2-NO MODEL
```

```
C.......HL=HUB LOSS MODEL CONTROLLER
C.............0-NONE
C.............1-PRANDTL
C
C.......APP=ANGULAR INTERFERENCE LOCKOUT
C.............0-FACTOR CALCULATED
C.............1-FACTOR=0.
C
C.......RR(I)    =% RADIUS FOR STATIONS
C.......CI(I)    =CHORD    FOR STATIONS  (METER)
C.......THETI(I)=TWIST ANGLE FOR STATIONS (DEGREE)
C.......TH       =MAXIMUM THICKNESS / CHORD RATIO
C.......ALO      =ANGLE OF ATTACK FOR ZERO LIFT  (DEGREE)
C.......CLT(I)   =CO-EFFICIENT OF LIFT DATA
C.......CDT(I)   =CO-EFFICIENT OF DRAG DATA
C.......AAT(I)   =ANGLE OF ATTACK  (DEGREE)
C
C.......VAL=VALUES OF H,HH,XETA,HB.
C.............1-SPECIFIED
C.............2-NOT SPECIFIED
C____________________________________________________________________
      READ(60,7001)T,PRINT
      WRITE(61,8000)
      WRITE(61,8001)T,PRINT
      READ(60,7003)VM,P,DTSR,SH,PITCH
      WRITE(61,8003)VM,P,DTSR,SH,PITCH
      READ(60,7004)HH,XETA,HB,H,VAL
      WRITE(61,8004)HH,XETA,HB,H,VAL
      GO TO 15
   15 CONTINUE
      YA=0.
      TILT=0.
      Z1=0
      Z2=0.
      IF(T.EQ.2)GO TO 1
      IF(T.EQ.1)COAN=9.
      I11=1
      HI1=0.
  101 THETP=HI1
      IF(PITCH.EQ.1)THETP=0.
      IF(DTSR.LE.3.)B=3.
      IF(DTSR.GE.4.)B=2.
      XMIN=2.
      XMAX=13.
      DX=0.5
      APP=0
      X=DTSR
      GO=0
      HL=1
      PI=3.1415926536
      DR=0.02
      RHOS=1.225
      SI=0.
      CP1=0.45
      NF=12
```

```
      R=SQRT((2.*P*1000.)/(PI*RHOS*VM**3*CP1))                          HAD01590
      RPM=(X*60.*VM)/(R*2.*PI)                                          HAD01600
      IF(VAL.EQ.1)GO TO 666                                             HAD01610
      HH=2.*R                                                           HAD01620
      H=2.*R                                                            HAD01630
      XETA=1./6.                                                        HAD01640
      HB=0.10*R                                                         HAD01650
  666 CONTINUE                                                          HAD01660
      RL=R                                                              HAD01670
      IF(DTSR.GE.4.)NPROF=4418                                          HAD01680
      IF(NPROF.EQ.4418)TH=0.18                                          HAD01690
      ALO=0.                                                            HAD01700
  288 CONTINUE                                                          HAD01710
      IF(I11.GT.1)GO TO 1                                               HAD01720
      IF(NPROF.EQ.4418)GO TO 93                                         HAD01730
   93 ALPH=7.                                                           HAD01740
      CL=1.07                                                           HAD01750
      DO 11 I=1,NF                                                      HAD01760
      I3=NF                                                             HAD01770
      I2=NF-1                                                           HAD01780
      IF(I.GE.I2)GO TO 13                                               HAD01790
      RR(I)=110.-I*10.                                                  HAD01800
      IF(SH.GE.2)GO TO 16                                               HAD01810
      XL=RL/R*X                                                         HAD01820
      ZHI=2./3.*ATAN(1./XL)                                             HAD01830
      CI(I)=((8.*PI*RL)/(B*CL))*(1.-COS(ZHI))                           HAD01840
      ZHI=ZHI*180./PI                                                   HAD01850
      THETI(I)=ZHI-ALPH                                                 HAD01860
      GO TO 17                                                          HAD01870
   16 XL=0.5*X                                                          HAD01880
      XHI5=2./3.*ATAN(1./XL)                                            HAD01890
      C5=((8.*PI*0.5*R)/(B*CL))*(1.-COS(XHI5))                          HAD01900
      B5=XHI5*180./PI-ALPH                                              HAD01910
      XL=0.9*X                                                          HAD01920
      XHI9=2./3.*ATAN(1./XL)                                            HAD01930
      C9=((8.*PI*0.9*R)/(B*CL))*(1.-COS(XHI9))                          HAD01940
      B9=XHI9*180./PI-ALPH                                              HAD01950
      CI(I)=(((C9-C5)/(0.4*R))*RL+2.25*C5-1.25*C9)                      HAD01960
      THETI(I)=((B9-B5)/(0.4*R))*RL+2.25*B5-1.25*B9                     HAD01970
      IF(SH.EQ.3)THETI(I)=0.                                            HAD01980
   17 RL=R-I/10.*R                                                      HAD01990
   11 CONTINUE                                                          HAD02000
   13 RR(I2)=(HB-0.0001*R)/R*100.                                       HAD02010
      CI(I2)=0.                                                         HAD02020
      THETI(I2)=0.                                                      HAD02030
      RR(I3)=0.                                                         HAD02040
      CI(I3)=0.                                                         HAD02050
      THETI(I3)=0.                                                      HAD02060
      R5=R                                                              HAD02070
    1 PI=3.1415926536                                                   HAD02080
      IF(T.NE.2)GO TO 63                                                HAD02090
      R6=R                                                              HAD02100
      THETP=0.                                                          HAD02110
  106 CONTINUE                                                          HAD02120
      DR=0.02                                                           HAD02130
```

```
      HB=0.1*R
      H=2.*R
      HH=H
      XMIN=2.
      XMAX=13.
      DX=0.5
      HL=1
      SI=0.
      GO=0
      APP=0
      TH=0.18
      ALO=0.
      NFS=NF
      NPROF=4418
   63 IF(T.EQ.1)X=DTSR
      IF(T.EQ.2)X=XMIN
      OMEGA=RPM*PI/30.
      V=R*OMEGA/X
      H1=H
      HH1=HH
      R1=R
      HB1=HB
      DR1=DR
      Z1=0
      IF(Z1.EQ.0)COAN=9.
      IF(Z1.EQ.1)GO TO 62
C-------------------------------------------------------------------]
C     ...........PRINT INPUT AND TITLES FOR OUTPUT.....             ]
C___________________________________________________________________]
  622 CALL TITLES(RR,CI,THETI,NF,SOLD,RPM)
   62 CONTINUE
      RHO=1.225
C______________________________________________________________________
C     ...........STARTING TORQUE COEFFICIENT.............             ]
C_____________________________________________________________________]
      IF(PITCH.EQ.1)GO TO 255
      NS=20
      NS1=NS+1
      DRST=(R-HB)/FLOAT(NS)
      DRST2=DRST/2.
      SFX=0.
      SQX=0.
      DO 250 K=1,NS1
      QU=1.
      RL=HB+FLOAT(K-1)*DRST
      CALL SEARCH(RL,RR,CI,THETI,NS,C,THET)
      ALF=PI/2.-(THET+THETP*PI/180.)
      CALL CLCD(ALF,ALFD,CL,CLD,CLT,CD,CDD,CDT,RL,AAT,NFS,SOLD)
      SFX=SFX+(2-(1/K+K/NS1))*CL*C*DRST2/(PI*R**2)
      SQX=SQX+(2-(1/K+K/NS1))*CL*C*RL*DRST2/(PI*R**3)
  250 CONTINUE
      SQC(I11)=SQX*B
      ANGL(I11)=THETP
      QU=2.
  255 CONTINUE
```

```
C_____________________________________________________________________      HAD02690
C      .......INITIALIZATION AND CONST. PARAMETER CALCULATIONS....]          HAD02700
C_____________________________________________________________________]     HAD02710
      X=XMIN                                                                HAD02720
      M=1                                                                   HAD02730
      MM=1                                                                  HAD02740
      BF=0.                                                                 HAD02750
      STRESM=0.                                                             HAD02760
      FX8(1)=0.                                                             HAD02770
      FY8(1)=0.                                                             HAD02780
      TILT=0.                                                               HAD02790
  100 CONTINUE                                                              HAD02800
      C11(1)=0.                                                             HAD02810
      C12(1)=0.                                                             HAD02820
      JMAX=1                                                                HAD02830
      IF(Z1.EQ.1)JMAX=21                                                    HAD02840
      DO 166 J=1,JMAX                                                       HAD02850
      IF(Z1.EQ.1)THETP=0.+FLOAT(J-1)                                        HAD02860
      IF(Z1.EQ.1)THETP1(J)=THETP                                            HAD02870
      IMAX=1                                                                HAD02880
      IF(Z1.EQ.1)IMAX=13                                                    HAD02890
      DO 167 I=1,IMAX                                                       HAD02900
      IF(Z1.EQ.1)V3(I)=8.+FLOAT(I-1)                                        HAD02910
      ITOT=-1                                                               HAD02920
      DR=0.02                                                               HAD02930
      R=R1                                                                  HAD02940
      IF(T.EQ.1.AND.PITCH.EQ.2)R=R5                                         HAD02950
      IF(T.EQ.2.AND.PITCH.EQ.2)R=R6                                         HAD02960
      IF(Z1.EQ.3.OR.Z1.EQ.4)X=DTSR                                          HAD02970
      IF(Z1.EQ.3.OR.Z1.EQ.4)THETP=0.                                        HAD02980
      IF(Z1.EQ.3.OR.Z1.EQ.4)VXV=R*OMEGA/DTSR                                HAD02990
      IF(Z1.EQ.5)X=XV                                                       HAD03000
      IF(Z1.EQ.5)THETP=0.                                                   HAD03010
      IF(Z1.NE.1)V=R*OMEGA/X                                                HAD03020
      IF(Z1.EQ.1)V=V3(I)                                                    HAD03030
      J1=1                                                                  HAD03040
      IK=1                                                                  HAD03050
      IF(Z1.GE.3)GO TO 302                                                  HAD03060
      IF(VAL.EQ.1.AND.ABS(Z2-1.).LT.1.E-25)GO TO 302                        HAD03070
      H=H1                                                                  HAD03080
      HB=HB1                                                                HAD03090
      HH=HH1                                                                HAD03100
  302 KL=1                                                                  HAD03110
      T7=0.                                                                 HAD03120
      T8=0.                                                                 HAD03130
      XMXX=0.                                                               HAD03140
      XMYX=0.                                                               HAD03150
      XMYA=0.                                                               HAD03160
      XMPI=0.                                                               HAD03170
      QX=0.                                                                 HAD03180
      AC=0.38                                                               HAD03190
      AK=0.                                                                 HAD03200
      SFAC=1.25                                                             HAD03210
      TX=0.                                                                 HAD03220
      FXXP1=0.                                                              HAD03230
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      FYXP1=0.                                                          HAD03240
      QY=0.                                                             HAD03250
      TY=0.                                                             HAD03260
      PY=0.                                                             HAD03270
      FX=0.                                                             HAD03280
      FY=0.                                                             HAD03290
      XMXY=0.                                                           HAD03300
      XMYY=0.                                                           HAD03310
      XMYAW1=0.                                                         HAD03320
      XMPIT1=0.                                                         HAD03330
      TFXT=0.                                                           HAD03340
      TFYT=0.                                                           HAD03350
      TFZT=0.                                                           HAD03360
      ASTOP=0.                                                          HAD03370
      A=0.                                                              HAD03380
      NFIT=4                                                            HAD03390
      AP=0.                                                             HAD03400
      COAN1=COAN*PI/180.                                                HAD03410
      CNTROL=3                                                          HAD03420
      SI=SI*PI/180.                                                     HAD03430
      REF=R*COS(SI)                                                     HAD03440
      THETP=THETP*PI/180.                                               HAD03450
      ALO=ALO*PI/180.                                                   HAD03460
      RHO=1.225                                                         HAD03470
      VIS=(0.0000003719-0.00000000204*H/3280.)*32.174*4.88243           HAD03480
      AN=(R-HB)/(R*DR)+1.                                               HAD03490
      NN=IFIX(AN)                                                       HAD03500
      NN1=NN-1                                                          HAD03510
      XMS=0.                                                            HAD03520
      IF(Z1.NE.1.OR.Z1.NE.3)GO TO 743                                   HAD03530
      IF(X.NE.DTSR  .AND. THETP.NE.0.0)GO TO 743                        HAD03540
      DIMAS=0.                                                          HAD03550
      DIMS=0.                                                           HAD03560
      DIM2=0.                                                           HAD03570
      DIM=0.                                                            HAD03580
  743 CONTINUE                                                          HAD03590
      AIYY=0.                                                           HAD03600
      AIXX=0.                                                           HAD03610
      EAL=7.33944954E09*9.81                                            HAD03620
      RHAL=2700.                                                        HAD03630
      GAL=2.6759E10                                                     HAD03640
      RHOM=RHAL                                                         HAD03650
      FR=0.98                                                           HAD03660
      N9=1                                                              HAD03670
      RX=R                                                              HAD03680
      RLB=(1.-DR)*RX                                                    HAD03690
      DR=(RX-RLB)*COS(SI)                                               HAD03700
      DRO=DR                                                            HAD03710
      R=R*COS(SI)                                                       HAD03720
      HB=HB*COS(SI)                                                     HAD03730
      RL=R                                                              HAD03740
C_______________________________________________________________________ HAD03750
C      .......THREE PASS-NUMERICAL INT -TIP TO HUB..........           ] HAD03760
C______________________________________________________________________] HAD03770
      CAT=1                                                             HAD03780
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```
      IF(GO.EQ.2)CAT=2
      CLFA=1.
      IF(GO.EQ.3)CLFA=0.
      IF(GO.LT.2)GO TO 2
      CALL SEARCH(RL,RR,CI,THETI,NF,C,THET)
      CALL CALC(RL,C,THET,FXXP1,FYXP1,XMXX,XMYX,QX,TX,RE,PHIR,CL,CD,CX,
     &CY,A,AP,XL,AK,ALPHA,F,CLFA,CAT,AAT,CLT,CDT,NFS,SOLD,XMYA,XMPI,
     &TFX1,TFY1,TFZ1)
    2 A=0.
      CAT=0
      DO 8 L=1,NN
      TIP=0.
      IF((RL-HB)  .GE.  DR)GO TO 3
      ASTOP=ASTOP+1.
      IF(ASTOP.GE.2.)GO TO 9
      DR=(RL-HB)
    3 IF(GO.LT.3)GO TO 5
      IF(CNTROL  .EQ.  0)GO TO 5
      TIP=RL-DR
      IF(TIP .GT.REF)GO TO 4
      IF(CNTROL  .EQ. 2)GO TO 5
      DR=(RL-REF)
      CLFO=(REF-TIP)/(RL-TIP)
      CLF=0.5*CLFO
      CNTROL=1
      GO TO 5
    4 CLF=0.
    5 DR2=DR/2.
      DT6=DR/6.
      RL=RL-DR2
      IF(CNTROL  .EQ. 0)CLF=1.
      IF(CNTROL  .EQ. 2)CLF=(CLFO+1.)/2.
      AK=1.
      CALL SEARCH(RL,RR,CI,THETI,NF,C,THET)
      CALL CALC(RL,C,THET,FXXP1,FYXP1,XMXXP1,XMYXP1,QXP1,TXP1,RE,
     &PHIR,CL,CD,CX,CY,A,AP,XL,AK,ALPHA,F,CLF,CAT,AAT,CLT,CDT,NFS,
     &SOLD,XMYA1,XMPI1,TFX2,TFY2,TFZ2)
      IF(Z1.EQ.1.OR.Z1.GE.3)GO TO 185
      IF(T.EQ.2)GO TO 185
      IF(THETP.NE.0.)GO TO 185
      IF(X.NE.DTSR)GO TO 185
      XMS1=CL*CONST/((OMEGA**2*RL*COS(COAN1)*SIN(COAN1)))*SFAC
      AXMS1=XMS1/(RHAL)
      ATH1=AXMS1/(2.6206*C)
      AIYY1=0.05172*ATH1**3+0.19175*C**3*ATH1
      AIXX1=0.66666*C*ATH1**3+0.0104389*C**3*ATH1-0.12434*C**2*ATH1**2
  185 RL=RL-DR2
      IF(CNTROL.EQ.0)CLF=1.
      IF(CNTROL.EQ.1)CLF=CLFO
      IF(CNTROL.EQ.2)CLF=1.
      AK=0.
      CALL SEARCH(RL,RR,CI,THETI,NF,C,THET)
      CALL CALC(RL,C,THET,FXY,FYY,XMXXP,XMYXP,QXP,TXP,RE,PHIR,CL,CD,
     &CX,CY,A,AP,XL,AK,ALPHA,F,CLF,CAT,AAT,CLT,CDT,NFS,SOLD,
     &XMYA2,XMPI2,TFX3,TFY3,TFZ3)
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          QYX=DT6*(QX+4.*QXP1+QXP)                                        HAD04340
          QY=QY+QYX                                                       HAD04350
      IF(Z1.EQ.1.OR.Z1.GE.3)GO TO 111                                     HAD04360
      IF(T.EQ.2)GO TO 240                                                 HAD04370
      IF(THETP.NE.0.)GO TO 240                                            HAD04380
      IF(X.NE.DTSR)GO TO 240                                              HAD04390
      IF(N9.GT.NN)GO TO 240                                               HAD04400
      XMS2=CL*CONST/((OMEGA**2*RL*COS(COAN1)*SIN(COAN1)))*SFAC            HAD04410
      AXMS2=XMS2/RHAL                                                     HAD04420
      ATH2=AXMS2/(2.6206*C)                                               HAD04430
      IF(N9.EQ.1)GO TO 241                                                HAD04440
  241 CONTINUE                                                            HAD04450
      XMS=XMS2                                                            HAD04460
      AIYY=AIYY2                                                          HAD04470
      AIXX=AIXX2                                                          HAD04480
      N9=N9+1                                                             HAD04490
  240 CONTINUE                                                            HAD04500
      TYX=DT6*(TX+4.*TXP1+TXP)                                            HAD04510
        TY=TY+TYX                                                         HAD04520
      IF(THETP.NE.0.)GO TO 111                                            HAD04530
      IF(X.NE.DTSR)GO TO 111                                              HAD04540
         TY4(KL)=TYX                                                      HAD04550
         PY4(KL)=OMEGA*QYX                                                HAD04560
         QY2(KL)=QYX                                                      HAD04570
         FY8(KL)=DT6*(FYXP1+4.*FYXP1+FYY)                                 HAD04580
      C11(KL)=TH/2.*C                                                     HAD04590
         FX8(KL)=DT6*(FXXP1+4.*FXXP1+FXY)                                 HAD04600
      C12(KL)=0.75*C                                                      HAD04610
  111     PY=PY+OMEGA*QYX                                                 HAD04620
      IF(Z1.EQ.6) TY=TY+DT6*(TX+4.*TXP1+TXP)                              HAD04630
      IF(Z1.NE.3)GO TO 8691                                               HAD04640
          TY=TY+DT6*(TX+4.*TXP1+TXP)                                      HAD04650
 8691 IF(Z1.GE.3)GO TO 869                                                HAD04660
      IF(Z1.EQ.1)GO TO 161                                                HAD04670
C.....................................................................    HAD04680
C          FOLLOWING 10 LINES GIVES DEL-POWER,-TORQUE,-THRUST,-AXIAL  ]   HAD04690
C  BENDING MOMENT,TANGENTIAL BENDING MOMENT, & -AXIAL FORCE WITH      ]   HAD04700
C  RADIAL VARIATION.                                                  ]   HAD04710
C.....................................................................    HAD04720
         TY4(KL)=TYX                                                      HAD04730
         PY4(KL)=OMEGA*QYX                                                HAD04740
         QY2(KL)=QYX                                                      HAD04750
         FY8(KL)=DT6*(FYXP1+4.*FYXP1+FYY)                                 HAD04760
         FX8(KL)=DT6*(FXXP1+4.*FXXP1+FXY)                                 HAD04770
         CPCONT(KL)=(OMEGA*QYX)/(0.5*RHO*V**3*PI*RX**2)                   HAD04780
         CQCONT(KL)=CPCONT(KL)/X                                          HAD04790
         CTCONT(KL)=TY4(KL)/(.5*RHO*V**2*PI*RX**2)                        HAD04800
      FX=FX+DT6*(FXXP1+4.*FXXP1+FXY)                                      HAD04810
      FY=FY+DT6*(FYXP1+4.*FYXP1+FYY)                                      HAD04820
  869 XMXY=XMXY+DT6*(XMXX+4.*XMXXP1+XMXXP)                                HAD04830
      XMYY=XMYY+DT6*(XMYX+4.*XMYXP1+XMYXP)                                HAD04840
      IF(Z1.NE.4)GO TO 161                                                HAD04850
C -------------------------------------------------------------------------]HAD04860
C       FOLLOWING 5 LINES GIVES THE TOWER TOP AXIAL,VERTICAL, AND          ]HAD04870
C  TANGENTIAL FORCE. ALSO GIVES YAWING & PITCHING MOMENT CO-EFFICIENTS     ]HAD04880
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C  WITH YAW ANGLE VARIATIONS AT DIFFERENT BLADE POSITIONS(Z1=4)          ]HAD04890
C -----------------------------------------------------------------------HAD04900
         XMYAW1=XMYAW1+DT6*(XMYA+4.*XMYA1+XMYA2)                          HAD04910
         XMPIT1=XMPIT1+DT6*(XMPI+4.*XMPI1+XMPI2)                          HAD04920
         TFXT=TFXT+DT6*(TFX1+4.*TFX2+TFX3)                                HAD04930
         TFYT=TFYT+DT6*(TFY1+4.*TFY2+TFY3)                                HAD04940
         TFZT=TFZT+DT6*(TFZ1+4.*TFZ2+TFZ3)                                HAD04950
  161 IF(CNTROL.EQ.2) CNTROL=0                                            HAD04960
      IF(CNTROL.EQ.0)GO TO 6                                              HAD04970
      IF(ABS(RL-TIP).LT.1.E-25)GO TO 6                                    HAD04980
      IF(CNTROL.EQ.1) DR=REF-TIP                                          HAD04990
      IF(CNTROL.EQ.1) CNTROL=2                                            HAD05000
      GO TO 7                                                             HAD05010
    6 DR=DRO                                                              HAD05020
    7 CONTINUE                                                            HAD05030
      QX=QXP                                                              HAD05040
      IF(Z1.GE.3)GO TO 778                                                HAD05050
      IF(Z1.EQ.1)GO TO 777                                                HAD05060
  778 TX=TXP                                                              HAD05070
      XMXX=XMXXP                                                          HAD05080
      XMYX=XMYXP                                                          HAD05090
      XMYA=XMYA2                                                          HAD05100
      XMPI=XMPI2                                                          HAD05110
      TFX1=TFX3                                                           HAD05120
      TFY1=TFY3                                                           HAD05130
      TFZ1=TFZ3                                                           HAD05140
      FXY=FXXP1                                                           HAD05150
      FYY=FYXP1                                                           HAD05160
      IF(Z1.EQ.3)GO TO 777                                                HAD05170
      CTY=TY/(0.5*RHO*V**2*PI*RX**2)                                      HAD05180
      CPY=PY/(0.5*RHO*V**3*PI*RX**2)                                      HAD05190
      RAT=CTY/CPY                                                         HAD05200
  777    TP=PY/1000.                                                      HAD05210
      PHIO=PHIR*180./PI                                                   HAD05220
      ALPHA=ALPHA*180./PI                                                 HAD05230
      PR=RL/(RX*COS(SI))                                                  HAD05240
      IF(Z1.EQ.1.OR.Z1.GE.3)GO TO 8                                       HAD05250
         QY1=QY                                                           HAD05260
         CQ=CPY/X                                                         HAD05270
      TY1=TY                                                              HAD05280
      FX1=FX                                                              HAD05290
      FY1=FY                                                              HAD05300
      XMXY1=XMXY                                                          HAD05310
      XMYY1=XMYY                                                          HAD05320
C______________________________________________________                   HAD05330
C...............PRINT OUTPUT.............                    ]            HAD05340
C_____________________________________________________]                   HAD05350
      VOUT=V                                                              HAD05360
      PCCR=RL/(RX*COS(SI))                                                HAD05370
      IF(X.EQ.DTSR.AND.THETP.EQ.0.) RR1(KL)=RL                            HAD05380
      PCCCR(KL)=PCCR                                                      HAD05390
      KL=KL+1                                                             HAD05400
      RCD=CD/CL                                                           HAD05410
      IF(PRINT.NE.3)GO TO 8                                               HAD05420
C -----------------------------------------------------------------]     HAD05430
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C  WRITES DIFFERENT AERODYNAMIC PARAMETERS WITH RADIAL VARIATIONS. ]
C_______________________________________________________________________]
      WRITE(61,14)PCCR,A,AP,CL,CD,PHIO,ALPHA,F,RE,CTY,CPY,RCD
  8   CONTINUE
      IF(Z1.LT.4)GO TO 77
C ------------------------------------------------------------]
C   FOLLOWING 4 LINES GIVES POWER,TORQUE & THRUST CO-EFFICIENTS ]
C   WITH YAW ANGLE VARIATIONS ( Z1=6 ).                         ]
C ------------------------------------------------------------]
      IF(Z1.EQ.6) POYA(IA,IB)=CPY
      IF(Z1.EQ.6) TOYA(IA,IB)=CTY
      IF(Z1.EQ.6) QOYA(IA,IB)=CPY/X
      CQ1=CPY/X
      IF(Z1.EQ.5) T3P(IA)=PY/1000.
      IF(Z1.EQ.5.OR.Z1.EQ.6)GO TO 55
      IF(Z1.NE.4)GO TO 77
C-----------------------------------------------------------------------
C          NEXT 5  LINES  ARE FOR  TOWER TOP AXIAL,TANGENTIAL&        ]
C   VERTICAL FORCE  AND  YAWING & PITCHING MOMENT CO-EFFICIENT        ]
C   CALCULATION WITH YAW ANGLE &TOWER SHADOW FACTOR VARITIONS(Z1=4).]
C ---------------------------------------------------------------------
      TOFX(IA,IB)=TFXT
      TOFY(IA,IB)=TFYT
      TOFZ(IA,IB)=TFZT
      XMYAW(IA,IB)=XMYAW1
      PIM(IA,IB)=XMPIT1
      GO TO 55
   77 IF(T.EQ.2)GO TO 637
      IF(Z1.NE.3)GO TO 244
C ----------------------------------------------------------------------
C         FOLLWING 7 LINES GIVES FLAPWISE & EDGEWISE BENDING MOMENT,  ]
C POWER & TORQUE CO-EFFICIENT WITH YAW ANGLE AND TOWER SHADOW FACTOR ]
C VARIATIONS AT DIFFERENT BLADE POSITIONS.                            ]
C ----------------------------------------------------------------------
      FLAP1(IA,IB)=XMYY/1000.
      EDGE1(IA,IB)=XMXY/1000.
      CPYS1(IA,IB)=PY/(0.5*RHO*PI*R**2*VXV**3)
      CTYS1(IA,IB)=TY/(0.5*RHO*PI*R**2*VXV**2)
      WRITE(61,213)SIA,FLAP1(IA,IB),EDGE1(IA,IB),CPYS1(IA,IB),CTYS1(IA,
     &IB)
      IF(X.NE.DTSR)GO TO 55
      IF(IA.GT.1)GO TO 55
      DO 741 L1=10,370,10
      SPA=FLOAT(L1-10)
      SIA3(L1/10)=SPA
  741 CONTINUE
      GO TO 55
  244 IF(Z1.EQ.1)GO TO 637
      IF(Z1.GE.3)GO TO 55
      IF(THETP.NE.0.)GO TO 637
      IF(X.NE.DTSR)GO TO 637
      STRESM=0.
      DO 639 IM=1,44
  639 CONTINUE
      GO TO 637
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  640 COAN=COAN-1.                                                     HAD05990
      X=DTSR                                                           HAD06000
      ZOAN1=COAN                                                       HAD06010
      GO TO 55                                                         HAD06020
  637 IF(Z1.NE.1)GO TO 9                                               HAD06030
      T1P(I,J)=TP                                                      HAD06040
      GO TO 55                                                         HAD06050
    9 HP=TP*1.3410                                                     HAD06060
      IF(PRINT.NE.3)GO TO 53                                           HAD06070
      WRITE(61,33)X                                                    HAD06080
      WRITE(61,99)VOUT                                                 HAD06090
      WRITE(61,36)CPY                                                  HAD06100
   53 XX(M)=X                                                          HAD06110
      IF(VOUT.LE.20.) V2(MM)=VOUT                                      HAD06120
      V1(M)=VOUT                                                       HAD06130
      PISTEL=T7/(PI*RX**5)                                             HAD06140
      PI2TEL=T8*X/(PI*RX**6)                                           HAD06150
      CPAV=CPY                                                         HAD06160
      IF(PRINT.NE.3)GO TO 54                                           HAD06170
      WRITE(61,37)CPAV                                                 HAD06180
      WRITE(61,38)CTY                                                  HAD06190
      WRITE(61,51)PISTEL                                               HAD06200
      WRITE(61,52)PI2TEL                                               HAD06210
   54 II=I11                                                           HAD06220
      TP2(MM,II)=0.                                                    HAD06230
      CQY(M,II)=CQ                                                     HAD06240
      CP(M,II)=CPAV                                                    HAD06250
      TP1(M,II)=TP                                                     HAD06260
      IF(VOUT.LE.20.) TP2(MM,II)=TP                                    HAD06270
      IF(TP2(MM,II).GE.P) TP2(MM,II)=P                                 HAD06280
      IF(TP2(MM,II).LE.0.) TP2(MM,II)=0.                               HAD06290
      CT(M,II)=CTY                                                     HAD06300
      RA(M,II)=RAT                                                     HAD06310
      XMYY2(M,II)=XMYY1/1000.                                          HAD06320
      M=M+1                                                            HAD06330
      IF(VOUT.GT.20.)GO TO 862                                         HAD06340
      MM=MM+1                                                          HAD06350
  862 IF(PRINT.EQ.2.OR.PRINT.EQ.3)GO TO 86                             HAD06360
      IF(X.GT.XMIN)GO TO 861                                           HAD06370
      WRITE(61,118)HP                                                  HAD06380
      WRITE(61,120)SQC(I11)                                            HAD06390
      WRITE(61,81)                                                     HAD06400
      WRITE(61,82)                                                     HAD06410
      WRITE(61,83)                                                     HAD06420
  861 IF(PRINT.EQ.2.OR.PRINT.EQ.3)GO TO 86                             HAD06430
C -------------------------------------------------------------------- HAD06440
C       WRITES DIFFERENT AERODYNAMIC PARAMETERS WITH TIP SPEED RATIO. ] HAD06450
C ____________________________________________________________________] HAD06460
      WRITE(61,84)X,VOUT,TP,CPAV,QY1,CQ,TY1,CTY,XMXY1,XMYY1,FX1,FY1    HAD06470
   86 X=X+DX                                                           HAD06480
      IF(PRINT.NE.3)GO TO 55                                           HAD06490
C--------------------------------------------------------------------- HAD06500
C     FOLLOWING 11 LINES WRITES DEL-POWER,TORQUE,THRUST,AXIAL & TANGE ] HAD06510
C -NTIAL FORCE,AXIAL & TANGENTIAL MOMENT,DEL-POWER,TORQUE & THRUST    ] HAD06520
C  CO-EFFICIENTS WITH RADIAL VARIATIONS.                              ] HAD06530
```

```
C--------------------------------------------------------------------
      WRITE(61,224)
      KKL=KL-1
      DO 222 JJ=1,KKL
      WRITE(61,223)PCCCR(JJ),CPCONT(JJ),PY4(JJ),QY2(JJ),TY4(JJ),FX8(JJ)
     &,FY8(JJ)
  222 CONTINUE
      WRITE(61,204)
      DO 206 MK=1,KKL
      WRITE(61,205)PCCCR(MK),CPCONT(MK),CQCONT(MK),CTCONT(MK)
  206 CONTINUE
   55 SI=SI*180./PI
      THETP=THETP*180./PI
      ALO=ALO*180./PI
  167 CONTINUE
  166 CONTINUE
      IF(Z1.EQ.6)GO TO 988
      IF(Z1.LT.3)GO TO 986
      IF(Z1.NE.4)GO TO 990
      SIA=SIA+10.
      GO TO 988
  990 IF(SIA.LE.15..OR.SIA.GE.345.) GO TO 987
      SIA=SIA+15.
      GO TO 988
  987 SIA=SIA+5.
  988 IF(Z1.GE.3)  IA=IA+1
      IF(Z1.EQ.6)GO TO 993
      IF(SIA.LE.360.)GO TO 587
      IF(Z1.EQ.4)GO TO 605
      IF(Z1.EQ.3)GO TO 603
  587 IF(Z1.GE.3)GO TO 100
  986 IF(ABS(X-DTSR).LT.1.E-25  .AND.  STRESM.GE. 98.)GO TO 100
      IF(Z1.EQ.1)GO TO 159
      IF(X.GT.XMAX)GO TO 102
      IF(PRINT.NE.3)GO TO 56
      WRITE(61,47)
      WRITE(61,57)
   56 GO TO 100
  102 CONTINUE
      IF(T.EQ.2)GO TO 103
      IF(PITCH.EQ.1)GO TO 103
      I11=I11+1
      HI1=HI1+2.5
      IF(HI1.LT.14.)GO TO 101
  103 CONTINUE
      IF(T.NE.2)GO TO 588
      IF(PITCH.EQ.1)GO TO 58
      IF(PITCH.EQ.2)THETP=THETP+2.
      IF(THETP.LE.12.)GO TO 106
      GO TO 58
  588 IF(PRINT.GE.3)GO TO 160
      IF(PITCH.EQ.1)GO TO 160
      Z1=1
      Z2=1.
      GO TO 100
```

```
  159 CONTINUE
C_____________________________________________________________________
C      .....NEXT 19 LINES ARE FOR CONST.POWER CALCULATIONS......     ]
C_____________________________________________________________________]
      P9=P
      P10=P9+0.4*P9
 1591 DO 168 J=1,21
      IF(T1P(1,J).GT.P)GO TO 168
      DO 169 I=1,13
      IF(T1P(I,J).LT.P)GO TO 169
      V33(J1,IK)=(V3(I)-V3(I-1))/(T1P(I,J)-T1P((I-1),J))*(P-T1P((I-1),J
     &))+V3(I-1)
      J1=J1+1
      GO TO 168
  169 CONTINUE
  168 CONTINUE
      N1(IK)=J1-1
      P=P+35.
      IF(P.GT.P10)GO TO 160
      J1=1
      IK=IK+1
      GO TO 1591
  160 CONTINUE
C-----------------------------------------------------------------------
C      * PUT CCCC BEFORE GO TO 58 * IF YAW REQD.( THIS LINE NO.IS 819)]
C______________________________________________________________________]
****  GO TO 58
C-----------------------------------------------------------------------
C          NEXT 20  LINES  ARE FOR  FLAPWISE,EDGEWISE BENDING MOMENT,]
C          POWER & THRUST CO-EFFICIENT CALCULATIONS WITH YAW ANGLE & ]
C          TOWER SHADOW VARIATIONS.                                  ]
CZ3---------------------------------------------------------------------
888   Z1=3
      IA=1
      IB=1
      BF=0.6
      YA=89.99
      SIA=0.
      WRITE(61,211)
      WRITE(61,214)DTSR,BF
      WRITE(61,210)YA
      WRITE(61,212)
      GO TO 100
  603 IA=1
      IB=IB+1
      BF=0.6
      SIA=0.
      YA=YA-6.
      IF(YA.LE.69.)GO TO 604
      WRITE(61,214)DTSR,BF
      WRITE(61,210)YA
      WRITE(61,212)
      PRINT*,YA,Z1
      GO TO 100
C-----------------------------------------------------------------------
```

```
C          NEXT 47  LINES  ARE FOR  TOWER TOP AXIAL,TANGENTIAL&        ]   HAD07640
C   VERTICAL FORCE  AND  YAWING & PITCHING MOMENT CO-EFFICIENT         ]   HAD07650
C   CALCULATION WITH YAW ANGLE &TOWER SHADOW VARITIONS.                ]   HAD07660
CZ4-------------------------------------------------------------------     HAD07670
  604 Z1=4                                                                 HAD07680
      SIA=0.                                                               HAD07690
      BF=0.6                                                               HAD07700
      IA=1                                                                 HAD07710
      IB=1                                                                 HAD07720
      YA=89.99                                                             HAD07730
      GO TO 100                                                            HAD07740
  605 IA=1                                                                 HAD07750
      Z1=4                                                                 HAD07760
      IB=IB+1                                                              HAD07770
      YA=YA-6.                                                             HAD07780
      BF=0.6                                                               HAD07790
      SIA=0.                                                               HAD07800
      IF(YA.LE.69.)GO TO 606                                               HAD07810
      PRINT*,YA,Z1                                                         HAD07820
      GO TO 100                                                            HAD07830
  606 CONTINUE                                                             HAD07840
C______________________________________________________________________     HAD07850
C     .....NEXT 3 LINES ARE FOR CENTRIFUGAL STRESS......                ]   HAD07860
C                                                                       ]   HAD07870
C..............................        REJECTED        ..............]   HAD07880
C     CENTS(1)=0.                                                       ]   HAD07890
C     DO 78 I=2,45                                                      ]   HAD07900
C  78 CENTS(I)=GG(46-I)/CROS(I)                                         ]   HAD07910
C ---------------------------------------------------------------------    HAD07920
      ARR=0.5*RHO*PI*R**3*V**2                                             HAD07930
      DO 88 J=1,4                                                          HAD07940
      DO 88 I=1,37                                                         HAD07950
      IF(I.GT.18)GO TO 909                                                 HAD07960
      XMYAH(I,J)=(XMYAW(18+I,J)+XMYAW(I,J))/ARR                            HAD07970
      PIM1(I,J)=(PIM(18+I,J)+PIM(I,J))/ARR                                 HAD07980
      TOFX1(I,J)=TOFX(18+I,J)+TOFX(I,J)                                    HAD07990
      TOFY1(I,J)=TOFY(18+I,J)+TOFY(I,J)                                    HAD08000
      TOFZ1(I,J)=TOFZ(18+I,J)+TOFZ(I,J)                                    HAD08010
      GO TO 88                                                             HAD08020
  909 XMYAH(I,J)=(XMYAW(I,J)+XMYAW(I-18,J))/ARR                            HAD08030
      PIM1(I,J)=(PIM(I,J)+PIM(I-18,J))/ARR                                 HAD08040
      TOFX1(I,J)=TOFX(I,J)+TOFX(I-18,J)                                    HAD08050
      TOFY1(I,J)=TOFY(I,J)+TOFY(I-18,J)                                    HAD08060
      TOFZ1(I,J)=TOFZ(I,J)+TOFZ(I-18,J)                                    HAD08070
   88 CONTINUE                                                             HAD08080
      DO 129 KKK=1,37                                                      HAD08090
      XMYAH(KKK,1)=XMYAH(KKK,1)*100.                                       HAD08100
 129  CONTINUE                                                             HAD08110
C--------------------------------------------------------------------      HAD08120
C          NEXT 16  LINES  WRITES  TOWER TOP AXIAL,TANGENTIAL&         ]   HAD08130
C   VERTICAL FORCE  AND  YAWING & PITCHING MOMENT CO-EFFICIENT         ]   HAD08140
C   FOR DIFFERENT YAW ANGLE &TOWER SHADOW FACTORS.                     ]   HAD08150
C--------------------------------------------------------------------      HAD08160
      WRITE(61,211)                                                        HAD08170
      WRITE(61,214)DTSR,BF                                                 HAD08180
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```
      WRITE(61,215)
      WRITE(61,1000)
      WRITE(61,1001)
          DO 216 J=1,4
      WRITE(61,1008)J
          DO 216 I=1,37
          WRITE(61,1009)I,XMYAH(I,J),PIM1(I,J),TOFY1(I,J),TOFX1(I,J),
     +TOFZ1(I,J)
 216    CONTINUE
C-----------------------------------------------------------------------
C   NEXT 29  LINES  ARE FOR  POWER,TORQUE,&THRUST  CO-EFFICIENT  ]
C   CALCULATION WITH YAW ANGLE  VARITIONS.                       ]
CZ6---------------------------------------------------------------------
  984 CONTINUE
      Z1=6
      IA=1
      IB=1
      SIA=0.
      X=XMIN
      BF=0.
      THETP=0.
          YA=84.
          XETA=0.167
         WRITE(61,209)
       WRITE(61,217)YA
         WRITE(61,208)
      GO TO 100
 993  CONTINUE
      WRITE(61,207)X,CPY,CTY,CQ1
      X=X+1.
      IF(X.GT.XMAX)GO TO 992
      GO TO 100
  992 IA=1
      IB=IB+1
      X=XMIN
      YA=YA-6.
      IF(YA.LT.69.)GO TO 994
       WRITE(61,217)YA
         WRITE(61,208)
      PRINT*,YA,Z1
      GO TO 100
  994 CONTINUE
       PRINT*,'******  END  OF EXECUTION *******  '
   58 CONTINUE
C-----------------------------------------------------------------------
C      ......FORMAT FOR INPUT/OUTPUT............
CM______________________________________________________________________
   10  FORMAT(3F10.4)
   14  FORMAT(F5.3,1X,F7.4,3X,F6.4,1X,F5.2,3X,F5.3,2X,F6.2,2X,
     &F6.2,2X,F6.4,2X,E10.3,1X,F7.4,2X,F8.4,2X,F8.5)
   33  FORMAT(///,' PERFORMANCE SUMMARY= ',30X,'TIP SPEED RATIO=',F8.3)
   36  FORMAT(/,15X,27H TOTAL POWER COEFFICIENT = ,F7.5)
   37  FORMAT(/,15X,43H AVERAGE POWER COEFFICIENT WITH GRADIENT = ,F10.5)
   38  FORMAT(/,15X,28H TOTAL THRUST COEFFICIENT = ,F7.4)
   39 FORMAT(7F10.4)
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CM ------------------------------------------------------------------------ HAD08740
 8000 FORMAT(22X,' I N P U T E D  V A L U E S   ',/22X,28('_')//)           HAD08750
 7001 FORMAT(2I2)                                                           HAD08760
 8001 FORMAT(5X,'T=',I2,'      PRINT=',I2)                                  HAD08770
 7002 FORMAT(I2)                                                            HAD08780
 8002 FORMAT(25X,'NFIT=',I2)                                                HAD08790
 7003 FORMAT(3F10.5,2I2)                                                    HAD08800
 8003 FORMAT(5X,'VM=',F5.2,'   P=',F6.2,'   DTSR=',F4.2,'     SH=',I2,'     HAD08810
     &   PITCH=',I2)                                                        HAD08820
 7004 FORMAT(4F10.5,I2)                                                     HAD08830
 8004 FORMAT(5X,'HH=',F4.2,'   XETA=',F6.4,'   HB=',F4.2,'   H=',F4.2,'     HAD08840
     &   VAL=',I2)                                                          HAD08850
CM ------------------------------------------------------------------------ HAD08860
 7005 FORMAT(4F10.5,I2)                                                     HAD08870
 8005 FORMAT(9X,'R=',F7.3,'   B=',F3.1,'   RPM=',F5.2,'   XETA=',F6.4,'     HAD08880
     &   PITCH=',I2)                                                        HAD08890
 7006 FORMAT(I2)                                                            HAD08900
 8006 FORMAT(25X,'NF=',I2)                                                  HAD08910
 7007 FORMAT(3F10.5)                                                        HAD08920
 9001 FORMAT(//,10X,'RR(I)          CI(I)          THETI(I)')               HAD08930
 8007 FORMAT(10X,3(F10.5,5X))                                               HAD08940
 7008 FORMAT(3F10.5)                                                        HAD08950
 9002 FORMAT(//,20X,'AAT(I)         CLT(I)         CDT(I)')                 HAD08960
 8008 FORMAT(20X,3(F10.5,5X))                                               HAD08970
C------------------------------------------------------------------------   HAD08980
  40  FORMAT(3F10.4)                                                        HAD08990
  41  FORMAT(2I2,I4)                                                        HAD09000
 47   FORMAT(//,5X,22H PERFORMANCE ANALYSIS:)                               HAD09010
  51  FORMAT(//,' TORQUE VARIATION DUE TO AERODYNAMIC SECOND DERIVATIVE     HAD09020
     &=',F15.4)                                                             HAD09030
  52  FORMAT(/,'   TORQUE VARIATION DUE TO SHEAR SECOND DERIVATIVE =',      HAD09040
     &F15.4)                                                                HAD09050
 57   FORMAT(////,1X,'PCCR',5X,'A',6X,'AP',5X,'CL',6X,'CD',5X,'PHI',        HAD09060
     &4X,'ALPHA',5X,'F',8X,' RE-NO.',6X,'CT',6X,'CP',5X,'CD/CL',/)          HAD09070
  99  FORMAT(/,15X,'WIND VELOCITY (METER/SEC) =',F8.2)                      HAD09080
 224  FORMAT(//,3X,'STATION',5X,'DEL-CP',7X,'POWER',8X,'TORQUE',8X,         HAD09090
     &'THURST',8X,'X-FORCE',5X,'Y-FORCE',6X,'X-MOMENT',4X,'Y-MOMENT'/)      HAD09100
 223  FORMAT(F8.2,5X,F10.5,7(3X,F10.3),/)                                   HAD09110
   81 FORMAT(///,3X,'TIP SPEED  WIND   POWER   POWER  TORQUE   TORQUE  THAD09120
     &HRUST    THRUST    MOMENT/BLADE MOMENT/BLADE FORCE/BLADE FORCE/BLAHAD09130
     &DE')                                                                  HAD09140
   82 FORMAT(3X,'  RATIO    VEL.          COFF.           COFF.             HAD09150
     &      COFF.      X-DIRECTION  Y-DIRECTION  X-DIRECTION Y-DIRECTION    HAD09160
     &   ')                                                                 HAD09170
                                                                            HAD09180
   83 FORMAT(3X,'          (M/SEC)  (KW)    (CP)    (N-M)    (CQ)      (N)HAD09190
     &      (CT)           (N-M)          (N-M)          (N)         (N)'/) HAD09200
   84 FORMAT(5X,F5.2,2X,F5.2,2X,G8.2,2X,G8.2,1X,G9.3,2X,G10.3,              HAD09210
     &1X,G9.3,2X,G9.3,1X,G10.4,3X,G10.4,2X,G11.5,3X,G11.5)                  HAD09220
 118  FORMAT(/,15X,'HORSE   POWER ,          H.P.  =     ',F10.2//)         HAD09230
 120  FORMAT(15X,'STARTING TORQUE CO-EFFICIENT= ',G10.3//)                  HAD09240
 204  FORMAT(//,3X,'STATION',5X,'DEL-CP',7X,'DEL-CQ',5X,'DEL-CT')           HAD09250
 205  FORMAT(F8.2,7X,F8.6,5X,F8.6,3X,F8.6/)                                 HAD09260
CFYAW------- FORMAT FOR YAWING ANGLE VARIATIONS  --------------------       HAD09270
 1000 FORMAT(///,10X,'ADDED VALUES OBTAINED FROM THE LOOP Z1 = 4 ',/        HAD09280
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     & 10X,'YAWING MOMENT CO-EFF.,PITCHING MOMENT CO-EFF.,'/)
 1001 FORMAT(10X,'TOWER TOP AXIL,VERTICAL & TANGENTIAL FORCE.'///)
 1007 FORMAT(3X,I3,6X,F10.6,/)
 1008 FORMAT(//,I2,8X,'YAW MOMENT CO-EFF.',2X,'PITCH MOMENT CO-EFF.',3X
     +,'AXIL FORCE',3X,'TANGENTIAL FORCE',3X,'VERTICAL FORCE'//)
 1009 FORMAT(I3,5X,5(E16.8,3X))
C----------- FORMAT FORMAT FORMAT FORMAT FORMAT-------------------
 207  FORMAT(4X,F5.2,5X,F8.5,4X,F8.5,3X,F8.5/)
 208  FORMAT(/5X,'T.S.R.',7X,'CP',10X,'CT',8X,'CQ'/)
 209  FORMAT(//10X,'EFFECT OF YAWING ANGLE ON POWER,TORQUE & THRUST CO-E
     &FF.:- ',/10X,53('-')//)
 210  FORMAT(/30X,'   YAW ANGLE = ',F5.2//)
 211  FORMAT(//10X,'EFFECT OF YAWING ANGLE & TOWER SHADOW:-',/10X,37(
     &'-')//)
 212  FORMAT(/3X,'THETA',6X,'FLAP',9X,'EDGE',13X,'CP',12X,'CT'/)
 213  FORMAT(1X,F5.1,5X,G11.5,4X,G11.5,3X,G11.5,5X,G11.5/)
 214  FORMAT(//10X,'DESIGN TIP SPEED RATIO=',F5.2,17X,'TOWER SHADOW FACT
     &OR=',F5.3,//)
 215  FORMAT(///,25X,'YAW ANGLE INTERVAL  = 6.0'///)
 217  FORMAT(/15X,'   YAW ANGLE = ',F5.2//)
      STOP
      END
CS=1 ----------------------------
C     SUBROUTINE SEARCH          ]
C     _________________          ]
C                                ]
C ------------------------------]
      SUBROUTINE SEARCH(RL,RR,CI,THETI,NF,C,THET)
C___________________________________________________________________
C  ....SEARCH DETERMINES CHORD AND TWIST ANGLE AT A GIVEN RADIUS    ]
C      ALONG THE SPAN BY LINEAR INTERPOLATION....                   ]
C___________________________________________________________________
      DIMENSION RR(25),CI(25),THETI(25)
      COMMON/C1/R,DR,HB,B,V,X,THETP,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,W
     &,NPROF,APP,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,ALO,AC,TH
      DO 1 I=1,NF
      RRV=RL/(RX*COS(SI))*100.
CEQ   IF(RRV.EQ.RR(I))GO TO 4
      IF(ABS(RRV-RR(I)).LT.1.E-25)GO TO 4
      IF(RRV.GE.RR(I))GO TO 2
      IF(I.EQ.NF)GO TO 3
    1 CONTINUE
    2 J=I+1
      PER=(RRV-RR(J-1))/(RR(J-2)-RR(J-1))
      C=PER*(CI(J-2)-CI(J-1))+CI(J-1)
      THET=PER*(THETI(J-2)-THETI(J-1))+THETI(J-1)
      GO TO 5
    3 C=CI(NF)
      THET=THETI(NF)
      GO TO 5
    4 C=CI(1)
      THET=THETI(1)
    5 THET=THET*PI/180.
      RETURN
      END
```

```
CS=2 -----------------------------
C      SUBROUTINE TITLES           ]
C      _________________           ]
C                                  ]
C ------------------------------]
      SUBROUTINE TITLES(RR,CI,THETI,NF,SOLD,RPM)
C_______________________________________________________________________
C     .....TITLES  PRINTS OUT INPUT DATA, SYMBOLS/TITLES FOR OUTPUT  ]
C_______________________________________________________________________
      INTEGER PRINT,GO,HL,APP
      DIMENSION RR(25),CI(25),THETI(25)
      COMMON/C1/R,DR,HB,B,V,X,THETP,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,W,
     &NPROF,APP,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,ALO,AC,TH
      COMMON/C2/DTSR,PRINT,T
      IF(PRINT.EQ.2)GO TO 61
      WRITE(61,12)
      WRITE(61,13)
      WRITE(61,15)
      WRITE(61,14)
      WRITE(61,16)XETA
      WRITE(61,17)HH
      WRITE(61,18)H
      WRITE(61,117)TH
      WRITE(61,119)RPM
      WRITE(61,19)OMEGA
      IF(T.EQ.1)GO TO 49
      WRITE(61,20)X
      GO TO 51
   49 DTSR=X
      WRITE(61,50)DTSR
   51 WRITE(61,21)THETP
   61 RX=R
      CSI=SI
      SI=SI*PI/180.
      CRL=0.75*R
      CALL SEARCH(CRL,RR,CI,THETI,NF,C,THET)
      SI=CSI
      BANG=THET*180./PI+THETP
      IF(PRINT.EQ.2)GO TO 62
      WRITE(61,22)BANG
      WRITE(61,23)SI
      WRITE(61,24)
      WRITE(61,25)B
      WRITE(61,26)R
      WRITE(61,27)HB
   62 CALL SOLIDT(RR,CI,NF,B,R,PI,SOLD)
      IF(PRINT.EQ.2)GO TO 63
      WRITE(61,28)SOLD
   63 CALL ACTIVI(RR,CI,NF,B,R,PI,ACF)
      IF(PRINT.EQ.2)GO TO 60
      WRITE(61,29)ACF
      WRITE(61,30)NPROF
      GO TO 55
   55 WRITE(61,31)NF
      WRITE(61,32)
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      WRITE(61,33)                                                      HAD10390
      WRITE(61,34)(RR(I),CI(I),THETI(I),I=1,NF)                         HAD10400
      WRITE(61,35)                                                      HAD10410
      WRITE(61,36)DR                                                    HAD10420
      IF(APP.EQ.0)GO TO 1                                               HAD10430
      WRITE(61,37)                                                      HAD10440
    1 CONTINUE                                                          HAD10450
    2 WRITE(61,39)                                                      HAD10460
    3 IF(B.GT.2.)GO TO 4                                                HAD10470
      IF(GO.EQ.0)GO TO 8                                                HAD10480
      IF(GO.EQ.1)GO TO 5                                                HAD10490
      IF(GO.EQ.2)GO TO 6                                                HAD10500
      IF(GO.EQ.3)GO TO 7                                                HAD10510
      GO TO 9                                                           HAD10520
    4 IF(GO.EQ.2)GO TO 6                                                HAD10530
      IF(GO.EQ.3)GO TO 7                                                HAD10540
      GO TO 8                                                           HAD10550
    5 WRITE(61,41)                                                      HAD10560
      GO TO 9                                                           HAD10570
    6 WRITE(61,42)                                                      HAD10580
      GO TO 9                                                           HAD10590
    7 WRITE(61,43)                                                      HAD10600
      GO TO 9                                                           HAD10610
    8 WRITE(61,40)                                                      HAD10620
    9 IF(HL.EQ.0)GO TO 10                                               HAD10630
      WRITE(61,45)                                                      HAD10640
      GO TO 11                                                          HAD10650
   10 WRITE(61,44)                                                      HAD10660
   11 CONTINUE                                                          HAD10670
      WRITE(61,46)                                                      HAD10680
      WRITE(61,47)                                                      HAD10690
      IF(PRINT.NE.3)GO TO 60                                            HAD10700
      WRITE(61,48)                                                      HAD10710
   60 RETURN                                                            HAD10720
C ______________________________________________________________________HAD10730
   12 FORMAT(1H1)                                                       HAD10740
   13 FORMAT(12X,46H PERFORMANCE OF A HORIZONTAL AXIS WIND TURBINE)     HAD10750
   15 FORMAT(18X,29H CALCULATED BY HAWTDP PROGRAM)                      HAD10760
   14 FORMAT(//,5X,22H OPERATING CONDITIONS:)                           HAD10770
   16 FORMAT(/,15X,'WIND VELOCITY GRADIENT EXPONENT=',F7.4)             HAD10780
   17 FORMAT(/,15X,'HUB HEIGHT ABOVE GROUND LEVEL(METER)=',F12.4)       HAD10790
   18 FORMAT(/,15X,'ALTITUDE OF HUB ABOVE SEA LEVEL(METER)=',F15.4)     HAD10800
  117 FORMAT(/,15X,'MAXIMUM THICKNESS/ CHORD RATIO, =  ',F6.2)          HAD10810
  119 FORMAT(/,15X,'REVOLUTIONS PER MINUTE,R.P.M.    =  ',F7.2)         HAD10820
   19 FORMAT(/,15X,'ANGULAR VELOCITY(RADIAN PER SECOND)=',F10.4)        HAD10830
   20 FORMAT(/,15X,'TIP SPEED RATIO=',F7.4)                             HAD10840
   50 FORMAT(/,15X,'DESIGN TIP SPEED RATIO=',F7.4)                      HAD10850
   21 FORMAT(/,15X,'PITCH ANGLE FROM NOMINAL TWIST(DEGREES)=',F10.4)    HAD10860
   22 FORMAT(/,15X,'PITCH ANGLE AT 0.75 RADIOUS(DEGREES)=',G14.8)       HAD10870
   23 FORMAT(/,15X,'CONING ANGLE(DEGREES)=',F10.4)                      HAD10880
   24 FORMAT(/,10X,'BLADE DESIGN:')                                     HAD10890
   25 FORMAT(/,15X,'NUMBER OF BLADES=',F3.0)                            HAD10900
   26 FORMAT(/,15X,'TIP RADIUS(METER)=',F7.4)                           HAD10910
   27 FORMAT(/,15X,'HUB RADIUS(METER)=',F7.4)                           HAD10920
   28 FORMAT(/,15X,'SOLIDITY=',F12.8)                                   HAD10930
```

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   29  FORMAT(/,15X,'ACTIVITY FACTOR=',G14.8)                               HAD10940
   30  FORMAT(/,15X,'NACA PROFILE=',I5)                                     HAD10950
   52  FORMAT(/,15X,' NACA PROFILE=0012 ')                                  HAD10960
   53  FORMAT(/,15X,' NACA PROFILE=0015 ')                                  HAD10970
   31  FORMAT(/,15X,'NUMBER OF STATIONS ALONG SPAN=',I4)                    HAD10980
   32  FORMAT(/,10X,'CHORD AND TWIST DISTRIBUTION')                         HAD10990
   33  FORMAT(/,15X,'PERCENT RADIUS',5X,'CHORD(METER)',10X,'TWIST(DEG)')   HAD11000
   34  FORMAT(/,20X,F6.2,8X,F10.5,10X,F10.5)                                HAD11010
   35  FORMAT(//,5X,'PROGRAM OPERATING CONDITIONS:')                        HAD11020
   36  FORMAT(/,15X,'INCREMENTAL PERCENTAGE=',F7.4)                         HAD11030
   37  FORMAT(/,15X,'ANGULAR INTERFERENCE FACTOR,AP=0.0')                   HAD11040
   39  FORMAT(/,15X,'STANDARD AXIAL INTERFERENCE METHOD USED')              HAD11050
   40  FORMAT(/,15X,'TIP LOSSES MODELED BY PRANDTLES FORMULA')              HAD11060
   41  FORMAT(/,15X,'TIP LOSSES MODELED BY GOLDSTEINS FORMULA')             HAD11070
   42  FORMAT(/,15X,'NO TIP LOSS MODEL')                                    HAD11080
   43  FORMAT(/,15X,'TIP LOSS MODEL  -->  N A S A  VERSION')                HAD11090
   44  FORMAT(/,15X,'NO HUB LOSS MODEL USED')                               HAD11100
   45  FORMAT(/,15X,'HUB LOSSES MODELED BY PRANDTLE')                       HAD11110
   46  FORMAT(1H1)                                                          HAD11120
   47  FORMAT(//,3X,'PERFORMANCE ANALYSIS:-')                               HAD11130
   48  FORMAT(////,1X,'PCCR',5X,'A',6X,'AP',5X,'CL',6X,'CD',5X,'PHI',      HAD11140
     &4X,'ALPHA',5X,'F',8X,' RE-NO.',6X,'CT',6X,'CP',5X,'CD/CL'/)           HAD11150
       END                                                                  HAD11160
CS=3 ----------------------------                                           HAD11170
C      SUBROUTINE CALC              ]                                       HAD11180
C      _______________              ]                                       HAD11190
C                                   ]                                       HAD11200
C ----------------------------------]                                       HAD11210
       SUBROUTINE CALC(RL,C,THET,FXF,FYF,XMFXF,XMFYF,QF,TF,RE,PHIR,CL,CD    HAD11220
     &,CX,CY,A,AP,XL,AK,ALPHA,F,CLF,CAT,AAT,CLT,CDT,NFS,SOLD,XM1Y,XM1P,     HAD11230
     &TFX,TFY,TFZ)                                                          HAD11240
C___________________________________________________________________        HAD11250
C  ....CALC DETERMINES THE AXIAL AND ANGULAR INTERFERENCE FACTOR AT ]       HAD11260
C       A GIVEN RADIUS AND FUNCTIONS DEPENDENT UPON THESE PARAMETERS.]..    HAD11270
C____________________________________________________________________       HAD11280
       INTEGER T,GO,APP,Z1,CAT                                              HAD11290
       DIMENSION AAT(45),RR8(45),CLT(45),CDT(45)                            HAD11300
       COMMON/C1/R,DR,HB,B,V,X,THETP,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,W       HAD11310
     &,NPROF,APP,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,ALO,AC,TH             HAD11320
       COMMON/C2/DTSR,PRINT,T                                               HAD11330
       COMMON/C3/Z1,SH,CONST,ZOAN1                                          HAD11340
       COMMON/C6/SIA,BF,COAN1,YA                                            HAD11350
       COMMON/C4/QU,RR8,NFIT,RHOM,GAL                                       HAD11360
       COMMON/C9/TILT                                                       HAD11370
C ---------------------------------------------------------------------    HAD11380
       XREF=PI/12.                                                          HAD11390
       HREF=1.5*R                                                           HAD11400
       DDD=1.5*R                                                            HAD11410
       ATILT=PI/180.*TILT                                                   HAD11420
       IF(Z1.GE.3) ZOAN=PI/180.*ZOAN1                                       HAD11430
       IF(T.EQ.2) ZOAN=PI/180.*0.                                           HAD11440
       VX=V                                                                 HAD11450
       IF(Z1.LT.3.OR.Z1.EQ.6) SIA1=0.                                       HAD11460
       YA1=YA*PI/180.                                                       HAD11470
       IF(Z1.EQ.6)GO TO 242                                                 HAD11480
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      IF(Z1.NE.4)YA1=PI/180.*90.
      IF(Z1.EQ.3)YA1=YA*PI/180.
      IF(Z1.LT.3)GO TO 242
      VA=R*OMEGA/DTSR
      IF(Z1.EQ.5)VA=R*OMEGA/X
      SIA1=SIA*PI/180.
      XX=COS(ZOAN)*SIN(SIA1)
      YY=SIN(-ZOAN)*COS(ATILT)+COS(ZOAN)*COS(SIA1)*SIN(ATILT)
      YY2=-YY
      YY=YY2*RL-DDD*COS(ATILT)
      ZZ=-SIN(-ZOAN)*SIN(ATILT)+COS(ZOAN)*COS(SIA1)*COS(ATILT)
      ZZ2=ZZ
      ZZ=ZZ*RL
      ZZ=ZZ-DDD*SIN(ATILT)
      ZZZ=HH-(ZZ2*RL)
      VREF=VA/(HH/HREF)**XETA
      V=(ZZZ/HREF)**XETA*VREF
      XV=ABS(XX)
      IF(Z1.EQ.4)GO TO 243
      IF(SIA.LE.15.  .OR.  SIA.GE.345.) V=V*(1.-BF*(1.+COS(12.*SIA1)))
      GO TO 242
  243 IF(SIA.LT.20.  .OR.  SIA.GT.340.) V=V*(1.-BF*(1.+COS(9.*SIA1)))
  242 CONTINUE
      IF(ABS(V).LT.1.E-25) V=1.E-5
      IF(Z1.LT.3) ZOAN=0.
      XL=RL*COS(ZOAN)*OMEGA/V
      RH=HB
      DO 15 J=1,40
      JA=0
   88 CONTINUE
      BETA=A
      DELTA=AP
      CSI=COS(SI)
      IF(ABS(A-1.).LT.1.E-25) A=0.999999
      IF(ABS(AP+1.).LT.1.E-25) AP=-0.999999
      CC1=COS(ZOAN)*SIN(YA1)*COS(ATILT)+SIN(SIA1)*COS(YA1)*SIN(-ZOAN)
     &-A*SIN(YA1)*COS(ZOAN)*COS(ATILT)-SIN(YA1)*SIN(-ZOAN)*SIN(ATILT)
     &*COS(SIA1)
      CC1=CC1*V
      CC2=-((1.+AP)*RL*COS(ZOAN)*OMEGA)+COS(YA1)*COS(SIA1)*V
     &+V*SIN(YA1)*SIN(SIA1)*SIN(ATILT)
      FC1=-(CC1/CC2)
      PHI=ATAN(FC1)
      IF(PHI.LT.0.  .AND.  CC1.GT.0.) PHI=PHI+PI
      SPHI=SIN(PHI)
      CPHI=COS(PHI)
      PHIAA=ABS(PHI)
      XXL=COS(PHIAA)/SIN(PHIAA)
      XXLO=XXL*R/RL
      PHIR=PHI
      ALPHA=PHI-THET-THETP
      IF(ABS(XL).LT.1.E-25) XL=1.E-9
      DDTC=ATAN((1.-A)/(XL*(1.+2.*AP)))-ATAN((1.-A)/XL)
      DA1=DDTC/4.
      DA2=((4./15.)*(SOLD*TH)/X)/((1./X)**2+(RL/RX)**2)
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```
      DALPHA=DA1+DA2
      ALPHA=ALPHA-DALPHA
      ALPHAD=ALPHA+0.001
C_____________________________________________________
C......CAL OF SECTIONAL LIFT AND DRAG CO-EFF ........
C_____________________________________________________
      IF(NPROF.EQ.4418)GO TO 1
 1    CALL NACA44(ALPHA,CL,CD,ALO)
      CALL NACA44(ALPHAD,CLD,CDD,ALO)
      IF(GO.LT.3)GO TO 6
      CL=CLF*CL
      CLD=CLF*CLD
      F=1.
      GO TO 7
C____________________________________
C  ......CAL TIP & HUB LOSSES ........
C____________________________________
  6   IF(CAT.EQ.1) F=0.
      IF(CAT.EQ.1)GO TO 7
      CALL TIPLOS(XXL,XXLD,F,B,GO,HL,PI,R,RL,PHI,RH)
  7   CONTINUE
  70  CONTINUE
  71  CXX=CL*SPHI
      CYY=CL*CPHI
      CX=CXX
      CY=CYY
      IF(PHI.GE.1.5272)GO TO 322
      GO TO 323
 322  CONTINUE
      A=0.
      AP=0.
      CC1=COS(ZOAN)*COS(ATILT)*SIN(YA1)+SIN(SIA1)*COS(YA1)*SIN(-ZOAN)
     &-A*SIN(YA1)*COS(ZOAN)*COS(ATILT)-SIN(YA1)*SIN(-ZOAN)*
     &SIN(ATILT)*COS(SIA1)
      CC1=CC1*V
      CC2=-((1.+AP)*RL*COS(ZOAN)*OMEGA)+COS(YA1)*COS(SIA1)*V
     &+V*SIN(YA1)*SIN(SIA1)*SIN(ATILT)
      GO TO 325
 323  CONTINUE
      SIG=(B*C)/(PI*RL)
      IF(SIG.GE.1.)GO TO 61
      GO TO 8
 61   CONTINUE
      IF(XL.GE.35.)GO TO 8
      Z=SIG*CL/8.
      CSI4=CSI**4
      Z2=Z*Z
      BOB=Z2*CSI4*(1.-A)*(1.-A)/(XL*XL)
      IF(ABS(CL).LT.1.E-25)GO TO 62
      Z3=ABS(CL)
      SIGN1=CL/Z3
      GO TO 63
 62   SIGN1=1.
  63  ZZ=Z2+(1.-Z2)*BOB
      IF(ZZ.LT.0.) ZZ=ABS(ZZ)
```

```
      Z4=SQRT(ZZ)
      AP=SIGN1*(Z2+Z4)/(1.-Z2)
      RW=Z*(CSI**2)*CPHI*(CC1**2+CC2**2)/(COS(ZOAN)**2*COS(ATILT)**2
     &*SIN(YA1)**2*V**2)
      BIP=1.-4.*RW
      IF(BIP.LT.0.) BIP=0.
      A=0.5*(1.-SQRT(BIP))
      GO TO 9
    8 VBR=0.125*SIG*CYY*COS(ZOAN)
      VAR=0.125*SIG*CXX/COS(ZOAN)
      IF(ABS(VBR).LT.1.E-25) VBR=0.000001
      IF(ABS(F).LT.1.E-25) F=0.0000001
      TBR=SIG/8.*CYY*(CC1**2+CC2**2)
      TBR1=F*(V*COS(ATILT)*SIN(YA1))**2*COS(ZOAN)
      IF(ABS(A-1.).LT.1.E-25) A=0.99999999
      IF(ABS(TBR1).LT.1.E-25) TBR1=0.000000001
      A=TBR/TBR1/(1.-A)
      IF(A.LE.AC)GO TO 32
      VBR=VBR/(SPHI**2)
      B1=((1.-2.*AC)*F+2.*VBR)/VBR
      C1=(VBR-AC*AC*F)/VBR
      CANN=B1*B1-4.*C1
      IF(CANN.LT.0.) CANN=0.
      A=0.5*(B1-SQRT(CANN))
   32 CONTINUE
      IF(ABS(A-1.).LT.1.E-25) A=0.9999999
      TAR=SIG/8.*CX*(CC1**2+CC2**2)
      TAR1=F*V*COS(ATILT)*COS(ZOAN)*SIN(YA1)*OMEGA*RL
      TAR1=TAR1*COS(ZOAN)**2
      AP=TAR/TAR1/(1.-A)
      IF(APP.EQ.1) AP=0.
    9 PCCR=RL/(RX*CSI)
C ......DAMPING OF AXIAL & ANGULAR INTERFERENCEFACTOR ITERATIONS....
      IF(J.EQ.1   .OR.J.EQ.40)GO TO 133
   12 A=(A+BETA)*0.5
      AP=(AP+DELTA)*0.5
   13 CONTINUE
      IF(APP.EQ.1)GO TO 14
      IF(ABS(AP-DELTA).LE.0.001)GO TO 16
      GO TO 15
   14 IF(ABS(A-BETA).LE.0.001)GO TO 16
  133 CONTINUE
      IF(J.EQ.1)GO TO 15
      JA=JA+1
      IF(JA.EQ.2)GO TO 325
      A=0.
      AP=0.
      GO TO 88
   15 CONTINUE
   16 IF(AK.GE.1.)GO TO 18
      PCCR=RL/RX*CSI
C_________________________________________________________________
C .....CAL OF FUNCTIONS DEPENDENT UPON AXIAL & ANG.INTERF. FACTOR....
C_________________________________________________________________
   18 CONTINUE
```

```
  325 CONTINUE                                                          HAD13140
      W=SQRT(CC1**2+CC2**2)                                             HAD13150
      CX=CL*SPHI-CD*CPHI                                                HAD13160
      CY=CL*CPHI+CD*SPHI                                                HAD13170
      IF(Z1.EQ.1)GO TO 50                                               HAD13180
      RE=RHO*W*C/VIS                                                    HAD13190
      CONST=(0.5*RHO*(W**2)*C)                                          HAD13200
      FXF=CONST*CX                                                      HAD13210
      FYF=CONST*CY                                                      HAD13220
      IF(Z1.NE.4)GO TO 97                                               HAD13230
      TFX=FXF*COS(SIA1)+FYF*SIN(-ZOAN)*SIN(SIA1)                        HAD13240
      TFY=FYF*COS(ZOAN)*COS(ATILT)-(FYF*SIN(-ZOAN)*COS(SIA1)-FXF        HAD13250
     &*SIN(SIA1))*SIN(ATILT)                                            HAD13260
      TFZ=(FYF*SIN(-ZOAN)*COS(SIA1)-FXF*SIN(SIA1))*COS(ATILT)+          HAD13270
     &FYF*COS(ZOAN)*SIN(ATILT)                                          HAD13280
      XM1Y=TFY*XX*RL-TFX*YY                                             HAD13290
      XM1P=-TFY*ZZ+TFZ*YY                                               HAD13300
      GO TO 50                                                          HAD13310
   97 XMFXF=FXF*(RL-HB)/CSI                                             HAD13320
      XMFYF=FYF*(RL-HB)/CSI                                             HAD13330
   50 CT1=(0.5*RHO*B*C)*(W*W)                                           HAD13340
      QF=CT1*RL*CX*COS(ZOAN)                                            HAD13350
      IF(Z1.EQ.1)GO TO 51                                               HAD13360
      TF=CT1*CY*CSI*COS(ZOAN)                                           HAD13370
      CLA=(CLD-CL)/0.001                                                HAD13380
      CDA=(CDD-CD)/0.001                                                HAD13390
   19 CONTINUE                                                          HAD13400
      IF(A.LE.0.0001   .OR.  AP.LE.0.0001)GO TO 51                      HAD13410
      T7=T7+(((2.+CPHI**2)*CL*SPHI-CPHI**3*CD+2.*CPHI*                  HAD13420
     &CLA)*(1.-A)**2*RL**3*C)*DR/2.                                     HAD13430
      T8=T8+(((1.+SPHI**2)/CPHI*CL-CD*SPHI+CLA*SPHI)*                   HAD13440
     &(1.+AP)*(1.-A)*RL**4*C)*DR/2.                                     HAD13450
   51 RETURN                                                            HAD13460
      END                                                               HAD13470
CS=4 ----------------------------                                       HAD13480
C     SUBROUTINE TIPLOS          ]                                      HAD13490
C     _________________          ]                                      HAD13500
C                                ]                                      HAD13510
C -------------------------------]                                      HAD13520
      SUBROUTINE TIPLOS(U,UO,F,Q,GO,HL,PI,R,RL,PHI,RH)                  HAD13530
C_______________________________________________________________________HAD13540
C ....GIVES TIP & HUB LOSSES BASED ON 'GOLDSTEIN' OR 'PRANDTLE' OR   ]  HAD13550
C                                        FOR NO LOSS .............]    HAD13560
C_______________________________________________________________________HAD13570
      INTEGER GO,HL                                                     HAD13580
      SUM2=0.                                                           HAD13590
      SUM=0.                                                            HAD13600
      AK=1.                                                             HAD13610
      AMM=1.                                                            HAD13620
      AM=0.                                                             HAD13630
      IF(Q.GT.2.)GO TO 1                                                HAD13640
      IF(GO.EQ.0)GO TO 2                                                HAD13650
      IF(GO.EQ.1)GO TO 4                                                HAD13660
      IF(GO.EQ.2)GO TO 3                                                HAD13670
    1 IF(GO.EQ.2)GO TO 3                                                HAD13680
```

```
  2   CONTINUE
      P1=SIN(PHI)
      P1=P1*P1+0.0001
      SOMSAK=(Q*(R-RL))/(2.*R*SQRT(P1))
      IF(ABS(SOMSAK).LT.1.E-25) SOMSAK=1.E-10
      IF(SOMSAK.GE.10.)GO TO 3
      F=(2./PI)*ACOS(EXP(-SOMSAK))
      GO TO 5
  3   F=1.
      GO TO 5
  4   IF((ABS(SIN(PHI))).LT.0.0001) GO TO 2
C__________________________________________
C    ....GOLDSTEIN'S METHOD..........
C__________________________________________
      DO 45 M=1,3
      V=(2.*AM+1.)
      ZO=UO*V
      V2=V*V
      Z=U*V
      Z2=Z*Z
C__________________________________________
      CALL BESSEL(Z,V,AI)
      CALL BESSEL(ZO,V,AIO)
C__________________________________________
      IF(Z.GE.3.5)GO TO 25
      A=2.*2.
      B=4.*4.
      C=6.*6.
      D=8.*8.
      T1VZ=Z2/(A-V2)+(Z2*Z2)/((A-V2)*(B-V2))+(Z2**3)/((A-V2)*(B-V2)*
     &(C-V2))+(Z2**4)/((A-V2)*(B-V2)*(C-V2)*(D-V2))
      CT1VZ=(V*PI*AI)/(2.*SIN(0.5*V*PI))-T1VZ
      GO TO 30
 25   TO=(U*U)/(1.+U*U)
      T2=4.*U*U*(1.-U*U)/((1.+U*U)**4)
      T4=16.*U*U*(1.-14.*U*U+21.*U**4-4.*U**6)/((1.+U*U)**7)
      T6=64.*U*U*(1.-75.*U*U+603.*U**4-1065.*U**6+460.*U**8-36.*U**10)
     &/((1.+U*U)**10)
      CY1VZ=TO+T2/V2+T4/(V2**2)+T6/(V2**3)
 30   FVU=(U*U)/(1.+U*U)-CT1VZ
      SUM=SUM+FVU/V2
      IF(AM.NE.0.)GO TO 35
      E=-0.098/(UO**0.668)
 35   IF(AM.NE.1.)GO TO 40
      E=0.031/(UO**1.285)
 40   IF(AM.GT.1.) E=0.
      SUM2=SUM2+((UO*UO*AMM)/(1.+UO*UO)-E)*(AI/AIO)
      AM=AM+1.
      AK=((2.*AM-1.)*AK)/(2.*AM)
 45   AMM=AK/(2.*AM+1.)
      G=(U*U)/(1.+U*U)-(8./(PI*PI))*SUM
      CIRC=G-(2./PI)*SUM2
      F=((1.+U*U))*CIRC
C    ......HUB LOSS  CALCULATION...............
  5   IF(HL.EQ.1)GO TO 6
```

```
      FI=1.
      GO TO 7
  6   CONTINUE
      P1=SIN(PHI)
      P1=P1*P1+0.0001
      IF(RL.LE.RH) RL=0.00001+RH
      SOMSAK=(G*(RL-RH))/(2.*RH*SQRT(P1))
      IF(ABS(SOMSAK).LT.1.E-25) SOMSAK=1.E-10
      IF(SOMSAK.GE.10)GO TO 71
      FI=(2./PI)*ACOS(EXP(-SOMSAK))
      GO TO 7
 71   FI=1.
  7   F=F*FI
      RETURN
      END
CS=5 ---------------------------
C     SUBROUTINE BESSEL          ]
C     __________________         ]
C                                ]
C ------------------------------]
      SUBROUTINE BESSEL(Z,V,AI)
C-----------------------------------------------------------------
C  ...CALCULATES BESSEL FUNCTIONS FOR THE GOLDSTEIN TIP LOSS MODEL]...
C_________________________________________________________________
      S=0.
      AK=0.
      C=1.
      DO 3 K=1,10
      B=(0.25*Z*Z)**AK
      D=V+AK
      P=1.
  1   TK=D-1.
      IF(TK.LE.0.)GO TO 2
      P=D*TK*P
      D=D-2
      GO TO 1
  2   E=P
      S=B/(C*E)+S
      AK=AK+1.
      C=AK*C
  3   CONTINUE
      AI=((0.5*Z)**V)*S
      RETURN
      END
CS=6 ---------------------------
C     SUBROUTINE NACA44          ]
C     __________________         ]
C                                ]
C ------------------------------]
      SUBROUTINE NACA44(ALPHA,CL,CD,ALO)
C_________________________________________________________________
C  ...NACA DETERMINES THE --      CO-EFF OF LIFT & DRAG              ]
C                        AT A GIVEN ANGLE OF ATTACK;ALPHA            ]
C                        FOR A NACA 4418 AIRFOIL.                    ]
C                                                                    ]
C
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C     THE EQUATIONS WERE OBTAINED BY A ORTHOGONAL POLYNOMIAL        ]   HAD14790
C                      CURVE FIT OF NACA DATA PUBLISHED IN          ]   HAD14800
C                      NACA REPORT NO.824, PAGE 401.                ]   HAD14810
C___________________________________________________________________    HAD14820
      DIMENSION AAD(5,4),AADG(4,3),AAL(9,3),AALG(3,3)                   HAD14830
C___________________________________________________________________    HAD14840
C                                                                   ]   HAD14850
C                       DATA                                        ]   HAD14860
C                                                                   ]   HAD14870
C___________________________________________________________________    HAD14880
      DATA AAD/0.074402988,2.1595697,4.8372211,-13.874289,6.8311195     HAD14890
     &,         0.31059104,-0.72021174,2.6283879,-5.1881313,3.7312441   HAD14900
     &,         325.97302,-1448.9419,2417.5137,-1791.7397,497.47345     HAD14910
     &,         -1.2031252,11.246515,-14.671162,1.5634329,3.1053634/    HAD14920
      DATA AADG/180.,14.,14.,0.                                         HAD14930
     &,          167.472,27.91252,37.9335,180.0792                      HAD14940
     &,          1.,0.04,0.04,1./                                       HAD14950
      DATA AAL/0.230799449,8.1520996,-90.209305,483.0567,-1452.5042     HAD14960
     &,         2499.7095,-2470.749,1316.7717,-294.75653                HAD14970
     &,         -0.84696233,1.3011253,11.170555,-26.763992,48.592003    HAD14980
     &,         -56.189163,23.942741,0.,0.                              HAD14990
     &,         1.2345905,13.929703,-217.36517,1162.4739,-3131.7378     HAD15000
     &,         4476.8242,-3240.5244,935.38135,0./                      HAD15010
      DATA AALG/180.,14.,0.,159.30945,38.,179.8416,1.,1.,1./            HAD15020
C___________________________________________________________________    HAD15030
      ALP=ALPHA*180./3.141593                                           HAD15040
      IF((ALP.GE.-180.)  .AND.  (ALP.LT.-14.)) J=1                      HAD15050
      IF(ALP.GE.14.  .AND.  ALP.LT.-14.) J=2                            HAD15060
      IF(ALP.GE.14.  .AND.  ALP.LT.24.) J=3                             HAD15070
      IF(ALP.GE.24.  .AND.  ALP.LE.180.) J=4                            HAD15080
      ALFA=(ALP+AADG(J,1))/AADG(J,2)                                    HAD15090
      CD=AADG(J,3)*AAD(1,J)                                             HAD15100
      DO 100 JJ=2,5                                                     HAD15110
      CD=CD+AADG(J,3)*AAD(JJ,J)*ALFA**(JJ-1)                            HAD15120
  100 CONTINUE                                                          HAD15130
      IF(J.EQ.3) J=2                                                    HAD15140
      IF(J.EQ.4) J=3                                                    HAD15150
      ALFA=(ALP+AALG(J,1))/AALG(J,2)                                    HAD15160
      CL=AALG(J,3)*AAL(1,J)                                             HAD15170
      DO 200 JJ=2,9                                                     HAD15180
      CL=CL+AALG(J,3)*AAL(JJ,J)*ALFA**(JJ-1)                            HAD15190
  200 CONTINUE                                                          HAD15200
      IF((J.EQ.1)  .AND.  (ALP.GT.-20.69055))                           HAD15210
     &  CL=-0.29817-0.0804763*(20.69055+ALP)                            HAD15220
      RETURN                                                            HAD15230
      END                                                               HAD15240
CS=7 ----------------------------                                       HAD15250
C     SUBROUTINE CLCD             ]                                     HAD15260
C     _______________             ]                                     HAD15270
C                                 ]                                     HAD15280
C -------------------------------]                                      HAD15290
      SUBROUTINE CLCD(ALPHA,ALPHAD,CL,CLD,CLT,CD,CDD,CDT,RL,AAT,NFS     HAD15300
     &,SOLD)                                                            HAD15310
      DIMENSION AAT(45),RR8(45),CLT(45),CDT(45)                         HAD15320
      COMMON/C1/R,DR,HB,B,V,X,THETP,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,W    HAD15330
```

```
     &,NPROF,APP,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,ALO,AC,TH
      COMMON/C4/QU,RR8,NFIT,RHOM,GAL
      IF(NPROF.EQ.4418)GO TO 1
    1 CALL NACA44(ALPHA,CL,CD,ALO)
      CALL NACA44(ALPHAD,CLD,CDD,ALO)
      RETURN
      END
CS=8 ----------------------------
C     SUBROUTINE SOLIDT           ]
C     __________________          ]
C                                 ]
C ------------------------------]
      SUBROUTINE SOLIDT (RR,CI,NF,B,R,PI,SOLD)
C______________________________________________________________________
C  .....SOLIDT DETERMINES THE TOTAL SOLIDITY OF THE                    ]
C                                        WIND TURBINE DESIGN .....]...
C______________________________________________________________________
      DIMENSION RR(25),CI(25)
      NFX=NF-1
      S1=0.
      DO 1 I=1,NFX
      SOL=((CI(I+1)+CI(I))/2.)*(RR(I)-RR(I+1))*R/100.
      S1=S1+SOL
    1 CONTINUE
      SOLD=B*S1/(PI*R**2)
      RETURN
      END
CS=9 ----------------------------
C     SUBROUTINE ACTIVI           ]
C     __________________          ]
C                                 ]
C ------------------------------]
      SUBROUTINE ACTIVI(RR,CI,NF,B,R,PI,ACF)
C______________________________________________________________________
C ......ACTIVI DETERMINES THE ACTIVITY FACTOR OF THE
C                                  WIND TURBINE DESIGN ...............
C______________________________________________________________________
      DIMENSION RR(25),CI(25)
      NFX=NF-1
      S1=0.
      DO 1 I=1,NFX
      C1=(CI(I+1)+CI(I))/2.
      ROR=((RR(I)-RR(I+1))/2.+RR(I+1))/100.
      DROR=(RR(I)-RR(I+1))/100.
      FAC=(C1/(2.*R))*ROR**3*DROR
      S1=S1+FAC
    1 CONTINUE
      ACF=S1*100000./16.
      RETURN
      END
```